A Speed Control of Stepping Motor Using a Self-Tuning Regulator

Young-Tae Kim* · Sei-Yoon Kim

Abstract

In this paper, a self-tuning regulator for a speed control of a permanent magnet type stepping motor is proposed. The self-tuning theory provides a nonlinear modeling of a stepping motor drive system and can provide the controller with information regarding the reference variation and parameter variation of the stepping motor through the on-line estimation. The proposed self-tuning regulator organize the positive feedback loop and IP (Integral-Proportional) type. Therefore, the proposed self-tuning regulator has a robust control capabilities during dynamic operation. The availability of the proposed controller is verified through experimental results.

Key Words: Self-Tuning Regulator, Speed Control, Permanent Magnet Type Stepping Motor, On-Line Estimation, Robust Control

1. Introduction

According to the development of microprocessor, the permanent magnet type stepping motor is widely used in the industrial field. The advantage of stepping motor is the high-torque, fast-response and high-resolution. Also, unlike other motors, the stepping motor is able to control with digital equipment and keep the stop position. But, it is well-known that the stepping motor system is a nonlinear system and it is difficult to control the nonlinear system by using the traditional control theory. In recent years, many kinds of modern control theories exist in the control area of stepping motor. The aim of these researches minimize problem of a traditional controller by a nonlinear characteristic of stepping motor system and is decreasing output error by a reference variation and load variation for a fast response. However, most of the modern control approaches use off-line system identification. Because of this reason, it is impossible to effectively cope with system parameters that are changed dynamically during operation. Therefore, the on-line system identification during dynamic operation is desirable. The self-tuning theory can estimate system parameters during dynamic operation. The self-tuning theory is a good method for tuning the controller parameters.
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according to reference variations of the stepping motor.

The self-tuning theory can divide two control methods by minimum variance control method and pole placement control method. In a minimum variance control method, the cost function is consist of a output error and control input. Also, the controller is designed to minimize a cost function. Because of this reason, the controller is able to minimize the output error of a stepping motor system and the construction of the controller is simple and easy [1].

In this paper, the self-tuning regulator for a robust control of the stepping motor system is proposed. In the proposed self-tuning regulator, the estimated parameters of a stepping motor system contain the effect of approximated modeling and a reference or load variation. According to the estimated parameters the self-tuning regulator can change the gains of the regulator. To design the proposed self-tuning regulator, we use the minimum variance control algorithm and recursive least squares algorithm.

Also, for a robust control of a stepping motor system we make the ideal cost function. The cost function limits a control input and contains integral function. To verify availability of the proposed self-tuning regulator, we use the M-G (Motor-Generator) system.

2. Stepping motor system

2.1 System construction

Fig. 1 shows a block diagram of the stepping motor system for a speed control.

We operate the stepping motor system from following steps. First, using the parameter estimation algorithm we define the mathematical model of the stepping motor system under the reference variation and the load variation. Second, using the ideal cost function and minimum variance algorithm we define the new control input value to minimize the ideal cost function. Also, using the rotor position we get the step-command pulses with leading angle. Last, we drive the current type driver using the new control input value and the step-command pulses.

![Fig. 1. Block diagram of stepping motor system](image)

To control the current we use the PWM (pulse-width modulation) driver. The voltage at the load current pick-up is compared with the reference voltage. The reference voltage is the superposition of high-frequency (34kHz) triangle component [2].

2.2 System analysis

The dynamic equation of stepping motor system is expressed as an equation (1).

\[
\frac{d\omega_r}{dt} = \frac{T_M - T_L - B\omega_r}{J}
\]  

(1)

where,

\( \omega_r \): angular speed of rotor

\( T_M \): torque produced by the motor

\( T_L \): load torque

\( J \): inertia moment

\( B \): viscous frictional constant

The torque of stepping motor denotes following equation (2).
\[ T_M = K_r I_M \sin \rho \]  
\[ T_L = K_i \omega_r(t) \]  
\[ \rho = \frac{\theta_{ref}}{N_r} - \frac{\theta}{N_r} \]  
\[ A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + \delta(t) \]  
\[ A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_n q^{-n} \]  
\[ B(q^{-1}) = b_0 + b_1 q^{-1} + \cdots + b_m q^{-m}, \quad b_0 \neq 0 \]  

The term \( \sin \rho \) is not explicitly calculated by controller. Recently, to compensate the effect of the \( \sin \rho \) in the parameter variation. Thus, we analyze a stepping motor as a DC motor, approximately.

