Neuro-Fuzzy GMDH Model and Its Application to Forecasting of Mobile Communication

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1. Introduction

Recently, the performance modeling and performance analysis of mobile communication system with its unreliable factors is one of the special research issues. The mobile communications are categorized as three types as; PC-to-PC, PC-to-phone, and phone-to-phone. In this study, we focused on the phone-to-phone type which looks to be effective to the telecommunication companies. Generally, telecommunication problems involve a considerable capital investment and it is for these reasons that, in order to achieve the full benefit of the telecommunication, a very complex control problem has to be solved to ensure the proper performance needed. Mathematical models, in which many input variables are involved, require a range of input and output data since the number of parameters increases with the input variables. GMDH (Group Method of Data Handling) has been used for the identification of a mathematical model that has many input variables but limited data needs by using a hierarchical structure. The GMDH-type neural networks have been proposed and applied in medical problem by Kondo T.(1997, 1998). The GMDH-type neural networks have several advantages compared with conventional multi-layered networks. The GMDH-
type neural networks also have the ability of self-selecting a number of layers and a number of neurons in each layer and the ability of self-selecting useful input variables. Also, useless input variables are eliminated and useful input variables are selected automatically, because of this feature it is very easy to apply this algorithm to the identification problems of practical complex systems. This paper proposes a neuro-fuzzy GMDH model and its application to the forecasting of mobile communication subscribers are described. The GMDH-type neural networks have the both characteristics of the GMDH and the conventional multi-layered neural network and can automatically organize the optimum neural network architecture by using the heuristic self-organization method. In the GMDH-type neural networks, many types of neurons can be used to organize neural networks architecture and neurons characteristics which fit the complexity of the nonlinear system. Also we developed a computer program for computation using a neuro-fuzzy GMDH algorithm, and we applied it to an example for prediction of subscribers of mobile communications.

It is shown that the neuro-fuzzy GMDH can be applied easily and that it is a useful method for the complicated problems.

2. Heuristic Self-Organization Method

The architectures of the neuro-fuzzy GMDH are organized automatically by using the heuristic self-organization method which is used in the GMDH algorithm (Ivakhnenko A.G. 1994, 1995). The heuristic self-organization method is constructed by the following five procedures. This results in \( m(m-1)/2 \) second generation variables for predicting \( Y \) instead of the original variables. GMDH algorithm is implemented in the following steps:

Step 1: Data collection and dividing data into two sets, training set and checking set.

The original data are separated into training data and testing data. The training data are used for the estimation of the weights of the neural networks. The testing data are used for organizing the network architectures.

Step 2: Construction of new variables.

Independent variables are taken two at a time and this combinations take all the data points at each layer. For generating the combinations of the input variables in each layer, many combinations of \( r \) input variables are generated in each layer. The number of combinations is \( p^r/((p-r)!r!) \). Here, \( p \) is the number of input variables and the number of \( r \) is usually set to two.

Step 3: Rating the results of estimated dependent variable by a rate criterion using only checking data.

\[
r_j^2 = \frac{\sum_{i=1}^{n} (y_i - z_i)^2}{\sum_{i=1}^{n} y_i^2}, j = 1, 2, ..., n
\]

If \( r_j < R \), the new variables are passed to the next level of algorithm, where \( R \) is some predetermined value.

Step 4: Testing for optimality.

If RMIN, the smallest of \( r_j \)'s, of the layer at which analysis is being done is greater than at the previous design, then optimal Ivakhnenko polynomial has obtained as the following equation

\[
y_0 = a + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_i x_j + ......., \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} a_{ijk} x_i x_j x_k
\]

The heuristic self-organization method plays very important roles for the organization of a neuro-fuzzy GMDH model.

3. Neuro-Fuzzy GMDH

There are a number of ways fuzzy logic can be used with neural networks. One of the simple ways is to use a fuzzyifier function to pre-process or post-process date for a neural network. So far we have considered how fuzzy logic plays a role in neural networks. The converse relationship, neural networks in fuzzy systems, is also an active area of the researches. In this paper, we proposed a neuro-fuzzy GMDH model based on neuro-fuzzy networks to improve the conventional GMDH algorithms with improving in terms of identification accuracy.

3.1 Fuzzy Membership Function

For the fuzzy membership function, we used a fuzzy reasoning rule (Mamdani, 1976) which is
written as: if \( x_1 \) is \( F_{y_1} \) and \( F_{y_1} \), then output \( y \) is \( W_y \). We used Gaussian membership function, \( F_{y_k} \) for \( k \)th fuzzy rule in the domain of the \( j \)th input values \( x_j \) is given by \( F_{y_k}(x_j) = \exp \left( -\frac{(x_j - a_{y_k})^2}{b_{y_k}} \right) \), where, the parameters \( a_{y_k} \) and \( b_{y_k} \) are given for each rule.

The output \( y \) is given by \( y = \sum w_k w_k \), where \( w_k \), \( w_k \) is a real number of the concluding part of the \( k \)th rule.

