A Synthetic Exponentially Weighted Moving-average Chart for High-yield Processes

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Abstract. As charts to monitor the process fraction defectives, $P$, in the high-yield processes, Mishima et al. (2002) discussed a synthetic chart, the Synthetic CS chart, which integrates the CS (Confirmation Sample)$_{CCC}$ chart and the CCC-$r$ chart. The Synthetic CS chart is designed to monitor quality characteristics in real-time. Recently, Kotani et al. (2005) presented the EWMA (Exponentially Weighted Moving-Average)$_{CCC}$-$r$ chart, which considers combining the quality characteristics monitored in the past with one monitored in real-time. In this paper, we present an alternative chart that is more superior to the EWMA$_{CCC}$-$r$ chart. It is an integration of the EWMA$_{CCC}$-$r$ chart and the CCC-$r$ chart. In using the proposed chart, the quality characteristic is initially judged as either the in-control state or the out-of-control state, using the lower and upper control limits of the EWMA$_{CCC}$-$r$ chart. If the process is not judged as the in-control state by the EWMA$_{CCC}$-$r$ chart, the process is successively judged, using the CCC-$r$ chart to confirm the judgement of the EWMA$_{CCC}$-$r$ chart. We compare the ANOS (Average Number of Observations to Signal) of the proposed chart with those of other charts. From the numerical experiments, with the small size of inspection items, the proposed chart is the most sensitive to detect especially the small shifts in $P$ among other charts.

Keywords: High-yield process, Synthetic chart, EWMA (Exponentially Weighted Moving-average) chart, CCC (Cumulative Count of Conforming)-$r$ chart, ANOS (Average Number of Observations to Signal), Markov chain approach

1. INTRODUCTION

Recently, it is well known that the traditional Shewhart-type charts do not respond sensitively to the small or moderate shifts in quality characteristics obtained from the process (See Montgomery 2001). Roberts (1959, 1966) presented the EWMA (Exponentially Weighted Moving Average) chart to detect the small shifts in the process mean as an alternative of the Shewhart $\bar{x}$ chart. Many other EWMA charts for variables have been proposed for recent years (See Crowder, 1987; Ng and Case, 1989; Lucas and Saccucci, 1990, Domangue and Patch, 1991, Reynolds, 1996 and Steiner, 1998). As the EWMA chart for attributes, Gan (2002) and Borror et al. (1998) presented the EWMA chart to monitor the count of defects which follows the Poisson distribution, referred to as the EWMA$_c$ chart, as an alternative Shewhart $c$ chart. The Markov chain approach is used to calculate the Average
Run Length (ARL).

On the other hand, the manufacturing processes, especially in the electronic industries, have been automated with the aid of computers and the modern technologies. The items are produced and checked one by one (See Xie et al., 1998). Moreover, in the high-yield processes seen in the manufacturing environment of today, the process fraction defectives $P$ is substantially less than one percent. Few or even no items are nonconforming even for a fairly large sample. As one effective way, Xie et al. (1998) presented an alternative chart to monitor the process fraction defectives $P$ for the high-yield processes, referred to as the CCC (Cumulative Count of Conforming)-r chart. The quality characteristic is the cumulative count of item inspected until observing $r \geq 2$ nonconforming items. Furthermore, as an alternative of the CCC-r chart, Ohta and Kusukawa (2004) presented the CS (Confirmation Sample)CCC-r chart by applying the CS charting procedure proposed by Steiner (1999) to the CCC-r chart. It is illustrated that the CS CCC-r chart is more sensitive than the CCC-r chart to detect the small or moderate shifts in $P$ to both the upward and downward directions, which indicate the process deterioration and the process improvement, respectively. In order to enhance the performance of the CSCCC-r chart, Mishima et al. (2002) presented a synthetic chart which integrates the CCCC-r chart and the CCC-r chart, referred to as the Synthetic CS chart. The Synthetic CS chart has the higher detection powers for the small or moderate shifts in $P$ than the CCCC-r chart. However, above charts are designed to monitor quality characteristics in real-time. Neither the CCCC-r chart nor the Synthetic CS chart has the detection power required for the small or moderate shifts in $P$.

Recently, Kotani et al. (2005) presented the EWMA (Exponentially Weighted Moving-Average)CCC-r chart, which considers combining the quality characteristics monitored in the past with one monitored in real-time. The EWMACCC-r chart is constructed by applying the designing method of the EWMA chart for attributes (See Montgomery, 2001; Gan, 2002; Borror et al., 1998) to the CCC-r chart. The EWMACCC-r chart has the higher detection powers for any shifts in $P$ than the CCCC-r chart. In the real field, the fundamental aim of the monitoring and improving actions of the process quality is always to realize the zero-defects processes. From this viewpoint, in this paper, we focus on enhancing the detention power for especially the small shifts in $P$ to both the upward and downward directions which the process deterioration and the process improvement in the high-yield processes. Concretely, as a more superior chart to the EWMACCC-r chart and the Synthetic CS chart, we present a synthetic chart that is an integration of the EWMACCC-r chart and the CCC-r chart. In what follows, the proposed chart is referred to as the Synthetic EWMA chart. In using the Synthetic EWMA chart, the quality characteristic is initially judged as either the in-control state or the out-of-control state, using the lower and upper control limits of the EWMACCC-r chart. If the process is not judged as the in-control state by the EWMACCC-r chart, the process is successively judged, using the CCC-r chart to confirm the judgement of the EWMACCC-r chart.

In order to assess the performance of the detection power of the Synthetic EWMA chart, we compare the ANOS (Average Number of Observations to Signal) of the Synthetic EWMA chart with those of both the stand-alone EWMACCC-r chart and the Synthetic CS chart.

Each ANOS of the stand-alone EWMACCC-r chart and the Synthetic EWMA chart can be obtained based on the Markov chain approach discussed by Borror et al. (1998). Concretely, for the stand-alone EWMACCC-r chart, the Markov chain approach is used to calculate the ANOS based on the designing method of the EWMACCC-r chart. For the Synthetic EWMA chart, the Markov chain approach is used to calculate the ANOS based on the designing method of the EWMACCC-r chart. From the numerical experiments, with the small size of inspection items, the Synthetic EWMA chart is the most sensitive to detect especially small shifts in $P$ from the in-control state to both the upward and downward directions in the high-yield processes among the stand-alone EWMACCC-r chart and the Synthetic CS chart. The proposed process monitoring technique can be implemented by the on-line computers and the modern inspection equipment in the automated manufacturing environments in order to enhance the detection power for the small shifts in $P$.