Because of this reason, we can define that the dynamic equation of stepping motor system is a second order system.

### 3. Self-tuning regulator

#### 3.1 Control algorithm

In single-input single output system, the system equation denotes following equation (5).  
\[ A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + \delta(t) \]  

Where,  
\( y(t) \) : system output  
\( u(t) \) : system input  
\( \delta(t) \) : random variable  
\( t \) : sampling time \( (t = 1, 2, 3 \cdots) \)  
\( q^{-1} \) : backward shift operator  

\( A, B \) are polynomials in \( q^{-1} \) and denotes following equation (6).

\[ A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_n q^{-n} \]  
\[ B(q^{-1}) = b_0 + b_1 q^{-1} + \cdots + b_m q^{-m}, \quad b_0 \neq 0 \]  

The backward shift operator has the meaning  
\[ q^{-d}y(t) = y(t-d), \]  

Where,  
\( d \) : time delay \( (d \geq 1, \text{ integer}) \)

If we know the order of the system equation \((m, n)\) and the time delay \((d)\), we find the system parameters from the parameter estimation algorithm. Also, the system output must follow to reference input without error. Thus we define the cost function as a equation (7).

\[ J(u, t) = E[(y(t+d) - y_{ref}(t+d))^2] \]  
\[ + (\sigma_u (u(t))^2 + \sigma_e (e_e(t+d))^2)] \]  
\[ = E[\zeta(t)] \]  

Where,  
\( E[\cdot] \) : expected value at the sampling time \((t+d)\)  
\( y_{ref}(t) \) : reference value  
\( \sigma_u, \sigma_e \) : weighting coefficient

The weighting coefficient \( \sigma_u \) acts a forgetting
factor. As time $t_i$ increases, the effect of old data at time $t_0 < t_i$ is discounted gradually with the elapsed time $t_i - t_0$. From (7), we get the control input to minimize the cost function.

Because stepping motor system is second order system, we define the transfer function of the system model as equation (8).

$$G(q^{-1}) = \frac{q^{-1}(b_2 + b_1 q^{-1})}{1 + a_1 q^{-1} + a_2 q^{-2}}$$

(8)

Thus, the system output denotes following equation (9).

$$y(t) = -a_1 y(t-1) - a_2 y(t-2)$$

$$+ b_1 u(t-1) + b_2 u(t-2)$$

(9)

Also, We define the cost function as a equation (10).

$$J(u, t) = \frac{1}{2} (y(t+d))^2 + \frac{1}{2} \sigma_v (\nu_e(t+d))^2$$

$$+ \frac{1}{2} \sigma_u u(t)^2$$

(10)

For a robust control of a stepping motor system, the cost function contains the integral term, $\nu_e(t)$ of the output error in control loop. The integral term, $\nu_e(t)$ denotes following equation (11).

$$\nu_e(t) = \nu_e(t-1) + e(t)$$

$$e(t) = y_{ref}(t) - y(t)$$

(11)

Using (9), we get the control input from (11). The control input denotes the following equation (12).

$$u(t) = \frac{1}{h_0} [f_1 y(t) + f_2 y(t-1) + g_1 u(t-1)$$

$$+ g_2 W(t) + g_3 \nu_e(t)]$$

(12)

Where,

$$h_0 = b_2 (1 + \sigma_v) + \sigma_u$$

$$f_1 = (1 + \sigma_v) b_0 a_2$$

$$g_1 = -(1 + \sigma_v) b_0 b_1$$

$$g_2 = (1 + \sigma_v) b_0$$

$$g_3 = b_0 \sigma_u$$

The $W(t)$ is reference input, $y_{ref}$. Fig. 2 shows the block diagram of the proposed self-tuning regulator.