This simplified fuzzy reasoning model is used as the partial description of GMDH type adaptive learning network which is called neuro-fuzzy GMDH.

3.2 Neuro-Fuzzy GMDH-type Network Model

The neuro-fuzzy GMDH is an adaptive learning network(network-type of GMDH) in the hierarchical structure. Figure 1 shows the structure of neuro-fuzzy GMDH. The output from each partial description in a layer becomes the input variable in the next layer, respectively.

\[ \sum_{j} y = \sum_{i} w_i w_i \]

Figure 1. Sample structure of neuro-fuzzy GMDH.

The final output is given by the average of the outputs in the top layer. The procedure of model identification is constructed by the following five procedures:

1) Normalizing the data.
   Normalizing the input and output data into intervals [0,1]
2) Separating the original data into training and testing data.
3) Generating the optimal partial descriptions.
   According to the following procedure, each description is generated from the 1st layer upward, and corresponding output values are obtained. The \( m \)th model in the \( p \)th layer is the input values of the \( (m-1) \)th:
   \[ y^{m} = f(y^{m-1:m}, y^{m-1:m}) = \sum_{i} w_i w_i \]
   Let \( y^* \) be the observed value and the performance index of the error of the models is given by
   \[ Error = \frac{(y^* - y)^2}{2} \]

4) Criteria of accuracy
   Let \( \Delta_{p} \) be the error in the \( p \)th layer, using the checking data the mean square error between observed values \( y^* \) and the estimates \( y \) is determined for the top layer, \( p \), as
   \[ \Delta_{p}^2 = \sum_{d} (y_d^* - y_d)^2 / \sum_{d} y_d \]

5) Stopping rule
   When the errors of the checking data in each layer stop decreasing, the iterative computation is terminated, If, \( \Delta_{p+1}^2 \geq \Delta_{p}^2 \), then the iteration continue, and if \( \Delta_{p+1}^2 < \Delta_{p}^2 \), then, the iteration terminates and the models up to that layer are adopted. We developed the computer program for this model, and applied it in forecasting of the mobile communication problem. The follow chart of this model is shown in Figure 2. Using the above procedures, the neuro-fuzzy GMDH model can be constructed and Figure 2 shows the procedure for the optimal architectures which fit for the complexity of the nonlinear
system are automatically selected by using MSE. Therefore, many kinds of nonlinear systems can be automatically identified by using the neuro-fuzzy GMDH-type model.

4. An Application to Mobile Communication Forecasting Problem

To explain the applicability of the neuro-fuzzy GMDH model, we apply it to the forecasting of the amount of phone-to-phone mobile communication service subscribers in Korea. We compared the sample results of the neuro-fuzzy GMDH model with those of conventional GMDH model. Table 1 shows input-output data among 1984 ~ 2002. In Table 1, y is an output variable giving the amount of telephone subscribers (in hundred subscribers), xj, j = 1, 2, 3 are input variables:

- X1: amount of population (@ 1,000)
- X2: number of house holds (@ 1,000)
- X3: amount of average expenditure per house hold

Table 1. Input-output data

<table>
<thead>
<tr>
<th>No</th>
<th>Year</th>
<th>y</th>
<th>X1 (@ 1,000)</th>
<th>X2</th>
<th>X3</th>
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<tr>
<td>1</td>
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<td>7,093</td>
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<tr>
<td>5</td>
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<td>42,135</td>
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<td>42,721</td>
<td>9,795,550</td>
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<td>8</td>
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<td>166,198</td>
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<tr>
<td>9</td>
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<td>271,868</td>
<td>43,917</td>
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</tbody>
</table>

* Source: Ministry of Information & Communication and Korea National Statistical Office

The input and output data are divided into 10 for training and 9 for checking data.

The network is assumed to be 4 layers as shown in Figure 1. The sum of squares of error between in the estimates and the desired outputs for training and checking sets of both conventional GMDH and neuro-fuzzy GMDH model are shown in Figure 3. There is a little differences between neuro fuzzy GMDH and conventional GMDH method, but in neuro fuzzy GMDH model gives the better identification and forecasting accuracies, and faster convergence than conventional GMDH. Figure 4 shows the observed values and estimated fuzzy values of sample problem using neuro fuzzy GMDH model.

![Figure 3. Comparison of accuracy of neuro fuzzy GMDH with conventional model.](image-url)

5. Conclusions

Conventional regression analysis methods is constructed based on regression functions and with
many assumptions. Also the conventional GMDH model is also based on regression analysis. In this paper we proposed a neuro fuzzy GMDH model which is formulated from the view point of possible model GMDH. We developed the computer program for both conventional GMDH and neuro fuzzy GMDH model. The neuro fuzzy GMDH algorithm and its result of sample results in experimental analysis for the forecasting of mobile telephone subscribers have been illustrated. As a result, the extension of linear regression with probability concept to the neuro fuzzy GMDH can be said to be more efficient in the application aspect than in the theoretical aspect.

Reference