2. NOTATION

$P$: the process fraction defectives.
$P_0$: the in-control process fraction defectives.
$P_1$: the out-of-control process fraction defectives.
$\alpha$: the specified value of the overall probability of type I error for the stand-alone CCC-r chart, the stand-alone CCCC-r chart, the stand-alone EWMACCC-r chart, the Synthetic CS chart and the Synthetic EWMA chart.
$\alpha_{CS}$: the specified value of the probability of type I error for the CCCC-r chart in the Synthetic CS chart.
$\alpha_{CCC-r}$: the specified value of the probability of type I error for the CCC-r chart in the Synthetic CS chart.
$\alpha_{EWMA}$: the specified value of the probability of type I error for the EWMACCC-r chart in the Synthetic EWMA chart.
$\alpha_{EWMA}$: the specified value of the probability of type I error for the CCC-r chart in the Synthetic EWMA chart.
$\alpha_{EWMA}$: the specified value of the probability of type I error for the CCC-r chart in the Synthetic EWMA chart.
$r_i (z \in CCC-r, CS, EWMA)$: the number of nonconforming item observed before a point is plotted on the following chart: the CCC-r chart, the CCCC-r chart and the EWMACCC-r chart used in either the stand-alone chart or the Synthetic
chart (The recommended value is about 2-5 depending on the process fraction defectives and type of process being monitored (See Xie et al., 1998)).

\[
x_\beta: \text{the } t\text{th quality characteristic based on the cumulative count of item inspected until observing } r_{EWMA} \text{ nonconforming items from the process which follows the negative binomial distribution } NB (r_{EWMA}, P).
\]

\[
x_{\beta}: \text{the first quality characteristic of both the stand-alone CCC-}r \text{ chart and the } CS_{CCC-}r \text{ chart in the Synthetic CS chart based on the cumulative count of item inspected until observing } r_{CS} \text{ nonconforming items from the process which follows the negative binomial distribution } NB (r_{CS}, P).
\]

\[
x_{\beta}: \text{the second quality characteristic for confirmation of both the stand-alone } CS_{CCC-}r \text{ chart and the } CS_{CCC-}r \text{ chart in the Synthetic CS chart based on the cumulative count of item inspected until observing } r_{CS} \text{ nonconforming items from the process which follows the negative binomial distribution } NB (r_{CS}, P).
\]

LCL_{CS}: the lower confirmation control limit of the stand-alone } CS_{CCC-}r \text{ chart.
\]

UCL_{CS}: the upper confirmation control limit of the stand-alone } CS_{CCC-}r \text{ chart.
\]

LCL_{CS}: the lower confirmation control limit of the } CS_{CCC-}r \text{ chart in the Synthetic CS chart.
\]

UCL_{CS}: the upper confirmation control limit of the } CS_{CCC-}r \text{ chart in the Synthetic CS chart.
\]

Q_{LCL_{CS}}: the lower type I error of the } CS_{CCC-}r \text{ chart in the Synthetic CS chart.
\]

Q_{UCL_{CS}}: the upper type I error of the } CS_{CCC-}r \text{ chart in the Synthetic CS chart.
\]

N_{LCL_{CS}}: the cumulative count of } x_\beta \text{ inspected until observing } r_{CCC-}r \text{ nonconforming quality characteristics that both } x_\beta \text{ and } x_\beta \text{ exceed the lower confirmation control limit of the } CS_{CCC-}r \text{ chart in the Synthetic CS chart.
\]

N_{UCL_{CS}}: the cumulative count of } x_\beta \text{ inspected until observing } r_{CCC-}r \text{ nonconforming quality characteristics that both } x_\beta \text{ and } x_\beta \text{ exceed the upper confirmation control limit of the } CS_{CCC-}r \text{ chart in the Synthetic CS chart.
\]

LCL_{LCL_{CS}}: the lower control limit of the } CCC-}r \text{ chart in the Synthetic CS chart used in the second judgment of the process state for the lower detection on the } CS_{CCC-}r \text{ chart which detects changes to lower shift in } P_0 \text{ indicating the process improvement.
\]

LCL_{UCL_{CS}}: the lower control limit of the } CCC-}r \text{ chart in the Synthetic CS chart used in the second judgment of the process state for the upper detection on the } CS_{CCC-}r \text{ chart which detects changes to upper shift in } P_0 \text{ indicating the process deterioration.
\]

\[
Z_t^*: \text{ the } t\text{th plotting point of the stand-alone } EWMA_{CCC-}r \text{ chart.
\]

\[
h_U: \text{ the lower control limit of the stand-alone } EWMA_{CCC-}r \text{ chart.
\]

\[
h_U: \text{ the upper control limit of the stand-alone } EWMA_{CCC-}r \text{ chart.
\]

\[
N_{EWMA} = \sum_{i=1}^{n_{EWMA}} z_{NB} (i-1) P_0 (1-P_0)^{i-1} = \alpha/2 \quad (1)
\]

\[
P_{CCC-} = \sum_{i=1}^{n_{CCC-}} z_{NB} (i-1) P_0 (1-P_0)^{i-1} = 1-\alpha/2 \quad (2)
\]

See Xie et al., 1998). As } r_{CCC-}r \text{ is getting larger, the stand-alone } CCC-}r \text{ chart is more and more sensitive to detect the small or moderate shifts in } P \text{ to the upward direction, while the stand-alone } CCC-}r \text{ chart needs more and more observations to obtain a plotting point on the chart, then the cost is fairly high. The decision procedure in the stand-alone } CCC-}r \text{ chart is made as follows: in the stand-alone } CCC-}r \text{ chart, if } P < P_0 \text{ then the quality characteristic of the stand-alone } CCC-}r \text{ chart is expected to be larger, while if } P \text{ increases as like } P > P_0 \text{ then it is expected to be smaller. Therefore, according to the stand-alone } CCC-}r \text{ chart, if a plotted point is above the upper control limit, the proc-
ess is likely to have improved, while if a plotted point is below the lower control limit, the process has probably deteriorated.

