Design of a Power Factor Measurement System for Nonlinear Load

Md. Rifat Shahriar* · Ui-Pil Chong**

Abstract

This paper introduces and develops an efficient method for measuring power factor (PF) and its nature under nonlinear load current situations. The method is based on generating a pulse width modulated signal whose width correlates to the value of PF. This signal can then be employed as a feedback signal for controlling PF related power quantities in a system. This method has the advantages of its simple implementation, less computational complexity, and its allowable error of less than 4%, which is justified by the computer simulation results.

Key Words : Nonlinear Load Current, True Power Factor, Microcontroller, Pulse Width Modulated Wave

1. Introduction

Accurate measurements of power factors (PF) under nonlinear load currents merit increased attention due to the popularity and the growing numbers of uses for nonlinear loads and power saving devices in recent decades[1-3]. Acquisition of PF information is essential in many applications: power quality management, calculations of apparent power requirements and controlling the PF of a load system are examples. Precise measurements of PF under non-sinusoidal environments are needed at this time.

The most conventional approach to measuring PF is the use of an electromechanical power factor meter [4] which is only accurate for the sinusoidal case [5]. A number of analog electronic circuits for PF measurement are proposed in [6-9]; however, these systems are incapable of determining lead/lag discrimination of a PF nature. Another approach to measuring PF is reported in [10] which is only developed for sinusoidal cases.

Several techniques and algorithms for true PF measurement based on Discrete Fourier Transform (DFT) or its computationally efficient implementation named Fast Fourier Transform (FFT) are proposed in [11-14]. They are alsofollowed by the commercial PF measurement equipment [15]. However, this DFT-based approach may be erroneous due to aliasing, picket-fence effects, and
leakage which are analyzed and detailed in [16]. Measurement methods proposed in [17–18] based on wavelets are claimed to be successful, but their measurements and computations are considerably complex and their efficiency is highly dependent on the type of mother wavelet [19]. The least square algorithm based measurement method of electrical quantities is suggested in [19], which stated that this method is efficient, but that it requires the processing power of a DSP chip.

In this paper, we have developed a simple and computationally efficient method for real-time measurements of PF under nonlinear load current conditions. A system has been proposed that is capable of measuring a PF value and its nature from zero PF lagging through unity PF up to zero PF leading – contrasted to some of the early systems proposed in [6–9]. Unlike the commercial equipment available for this purpose [15], the proposed method does not require complex computations like those included in harmonic analysis. Basically, our method is based on a mixed-signal analysis approach, which outputs PF information in the form of a pulse width of a digital signal. Thus, a simplified implementation of the method proves its competitiveness with existing systems while computer simulation results guarantee its accuracy.

2. Power Factor Definition

Electric utilities generate and distribute nearly perfect fundamental sinusoidal voltages as reported in [30]. This fact, considered in [7, 18, 20, 33], is also verified in [31, 32] by case study and experimental results. Thus, obtained voltage waveform can be simply expressed as

\[ v(t) = \sqrt{2} V_{\text{rms}} \sin(\omega t). \]  

Considering the dc component in a steady state nonlinear load current, it can be expressed as

\[ i(t) = i_0 + i_1(t) + \sum_{n=2}^{\infty} i_n(t). \]  

(2)

Where \( i_0 \) is the dc component, \( i_1(t) \) is the fundamental component and \( i_n(t) \) is the component at the nth harmonic frequency. Equation (2) can also be expressed as

\[ i(t) = i_0 + \sqrt{2} I_1 \sin(\omega t + \theta_1) + \sum_{n=2}^{\infty} I_n \sin(n\omega t + \theta_n). \]  

(3)

Where \( I_1, I_n \) are the rms values and \( \theta_1, \theta_n \) are the phase of load current at the fundamental and the nth-order harmonic respectively. The average power \( P \) can be expressed as [21–23]

\[ P = \frac{1}{T} \int_0^T p(t)dt = \frac{1}{T} \int_0^T v(t)i(t)dt = V_{\text{rms}} I_1 \cos(\theta_1). \]  

(4)

According to the power triangle [24] apparent power can be expressed as

\[ S = V_{\text{rms}} I_{\text{rms}}. \]  

(5)

Power factor (PF) is generally defined as the ratio of the Real power (P) to the Apparent power (S) [21–23, 25].

\[ \text{PF} = \frac{P}{S} = \frac{V_{\text{rms}} I_1 \cos(\theta_1)}{V_{\text{rms}} I_{\text{rms}}} = \frac{I_1}{I_{\text{rms}}} \cos(\theta_1). \]  

(6)

The ratio of \( I_1 \) to \( I_{\text{rms}} \) is known as distortion power factor which indicates how the harmonic distortion of a load current decreases the average power transferred to the load [21].