![Fig. 2. Block diagram of a proposed self-tuning Regulator](image_url)

The proposed self-tuning regulator is composed an IP(integral–proportional) type controller to compensate the system error. Also, the propose regulator contains the positive feedback loop. Thus, the proposed self –tuning regulator has a robust control capabilities during dynamic operation.

### 3.2 Parameter estimation algorithm

To estimate the system parameter we use the recursive least squares method. We define the parameter vector as following equation (13).

$$\theta^T(t) = [a_1, a_2, b_0, b_1]$$

(13)

Also, we define the input and output vector as following equation (14).

$$\Phi^T(t) = [-y(t-1), -y(t-2), u(t-1), u(t-2)]$$

(14)

Using (15)–(17), we estimate the system parameter.
\[ \hat{\theta}(t) = \hat{\theta}(t-1) + \mu(t)[y(t) - \Phi^T(t-1)\hat{\theta}(t-1)] \quad (15) \]

\[ \mu(t) = \frac{p(t-1)\Phi(t)}{\lambda + \Phi^T(t)p(t-1)\Phi(t)} \quad (16) \]

\[ p(t) = \frac{1 - \mu^T(t)\Phi(t)}{\lambda}p(t-1) \quad (17) \]

Where,
\[ \lambda : \text{forgetting factor} \]

4. Experimental result

Fig. 3 shows the experimental equipment. To verify the availability of the proposed self-tuning regulator we manufactured experimental equipment using the double shaft type permanent magnet stepping motor(PK296-03B) with 1.8 step angle.

![Experimental equipment](image)

Fig. 3. Experimental equipment

Table 1 shows the parameters of the tested stepping motor.

Table 1. Parameters of stepping motor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>moment of inertia [10^-7 kg m^2]</td>
<td>2.2</td>
</tr>
<tr>
<td>rated current [A/phase]</td>
<td>4.5</td>
</tr>
<tr>
<td>rated voltage [V]</td>
<td>2</td>
</tr>
<tr>
<td>coil impedance [Ω/phase]</td>
<td>0.48</td>
</tr>
<tr>
<td>step angle [°]</td>
<td>1.8</td>
</tr>
</tbody>
</table>

From the speed response waveform, we can know that the response time is very fast within 10[ms].

Fig. 5 shows the current waveform during load variation. Fig. 5 (a) shows the current waveform before load variation. Fig. 5 (b) shows the current waveform during load variation. Fig. 5 (c) shows the current waveform after load variation.

From the current waveform, we can know that the current is increased to sustain with load torque through the proposed self-tuning regulator. Also, we can know that this system is operating with 34[kHz] in control switching frequency.

Also, from the experimental results, we can know that the response time is very fast within 10[ms]. It is the reason that unlike the traditional control theory using the speed pattern, the proposed self-tuning regulator can provide the controller with information regarding the reference variation and parameter variation of the stepping motor through the on-line estimation.

Thus, we can know that the proposed self-tuning regulator is a good controller for the stepping motor system and has a good performance in reference speed variation as well as in load variation.
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Fig. 4. Speed response waveform

5. Conclusion

In this paper, a self-tuning regulator for a speed control of a permanent magnet type stepping motor is proposed. To design the self-tuning regulator, we analyze a stepping motor with approximately DC motor and using the cost function. For a robust control, the cost function contains the integral term, $u_e(t)$ of the output error in control loop.

The designed self-tuning regulator is composed an IP (integral-proportional) type controller to
compensate the system error. Also, the proposed regulator contains the positive feedback loop. To verify the availability of the proposed self-tuning regulator, we manufactured the stepping motor system.

From the experimental results, we can know that the proposed self-tuning regulator has a good performance in reference speed variation as well as in load variation. Thus, the proposed self-tuning regulator is a good controller for the stepping motor system.

References


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Biography

Young-Tae Kim

He received the B.S., M.S., and Ph. D., degrees in electrical engineering from Hanyang University, Seoul, Korea, in 1984, 1989, and 1996, respectively. Since 1997, he has been with Kangneung-Wonju National University, Wonju, Korea, where he is currently an Associate Professor in the Department of Electrical Engineering. His research interests include switching techniques of power converters, and motor control applications.

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