4. OUTLINE OF THE STAND–ALONE CS\textsubscript{ccc-r} CHART

Figure 1 shows the decision procedure of the stand-alone CS\textsubscript{ccc-r} chart proposed by Ohta and Kusukawa (2004). In the stand-alone CS\textsubscript{ccc-r} chart, \(x_d\) is initially taken from the process. Then \(x_d\) is judged whether the process is in-control or out-of-control, using the lower and upper confirmation control limits of the stand-alone CS\textsubscript{ccc-r} chart, LCSL\textsubscript{CS} and UCSL\textsubscript{CS}, respectively. If \(x_d\) satisfies the following condition:

\[
\text{LCSL}_{\text{CS}} < x_d < \text{UCSL}_{\text{CS}}, \tag{3}
\]

it would be judged that the process is in-control. On the other hand, if \(x_d\) satisfies the following condition:

\[
x_d \leq \text{LCSL}_{\text{CS}} \text{ OR } \text{UCSL}_{\text{CS}} \leq x_d, \tag{4}
\]

the second quality characteristic for confirmation, \(x_B\), would be independently taken from the same process. Furthermore, the initial quality characteristic \(x_d\) and the confirmation quality characteristic \(x_B\) would be judged whether the process is in-control or out-of-control, using the same confirmation control limits. If both \(x_d\) and \(x_B\), denoted as \(x_d\) & \(x_B\), satisfy the following condition:

\[
x_d \text{ & } x_B \leq \text{LCSL}_{\text{CS}} \text{ OR } \text{UCSL}_{\text{CS}} \leq x_d \text{ & } x_B, \tag{5}
\]

it would be judged that the process is out-of-control. On the other hand, if both \(x_d\) and \(x_B\) do not satisfy equation (5), it would be judged that the process is in-control. The overall probability of the type I error assuming the fixed values for the confirmation control limits, \(\alpha\), can be obtained as follows:

\[
\alpha = P(x_d \& x_B \geq \text{UCSL}_{\text{CS}}) + P(x_d \& x_B \leq \text{LCSL}_{\text{CS}}) = P(x_d \geq \text{UCSL}_{\text{CS}})^2 + P(x_d \leq \text{LCSL}_{\text{CS}})^2. \tag{6}
\]

In this paper, it is assumed that type I error in either direction is equally undesirable. From equation (6), the lower and upper confirmation control limits of the stand-alone CS\textsubscript{ccc-r} chart, denoted as LCSL\textsubscript{CS} and UCSL\textsubscript{CS}, can be obtained as the solutions of the following equations:

\[
\sum_{i=r_{CS}}^{l_{CS}} \left(\frac{i}{r_{CS}} \right) P_r^i (1 - P_r)^{r_i - i} = \frac{\alpha}{2}, \tag{7}
\]

\[
\sum_{i=r_{CS}}^{l_{CS}} \left(\frac{i}{r_{CS}} \right) P_r^i (1 - P_r)^{r_i - i} = 1 - \sqrt{\alpha}/2. \tag{8}
\]

5. OUTLINE OF THE SYNTHETIC CS CHART

5.1 Decision Procedure of the Synthetic CS Chart for The Process

The decision procedure of the Synthetic CS chart proposed by Mishima et al. (2002) is shown as follows:

Step 1: For the CS\textsubscript{ccc-r} chart in the Synthetic CS chart, \(x_d\) is initially taken from the process.

Step 2: If \(x_d\) exceeds either the lower confirmation control limit or the upper confirmation control limit of the CS\textsubscript{ccc-r} chart in the Synthetic CS chart, the second independent quality characteristic for confirmation, \(x_B\), would be taken from the same process.

Step 3: If both \(x_d\) and \(x_B\) exceed the same side confirmation control limit of the CS\textsubscript{ccc-r} chart in the Synthetic CS chart, the process is furthermore successively judged, using the CCC-\(r\) chart in the Synthetic CS chart.

Step 4: For the CCC-\(r\) chart in the Synthetic CS chart, both \(x_d\) and \(x_B\) that exceed the same side confirmation control limit of the CS\textsubscript{ccc-r} chart in the Synthetic CS chart is counted as one nonconforming item of the CCC-\(r\) chart in the Synthetic CS chart.

Step 5: For shifts in the process fraction defectives, \(P\), from the in-control state to the upward direction, \(N_{\text{UCSIA}}\) is observed until both \(x_d\) and \(x_B\) exceed...
the upper confirmation control limit of the CS CCC-r chart in the Synthetic CS chart.

Step 6: For shifts in the process fraction defectives, \( P \), from the in-control state to downward direction, \( N_{LCSLs} \) is observed until both \( x_a \) and \( x_b \) exceed the lower confirmation control limit of the CS CCC-r chart in the Synthetic CS chart.

Step 7: If either \( N_{UCSLS} \) or \( N_{LCSLS} \) exceeds each of the lower control limits of the CCC-r chart in the Synthetic CS chart setting for \( N_{UCSLS} \) or \( N_{LCSLS} \), it would be judged that the process is out-of-control. Otherwise, it would be judged that the process is in-control.

### 5.2 Settings of Type I Error and The Control Limits of the Synthetic CS Chart

The lower and upper confirmation control limits of the CS CCC-r chart in the Synthetic CS chart can be designed based on the negative binomial distribution \( NB(r, P) \). As with equation (6), the probability of type I error, \( \alpha_{CS} \), of the CSCCC-r chart in the Synthetic CS chart can be obtained as

\[
\alpha_{CS} = P(x_a&x_b \geq UCXSLS) + P(x_a&x_b \leq LCSLS) = P(x_a \geq UCXSLS)^2 + P(x_a \leq LCSLS)^2. \quad (9)
\]

From equation (9), using the probability limit method based on the negative binomial distribution, the lower and upper confirmation control limits, denoted as LCSLS and UCSLS, of the CSCCC-r chart in the Synthetic CS chart can be obtained as the solutions of the following equations:

\[
\sum_{i=CS}^{LCSLS} \left( \frac{i-1}{r_{CS} - 1} \right) p_{cs}^i (1 - p_{cs})^{-iCS} = \sqrt{\alpha_{CS}/2}, \quad (10)
\]

\[
\sum_{i=CS}^{UCSLS} \left( \frac{i-1}{r_{CS} - 1} \right) p_{cs}^i (1 - p_{cs})^{-iCS} = 1 - \sqrt{\alpha_{CS}/2}. \quad (11)
\]

The CCC-r chart in the Synthetic CS chart can be designed based on the probabilities of the lower and upper type I errors for the CSCCC-r chart in the Synthetic CS chart. Here, the lower and upper type I errors of the CSCCC-r chart in the Synthetic CS chart, denoted as \( Q_{LCSLS} \) and \( Q_{UCSLS} \), can be obtained as the solutions of the following equations:

\[
Q_{LCSLS} = \left[ \sum_{i=CS}^{LCSLS} \left( \frac{i-1}{r_{CS} - 1} \right) p_{cs}^i (1 - p_{cs})^{-iCS} \right]^2, \quad (12)
\]

\[
Q_{UCSLS} = \left[ 1 - \sum_{i=CS}^{UCSLS} \left( \frac{i-1}{r_{CS} - 1} \right) p_{cs}^i (1 - p_{cs})^{-iCS} \right]^2. \quad (13)
\]

When the probability of type I error is specified as \( \alpha_{CS} \) for the CCC-r chart in the Synthetic CS chart, the lower control limit of the CCC-r chart in the Synthetic CS chart for the lower detection of the CS CCC-r chart in the Synthetic CS chart, \( LCL_{LCSLS} \), can be obtained as

\[
\sum_{i=LCSLS}^{LCSLS} \left( \frac{i-1}{r_{CS} - 1} \right) p_{cs}^i (1 - p_{cs})^{-iCS} = \alpha_{CS}. \quad (14)
\]

The lower control limit of the CCC-r chart in the Synthetic CS chart for the upper detection of the CSCCC-r chart in the Synthetic CS chart, \( LCL_{UCSLS} \), can be obtained as

\[
\sum_{i=UCSLS}^{UCSLS} \left( \frac{i-1}{r_{CS} - 1} \right) p_{cs}^i (1 - p_{cs})^{-iCS} = \alpha_{CS}. \quad (15)
\]

The overall probability of type I error for the Synthetic CS chart, \( \alpha \), can be obtained as

\[
\alpha = P(x_a \geq UCSLS)^2 P(N_{UCSLS} \leq LCL_{UCSLS}) + P(x_a \leq LCSLS)^2 P(N_{LCSLS} \leq LCL_{LCSLS}) = \alpha_{CS} \times \alpha_{CS}. \quad (16)
\]

It is revealed from the numerical experiments for the Synthetic CS chart investigated by Kusukawa and Ohta (2003) that the Synthetic CS chart is the most sensitive to detect any shifts in quality characteristics when the Synthetic CS chart is designed by the set values of type I error as

\[
\alpha_{CS} = \alpha_{CS} = \sqrt{\alpha}. \quad (17)
\]

### 6. OUTLINE OF THE STAND-ALONE EWMA CCC-r CHART

The stand-alone EWMA CCC-r chart proposed by Kotani et al. (2005) can be constructed by applying the designing method of the stand-alone EWMA chart for attributes (See Montgomery, 2001; Gan, 2002; Borror et al., 1998) to the CCC-r chart. Using the t-th cumulative count \( x_t \) of item inspected until observing \( r_{EWMA} \) nonconforming items on the process, a plotting point of the stand-alone EWMA CCC-r chart is obtained by

\[
Z_t = \lambda x_t + (1 - \lambda) Z_{t-1} (0 < \lambda \leq 1, \ t = 1, 2 \cdots), \quad (18)
\]

where \( \lambda \) is a smoothing constant such that \( 0 < \lambda \leq 1 \). When the process is the in-control state with the fraction defectives \( P_0 \), it assumes that the process obeys the negative binomial distribution \( NB(r_{EWMA}, P_0) \). Under the situation, mean and variance of the cumulative count \( x_t \) are
\begin{equation}
E[x] = \frac{r_{\text{EWMA}}}{p_0} \quad \text{and} \quad V[x] = \frac{r_{\text{EWMA}}(1 - p_0)}{p_0^2}, \quad (19)
\end{equation}

For a plotting point \( Z_t \) of the EWMA\_CCC-\_r chart after the \( t \) th observation, the initial plotting point \( Z_0 \) is defined as
\begin{equation}
Z_0 = E[x] = \frac{r_{\text{EWMA}}}{p_0} . \quad (20)
\end{equation}

Let \( L \) be the width between the center line which is mean and each of the control limits. The upper and lower control limits, \( h_U \) and \( h_L \), of the EWMA\_CCC-\_r chart are obtained as
\begin{align}
    h_U &= \frac{r_{\text{EWMA}}}{p_0} + L \sqrt{\frac{r_{\text{EWMA}}(1 - p_0)}{p_0^2} \frac{\lambda}{2 - \lambda}}, \quad (21) \\
    h_L &= \frac{r_{\text{EWMA}}}{p_0} - L \sqrt{\frac{r_{\text{EWMA}}(1 - p_0)}{p_0^2} \frac{\lambda}{2 - \lambda}} . \quad (22)
\end{align}

7. PROPOSAL OF THE SYNTHETIC EWMA CHART

7.1 Settings of Type I Error and Control Limits of The Synthetic EWMA Chart

For setting the control limits of the Synthetic EWMA chart that integrates the EWMA\_CCC-\_r chart and the CCC-\_r chart, the actual value of ANOS (Average Number of Observations to Signal) is used instead of that of type I error. The value of ANOS for the EWMA\_CCC-\_r chart is set to actualize ANOS (\( = 1/\alpha_{\text{EWMA}} \)). For the calculation of ANOS of the EWMA\_CCC-\_r chart in the Synthetic EWMA chart, the Markov chain approach proposed by Kotani et al. (2005) is used. The details on the calculation of ANOS of the Synthetic EWMA chart would be described in subsection 7.4.

Under type I error \( \alpha_{\text{EWMA}} \) of the EWMA\_CCC-\_r chart in the Synthetic EWMA chart, for a given value of \( \lambda \), \( L \) is set to actualize ANOS (=1/\( \alpha_{\text{EWMA}} \)). Substituting the set values of \( \lambda \) and \( L \) into equations (21) and (22), the lower and upper control limits, \( h_U^S \) and \( h_L^S \), of the EWMA\_CCC-\_r chart in the Synthetic EWMA chart can be obtained. Under type I error \( \alpha_{\text{CCC-}} \) of the CCC-\_r chart in the Synthetic EWMA chart, using type I error \( \alpha_{\text{CCC-}} \) of the EWMA\_CCC-\_r chart in the Synthetic EWMA chart, the lower control limit, LCL, of the CCC-\_r chart in the Synthetic EWMA chart can be obtained as the solution of the following equation:
\begin{equation}
\sum_{c^{\text{CCC-}} = 1}^{1 - \alpha_{\text{CCC-}}} (1 - c^{\text{CCC-}})^{i - 1} \alpha_{\text{CCC-}}^{1 - c^{\text{CCC-}}} = \alpha_{\text{CCC-}} . \quad (23)
\end{equation}

The overall probability of type I error for the Synthetic EWMA chart, \( \alpha \), can be given as
\begin{equation}
\alpha = \alpha_{\text{EWMA}} \times \alpha_{\text{CCC-}}. \quad (24)
\end{equation}

In this paper, using a parameter \( w \), we set each probability of type I error, \( \alpha_{\text{EWMA}} \) and \( \alpha_{\text{CCC-}} \), for the EWMA\_CCC-\_r chart and the CCC-\_r chart in the Synthetic EWMA chart. In the case that the overall probability of type I error for the Synthetic EWMA chart is set to \( \alpha \), each probability of type I error, \( \alpha_{\text{EWMA}} \) and \( \alpha_{\text{CCC-}} \), for the EWMA\_CCC-\_r chart and the CCC-\_r chart in the Synthetic EWMA chart can be set respectively as follows:
\begin{align}
    \alpha_{\text{EWMA}} &= \alpha w, \quad (25) \\
    \alpha_{\text{CCC-}} &= \alpha (1 - w) . \quad (26)
\end{align}

7.2 Decision Procedure of The Synthetic EWMA Chart for The Process

According to Figure 2, the decision procedure for the Synthetic EWMA chart is made as follows:

Step 1: In the Synthetic EWMA chart, for the given values of \( \alpha \), \( r_{\text{EWMA}} \), \( r_{\text{CCC-}} \), the lower and upper control limits, \( h_U^S \) and \( h_L^S \), of the EWMA\_CCC-\_r chart and the lower control limit, LCL, of the CCC-\_r chart are obtained from equations (21)-(23).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Decision procedure for the Synthetic EWMA chart.}
\end{figure}
Step 2: For the EWMA \_\_\_ chart in the Synthetic EWMA chart, \( Z^s \) is initially calculated as the plotting point from equation (18). For \( Z^s \), the process is judged, using the lower and upper control limits, \( h^s_L \) and \( h^s_U \), of the EWMA \_\_\_ chart in the Synthetic EWMA chart.

Step 3: If \( Z^s \) exceeds either the lower control limit or the upper control limit, \( h^s_L \) and \( h^s_U \), of the EWMA \_\_\_ chart in the Synthetic EWMA chart, \( Z^s \) is counted as one nonconforming item on the EWMA \_\_\_ chart in the Synthetic EWMA chart.

Step 4: Steps 2 and 3 are repeated until \( N_{EWMA} \) is observed on the EWMA \_\_\_ chart in the Synthetic EWMA chart.

Step 5: For \( N_{EWMA} \) observed in Step 4, the process is furthermore successively judged, using the lower control limit of the CCC-r chart in the Synthetic EWMA chart.

Step 6: If \( N_{EWMA} \) exceeds the lower control limit, LCL, of the CCC-r chart in the Synthetic EWMA chart, it would be judged that the process is out-of-control. Otherwise, it would be judged that the process is in-control.

7.3 Calculation of ANOS of the Chart Using Markov Chain Approach

Based on the calculation method to obtain ANOS of the stand-alone EWMA \_\_\_ chart proposed by Kotani et al. (2005), the Markov chain approach is used to calculate ANOS of the EWMA \_\_\_ chart in the Synthetic EWMA chart for any shifts in \( P \). The interval between the upper control limit \( h^s_U \) and the lower control limit \( h^s_L \), obtained from equations (21) and (22) is divided into \( N \) subintervals, as shown in Figure 3. The \( j \)th subinterval \((L_j, U_j)\) are given as

\[
L_j = h^s_U + \frac{(j-1)(h^s_U - h^s_L)}{N}, \tag{27}
\]

\[
U_j = h^s_U + \frac{j(h^s_U - h^s_L)}{N}. \tag{28}
\]

The midpoint \( m_i \) of the \( i \)th subinterval \((L_j, U_j)\) is given as

\[
m_i = h^s_U + \frac{(2i-1)(h^s_U - h^s_L)}{2N}. \tag{29}
\]

The \((N+1)\)th state is absorbing and represents the out-of-control region above and below the control limits. This region is considered absorbing because the process is stopped when an out-of-control signal is raised. The ANOS is thus the expected time to absorption of the Markov chain.

The transition probability, \( P_y \), is the probability of moving from state \( i \) to state \( j \) in one step and is given by

\[
P_y = P(L_j < Z^s_i < U_j \mid L_i < Z^s_{i-1} < U_i). \tag{30}
\]

This is the probability that \( Z^s_i \) is within the boundaries of state \( j \), conditioned on \( Z^s_{i-1} \) being equal to the midpoint of state \( i \). This transition probability is obtained as the solution of the following equation:

\[
P_y = P(L_j < Z^s_i < U_j \mid Z^s_{i-1} = m_j) = P(L_j < \lambda x_i + (1-\lambda)m_j < U_j) \]

\[
= P(L_j < \lambda x_i + (1-\lambda)m_j < U_j) = P(h^s_L + \frac{(j-1)(h^s_U - h^s_L)}{N}) < \lambda x_i + (1-\lambda)m_j < h^s_U + \frac{j(h^s_U - h^s_L)}{N}) \]

\[
= P(h^s_L + \frac{h^s_U - h^s_L}{2N \lambda} - (2j-1)(1-\lambda)(2j-1)) \]

\[
< x_i < h^s_U + \frac{h^s_U - h^s_L}{2N \lambda} - (2j-1)(1-\lambda)(2j-1)) \tag{31}
\]

In general, the left and right sides of equation (31) will not be integer. \( P_y \) is calculated based on the distribution function of the negative binomial distribution. For example

\[
P_y = P(4.6 < x_i < 6.1) = P(x_i = 5) + P(x_i = 6)
\]

\[
= \binom{5-1}{r-1} P^r (1-P)^{5-r} + \binom{6-1}{r-1} P^r (1-P)^{6-r}. \tag{32}
\]

Define the vector \( R \) to be
Let \( Q \) be the matrix obtained from the transition matrix \( P \) by deleting row \( N+1 \) and column \( N+1 \). In other words, \( Q \) is the transition matrix among the in-control states. ANOS vector \( R \) is the solution to the system

\[
(I - Q)R = 1, \tag{34}
\]

where \( I \) is \( N \times N \) identity matrix and \( I \) is a \( 1 \times N \) column vector of ones. ANOS given that \( Z_0 = \mu_0 \) is just the middle entry, that is, the \((N+1)/2\)th entry is the vector \( R \) (We must choose \( N \) to be odd so that there is a unique middle value).

Under type I error \( \alpha_{\text{EWMA}} \) of the EWMA-CCC-r chart in the Synthetic EWMA chart, parameter values, \( \lambda \) and \( L \) to construct the lower and upper control limits of the EWMA-CCC-r chart in the Synthetic EWMA chart are set so as to actualize ANOS\((1/\alpha_{\text{EWMA}})^{\lambda}) as much as possible.

Using the desired set values of \( \lambda \) and \( L \), the lower and upper control limits, \( h_0^L \) and \( h_0^U \), of the the EWMA-CCC-r chart in the Synthetic EWMA chart are obtained from equations (21) and (22). From equation (34), ANOS of the EWMA-CCC-r chart in the Synthetic EWMA chart can be finally calculated.

8. COMPARISON OF THE PERFORMANCE

In order to assess the performance of the detection power of the Synthetic EWMA chart, we compare ANOS (Average Number of Observations to Signal) of the Synthetic EWMA chart with those of the stand-alone EWMA-CCC-r chart and the Synthetic CS chart. Type I error \( \alpha \) is set to 0.0027. The in-control population distribution is the negative distribution \( NBR_{\text{EWMA}}(r_2, P_0) \) \((z \in \text{CS}, \text{CCC-r, EWMA})\) given as \( r_2 = (2, 5) \) and \( P_0 = 0.001 \). We may express the null hypothesis \( H_0 \) (the process is the in-control state) and the alternative hypothesis \( H_1 \) (the process is the out-of-control state) in a formal manner as

\[
H_0: P=P_0, \quad H_1: P=P_1=\kappa P_0. \tag{35}
\]

In equation (35), in the case that \( \kappa =1 \), it indicates the in-control state. For \( \kappa <1 \), it implies that the process changes from the in-control state to the downward direction which indicates the process improvement. Moreover, for \( \kappa >1 \), it implies that the process changes from the in-control state to the upward direction which indicates the process deterioration.

In order to confirm the detection power of the stand-alone EWMA-CCC-r chart, we first compare ANOS of the stand-alone EWMA-CCC-r chart with that of the stand-alone CS-CCC-r chart (See Kotani et al., 2005). ANOS of the stand-alone CS-CCC-r chart is defined as the expected number of the plotted point \( x_1 \) required for the stand-alone CS-CCC-r chart to detect the first out-of-control observation. Here, let \( L_{\text{CSLS}} \) denote the lower power of the stand-alone CS-CCC-r chart for \( P_1 \). Let \( U_{\text{CSLS}} \) denote the upper power of the stand-alone CS-CCC-r chart for \( P_1 \). Under each of the conditions of \( NBR_{\text{CS}}(r_2, P_1=\kappa P_0) \) for several values of \( \kappa \), ANOS of the stand-alone CS-CCC-r chart is obtained as the solution of the following equation (See Kusukawa and Kusukawa, 2004):

\[
\text{ANOS} = \frac{1}{P_{\text{CSLS}} + P_{\text{CSLS}}}, \tag{36}
\]

where \( P_{\text{CSLS}} = \left( \sum_{i_{\text{CS}}} \right) \left( \sum_{j_{\text{CS}}} \right) \left[ \frac{P_{\text{CS}}(1 - P_{\text{CS}})^{\gamma_{\text{CS}}} - 1}{P_{\text{CSLS}} - 1} \right]^2 \), \( P_{\text{CSLS}} = \left( \sum_{i_{\text{CS}}} \right) \left( \sum_{j_{\text{CS}}} \right) \left[ \frac{P_{\text{CS}}(1 - P_{\text{CS}})^{\gamma_{\text{CS}}} - 1}{P_{\text{CSLS}} - 1} \right]^2 \).

ANOS of the stand-alone EWMA-CCC-r chart is defined as the expected number of the plotted point \( x_1 \) required for the stand-alone EWMA-CCC-r chart to detect the first out-of-control observation. Here, let \( L_{\text{CSLS}} \) denote the lower power of the stand-alone CS-CCC-r chart for \( P_1 \). Let \( U_{\text{CSLS}} \) denote the upper power of the stand-alone CS-CCC-r chart for \( P_1 \). Under each of the conditions of \( NBR_{\text{CS}}(r_2, P_1=\kappa P_0) \) for several values of \( \kappa \), ANOS of the stand-alone CS-CCC-r chart is obtained as the solution of the following equations (See Ohta and Kusukawa, 2004):

\[
\text{Lower ANOS} = \frac{1}{P_{\text{CSLS}}} \times \sum_{i_{\text{CS}}} \left[ \frac{P_{\text{CS}}(1 - P_{\text{CS}})^{\gamma_{\text{CS}}} - 1}{P_{\text{CSLS}} - 1} \right]^2, \tag{37}
\]

Upper ANOS = \( \frac{1}{P_{\text{CSLS}}} \times \sum_{i_{\text{CS}}} \left[ \frac{P_{\text{CS}}(1 - P_{\text{CS}})^{\gamma_{\text{CS}}} - 1}{P_{\text{CSLS}} - 1} \right]^2 \), \( \text{Upper ANOS} = \frac{1}{P_{\text{CSLS}}} \times \sum_{i_{\text{CS}}} \left[ \frac{P_{\text{CS}}(1 - P_{\text{CS}})^{\gamma_{\text{CS}}} - 1}{P_{\text{CSLS}} - 1} \right]^2 \).

where the lower and upper ANOS of the stand-alone EWMA-CCC-r chart, \( L_{\text{CSLS}} \) and \( U_{\text{CSLS}} \) in the Synthetic CS chart can be obtained respectively as the solutions of the following equations:

\[
P_{\text{CSLS}} = \left[ \sum_{i_{\text{CS}}} \right] \left[ \frac{P_{\text{CS}}(1 - P_{\text{CS}})^{\gamma_{\text{CS}}} - 1}{P_{\text{CSLS}} - 1} \right]^2, \tag{39}
\]
\[ P_{\text{UCSL}} = \left[ \sum_{i=2}^{LCSL} \left( r_{\text{CS}}^{-1} \right)^{i-1} P_{\text{CS}} \left( 1 - P_{\text{CS}} \right)^{r_{\text{CS}}} \right]^2. \]  

(40)

ANOS of the Synthetic EWMA chart is defined as the expected number of the plotted point required for the EWMA CCC-\( r \) chart in the Synthetic EWMA chart to detect the first out-of-control item. Under each state of \( NB(P_{\text{EWMA}}, P_0 = \kappa P_0) \) for a given \( \kappa \), ANOS of the Synthetic EWMA chart can be obtained as the solution of equation (34).

Table 1 shows the lower and upper confirmation control limits and the actual values of type I error \( \alpha \) of the stand-alone CSCCCC-\( r \) chart (See Ohta and Kusukawa 2004).

The lower and upper confirmation control limits of the stand-alone CSCCCC-\( r \) chart are obtained respectively as the solutions of equations (7)-(8). Table 2 shows the lower and upper control limits and the actual values of type I error \( \alpha \) of the stand-alone EWMACCC-\( r \) chart set at parameters \( \lambda \) and \( L \) which give the specified value of type I error \( \alpha \) and have the high detection powers for any shifts in quality characteristic (See Kotani et al., 2005). The lower and upper control limits of the stand-alone EWMACCC-\( r \) chart are obtained respectively as the solutions of equations (21) and (22). Notice that in the stand-alone EWMA chart, a smaller value of \( \lambda < 0.1 \) is recommended to detect small shifts in quality characteristic. Through the numerical experiments (See Kotani et al., 2005), it is found that the stand-alone EWMACCC-\( r \) chart with \( \lambda = 0.06 \) has the high detection powers for any shifts in quality characteristics. For \( \lambda = 0.06 \), a value of \( L \) is chosen in the interval \( 2.5 \leq L \leq 3.0 \) to give the desirable in-control ANOS, in this case, \( 1/0.0027 \approx 370 \). Table 3 shows the lower and upper control limits and the actual values of type I error \( \alpha \) of the Synthetic CS chart (See Mishima et al., 2002). The lower and upper control limits of the Synthetic CS chart are obtained respectively as the solutions of equations (9)-(17). Table 4 shows the lower and upper control limits and the actual values of type I error \( \alpha \) of the Synthetic EWMA chart set at parameters \( w, \lambda \) and \( L \) which actualize the specified value of type I error \( \alpha \) and have the high detection powers for any shifts in quality.

<table>
<thead>
<tr>
<th>( P_0 = 0.001 )</th>
<th>Stand-alone CSCCCC-( r ) chart</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCSL_{CS}</td>
<td>UCCL_{CS}</td>
</tr>
<tr>
<td>( r_{\text{CS}} = 2 )</td>
<td>299</td>
</tr>
<tr>
<td>( r_{\text{CS}} = 5 )</td>
<td>1805</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( P_0 = 0.001 )</th>
<th>Stand-alone EWMACCC-( r ) chart</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda, L )</td>
<td>( h_L, h_U )</td>
</tr>
<tr>
<td>( r_{\text{EWMA}} = 2 )</td>
<td>0.06</td>
</tr>
<tr>
<td>( r_{\text{EWMA}} = 5 )</td>
<td>0.06</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( P_0 = 0.001 )</th>
<th>Synthetic CS chart</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCSL_{CS}</td>
<td>UCCL_{CS}</td>
</tr>
<tr>
<td>( r_{\text{CS}} = 2 )</td>
<td>715</td>
</tr>
<tr>
<td>( r_{\text{CS}} = 5 )</td>
<td>2856</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( P_0 = 0.001 )</th>
<th>Synthetic EWMA chart</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda, L )</td>
<td>( h_L^2, h_U^2 )</td>
</tr>
<tr>
<td>( r_{\text{EWMA}} = 2 )</td>
<td>0.06</td>
</tr>
<tr>
<td>( r_{\text{EWMA}} = 5 )</td>
<td>0.06</td>
</tr>
</tbody>
</table>
characteristic. The lower and upper control limits of the Synthetic EWMA (chart 21)-(26). From Tables 1 - 4, it is illustrated that all charts are designed so as to actualize the specific value of type I error, \( \alpha = 0.0027 \) as much as possible.

Table 5 shows ANOSs of both the stand-alone EWMA\(_{CCC}\) chart and the stand-alone CS\(_{CCC}\) chart for 0.5 \( \leq \kappa \leq 1.5 \). These results are extracted from the numerical experiments investigated by Kotani et al. (2005). Table 6 shows ANOSs of the Synthetic EWMA chart and the Synthetic CS chart for 0.5 \( \leq \kappa \leq 1.5 \). The results of ANOS of the Synthetic CS chart for 0.5 \( \leq \kappa \leq 1.5 \) are extracted from the numerical experiments investigated by Mishima et al. (2002). Here, the closer to 0 the value of ANOS of the chart is, the more sensitive the detection for any shifts in quality characteristics from the in-control state is, while the closer to \( \infty \) the value of ANOS of the chart, the less sensitive the detection for any shifts in quality characteristics from the in-control state is. Notice that the stand-alone CS\(_{CCC}\) chart and the Synthetic CS chart are designed to monitor quality characteristics in real-time, whereas the stand-alone EWMA\(_{CCC}\), chart and the Synthetic EWMA chart consider combining the quality characteristics monitored in the past with one monitored in real-time. It is demonstrated from Table 5 that ANOSs of the stand-alone EWMA\(_{CCC}\), chart are smaller than those of the stand-alone CS\(_{CCC}\), chart for the upward shifts in \( P \) (case that \( \kappa > 1 \)) from the in-control state and the downward shifts in \( P \) (case that \( \kappa < 1 \)) from the in-control state. From the results, it can be seen that the stand-alone EWMA\(_{CCC}\), chart is more sensitive to detect any shifts in \( P \) from the in-control state than the stand-alone CS\(_{CCC}\), chart. It is evident from Table 6 that for \( r_z \) (\( z \in \text{CS}, \text{CCC}-r, \text{EWMA} \)) = (2, 5), ANOSs of the Synthetic EWMA chart are smaller than those of the stand-alone EWMA\(_{CCC}\), chart for the small and moderate shifts in \( P \) from the in-control state to the upward direction (the cases that \( \kappa > 1 \)) and the small and moderate shifts in \( P \) from the in-control state to the downward direction (the cases that \( \kappa < 1 \)). From the results, it is demonstrated that the Synthetic EWMA chart is more sensitive to detect any shifts in \( P \) from the in-control state than the stand-alone EWMA\(_{CCC}\), chart. It is evident from Table 6 that for \( r_z \) (\( z \in \text{CS}, \text{CCC}-r, \text{EWMA} \)) = 5, there isn’t so much of a difference between ANOSs of the Synthetic EWMA chart and those of the Synthetic CS chart for 0.5 \( \leq \kappa \leq 1.5 \). For \( r_z \) (\( z \in \text{CS}, \text{CCC}-r, \text{EWMA} \)) = 5 and 0.9 \( \leq \kappa \leq 1.1 \) which indicates the small shifts in \( P \) from the in-control state to both the upward and downward directions, ANOSs of the Synthetic EWMA chart are reduced highly, compared to the stand-alone CS\(_{CCC}\), chart, the stand-alone EWMA\(_{CCC}\), chart and the Synthetic CS chart.

It is illustrated from Table 6 that even for \( r_z \) (\( z \in \text{CS}, \text{CCC}-r, \text{EWMA} \)) = 2, ANOSs of the Synthetic EWMA chart are smaller than those of the Synthetic CS chart for the small and moderate shifts to the upward direction in \( P \) (the cases that 1 \( < \kappa \leq 1.5 \)) and the small shifts in \( P \) from the in-control state to the downward direction (cases that 0.7 \( < \kappa < 1 \)). From the results, it can be seen that even for \( r_z \) (\( z \in \text{CS}, \text{CCC}-r, \text{EWMA} \)) = 2, the Syn-

### Table 5. Comparison of ANOSs of the stand-alone CS\(_{CCC}\), chart with those of the stand-alone EWMA\(_{CCC}\), chart.

<table>
<thead>
<tr>
<th>( \kappa )</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSCCC, Chart</td>
<td>( r_{z,z} = 2 )</td>
<td>13</td>
<td>28</td>
<td>60</td>
<td>127</td>
<td>246</td>
<td>0.2695*</td>
<td>406</td>
<td>355</td>
<td>285</td>
<td>224</td>
</tr>
<tr>
<td>EWMA(_{CCC}), Chart</td>
<td>( r_{z,EWMACCC} ) = 2</td>
<td>( \lambda = 0.06 )</td>
<td>8</td>
<td>12</td>
<td>20</td>
<td>39</td>
<td>104</td>
<td>0.2695*</td>
<td>366</td>
<td>138</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>( \frac{L = 2.563}{1} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CSCCC, Chart</td>
<td>( r_{z,z} = 5 )</td>
<td>4</td>
<td>10</td>
<td>26</td>
<td>71</td>
<td>197</td>
<td>0.2699*</td>
<td>332</td>
<td>207</td>
<td>125</td>
<td>79</td>
</tr>
<tr>
<td>EWMA(_{CCC}), Chart</td>
<td>( r_{z,EWMACCC} ) = 5</td>
<td>( \lambda = 0.06 )</td>
<td>5</td>
<td>7</td>
<td>11</td>
<td>22</td>
<td>65</td>
<td>0.2694*</td>
<td>140</td>
<td>48</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>( \frac{L = 2.556}{1} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

### Table 6. Comparison of ANOSs of the Synthetic CS chart with those of the Synthetic EWMA chart.

<table>
<thead>
<tr>
<th>( * : ) Actual value of type I error (%)</th>
<th>( \kappa )</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synthetic CS Chart</td>
<td>( r_{z,z} = r_{z,CCC} = 2 )</td>
<td>0.5</td>
<td>4</td>
<td>8</td>
<td>19</td>
<td>56</td>
<td>179</td>
<td>0.2595*</td>
<td>313</td>
<td>170</td>
<td>94</td>
<td>56</td>
</tr>
<tr>
<td>Synthetic EWMA Chart</td>
<td>( r_{z,EWMACCC} ) = 2</td>
<td>0.8</td>
<td>6</td>
<td>9</td>
<td>14</td>
<td>27</td>
<td>83</td>
<td>0.2681*</td>
<td>252</td>
<td>79</td>
<td>39</td>
<td>26</td>
</tr>
<tr>
<td>Synthetic CS Chart</td>
<td>( r_{z,z} = r_{z,CCC} = 5 )</td>
<td>0.5</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>10</td>
<td>45</td>
<td>0.2616*</td>
<td>93</td>
<td>24</td>
<td>11</td>
<td>7</td>
</tr>
<tr>
<td>Synthetic EWMA Chart</td>
<td>( r_{z,EWMACCC} ) = 5</td>
<td>0.75</td>
<td>3</td>
<td>5</td>
<td>8</td>
<td>14</td>
<td>37</td>
<td>0.2688*</td>
<td>71</td>
<td>24</td>
<td>16</td>
<td>12</td>
</tr>
</tbody>
</table>
thetic EWMA chart is more sensitive to detect the small shifts in $P$ from the in-control state than the Synthetic CS chart to both the upward and downward directions. It implies that even with less number of inspection items from the process, the Synthetic EWMA chart can enhance the detection power for any shifts as well as the small and moderate shifts from the in-control state to both the upward and downward directions. Therefore, it is verified that it is the most adequate to use the Synthetic EWMA chart in the high-yield process as the alternative chart to detect sensitively any shifts in quality characteristics from the in-control state with a small size of inspection items observed from the process monitored.

From the comparison of ANOS performances of each control chart, our purpose has been successfully achieved.

9. CONCLUSIONS

The manufacturing processes, especially in the electronic industries, had been automated with the aid of computers and the modern technologies. Moreover, in the real field, the fundamental aim of the monitoring and improving actions of the process quality was always to realize the zero-defects processes. From this viewpoint, in this paper, we focused on enhancing the detention power for especially the small shifts in the fraction defective to both the upward and downward directions which the process deterioration and the process improvement in the high-yield processes. Concretely, we presented a Synthetic EWMA chart that was an integration of the EWMA$_{ccc-r}$ chart and the CCC-$r$ chart to enhance the detection power for the small or moderate shifts in the process fraction defectives from the in-control state to both the upward and downward directions in the high-yield processes seen in the manufacturing environment of today. The Synthetic EWMA chart was constructed by applying the designing method of the Synthetic CS chart to the stand-alone EWMA$_{ccc-r}$ chart which considered combining the quality characteristics monitored in the past with one monitored in real-time. Based on the calculation method to obtain ANOS (Average Number of Observations to Signal) of the stand-alone EWMA$_{ccc-r}$ chart, the Markov chain approach was used to calculate ANOS of the Synthetic EWMA chart for any shifts in the process fraction defectives from the in-control state. Comparing ANOS of the Synthetic EWMA chart with those of the stand-alone EWMA$_{ccc-r}$ chart and the Synthetic CS chart, it was verified that the Synthetic EWMA chart was the most sensitive chart to detect the small or moderate shifts in the process fraction defectives to both the upward and downward directions in the high-yield processes among the stand-alone EWMA$_{ccc-r}$ chart and the Synthetic CS chart, even when the number of nonconforming item observed before a point is plotted on the chart, $r$, was set to a small value. As $r$ increases, there was not so much of a difference between ANOSs of the Synthetic EWMA chart and those of the Synthetic CS chart for the moderate shifts in the process fraction defective from the in-control state. From both the aspects of the operational cost of the chart and the detection power of the chart, it was the most adequate to use the Synthetic EWMA chart in the high-yield processes as the alternative chart. The proposed process monitoring technique could be implemented with the help of the on-line computers and the modern inspection equipment in the automated manufacturing environment in order to enhance the detection powers for the small shifts in the process fraction defectives with a small size of the inspection items observed from the process monitored.

As $r$ increases, the Synthetic EWMA chart is more and more sensitive to detect the small or moderate shifts in the process fraction defectives to both the upward and downward directions, while more and more observations are required to obtain a plotting point on the chart. From the trade-off problem, it is necessary to determine the economical designing parameters such as the number of nonconforming item observed before a point is plotted on the chart, the sampling (inspection) interval and the lower and upper control limits of the Synthetic EWMA chart. The issue of an economical design of the Synthetic EWMA chart will be left for future research.

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