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Distortion Power Factor $= \frac{I_1}{I_{rms}} \quad (7)$

The cosine of the phase angle between voltage and current $\cos(\theta)$ is defined as the displacement power factor (DPF) [21, 23, 36].

\[ \text{DPF} = \cos(\theta_i) \quad (8) \]

Therefore, the representation of PF in case of a nonlinear load current situation is

\[ \text{PF} = \frac{I_1}{I_{rms}} \cdot \text{DPF}. \quad (9) \]

The definition of PF expressed by (9) [7, 21, 23] is also known as "True PF" [3].

When the load is linear, load current $i(t)$ becomes pure sinusoidal and can be expressed as

\[ i(t) = \sqrt{2} I_1 \sin(\omega t + \theta_i). \quad (10) \]

In this case $I_1$ and $I_{rms}$ become equal. Consequently, the expression in (9) becomes

\[ \text{PF} = \text{DPF} = \cos(\theta_i). \quad (11) \]

Note that the expression of (11) is appropriate only for sinusoidal load current conditions.

3. Development and Analysis of Proposed Scheme

A block diagram of the proposed method for measuring PF is shown in Fig. 1. Here a current transformer is used for measuring the nonlinear load current $i(t)$ which is then applied to the input of a narrow band filter to extract the fundamental component $i_1(t)$. Narrow band filters of higher orders usually consist of cascaded second order band pass filters that use the Sallen-Key or the Multiple Feedback topology. A general transfer function of a second-order filter is [27]

\[ A(s) = \frac{A_m}{Q} \cdot \frac{1}{s + \frac{2}{Q} \cdot s + \frac{1}{Q^2}} \quad (12) \]

Here, the complex frequency variable $s = \sigma + j\omega$; $\sigma$ = damping factor; $A_m$ = the gain at mid frequency and $Q$ = quality factor.

Quality factor $Q$ is defined as the ratio of the mid frequency ($f_m$) to the bandwidth ($B$).

\[ Q = \frac{f_m}{B} = \frac{f_m}{f_2 - f_1} = \frac{1}{\Omega_2 - \Omega_1} = \frac{1}{\Delta \Omega} \quad (13) \]

In (13) $\Omega_2 = \frac{f_2}{f_m}$ and $\Omega_1 = \frac{f_1}{f_m}$ are the normalized frequencies at lower and upper $-3\text{db}$ points of pass band and $\Delta \Omega$ is normalized bandwidth. $Q$ represents the selectivity of a band pass filter; a rising $Q$ value results in stepper frequency response.

Between the two topologies for narrow band filters mentioned above, Multiple Feedback topology has advantages over the Sallen-Key due to the fact that it allows a designer to adjust $Q$, $A_m$, and $f_m$ independently [27].
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Fig. 1. Overview of the designed system for PF Measurement

A second order active band pass filter implemented by multiple feedback topology is shown in Fig. 2 which has the following transfer function.

\[ A(s) = \frac{-\frac{R_2R_3}{R_1 + R_3}C\omega_n s}{1 + \frac{2R_1R_3}{R_1 + R_3}C\omega_n s + \frac{R_1R_2R_3}{R_1 + R_3}C^2\omega_n^2 s^2} \]  

(14)

After comparing coefficients of (14) with that of (12) and using (13), the following expressions of filter design parameters can be obtained.

\[ f_m = \frac{1}{2\pi} \sqrt{\frac{R_1 + R_3}{R_1R_2R_3}} \]  

(15)

\[ -A_m = \frac{R_2}{2R_1} \]  

(16)

\[ Q = \pi f_m R_2 C \]  

(17)

\[ B = \frac{1}{\pi R_2 C} \]  

(18)

To attain desired filtering performance, the above design parameters should be set as \( f_m = 60[\text{Hz}] \), \( B = 5[\text{Hz}] \), \( A_m = -1 \). Choosing capacitor value \( C_1 = C_2 = C = 100[\text{nF}] \) in the circuit diagram of Fig. 2, and using (15)-(18), values of resistors can be determined as \( R_1 = 316[\text{k\Omega}] \), \( R_2 = 638[\text{k\Omega}] \), and \( R_3 = 1.105[\text{k\Omega}] \).

Output of this band-pass filter \( i_1(t) \) is a pure sine wave of fundamental frequency (60[Hz]) which is phase shifted by 180[°] with respect to the original fundamental component contained in \( i(t) \).

\( i_1(t) \) is then fed to a Zero Crossing Detector (ZCD) whose circuit diagram and input/output characteristics are displayed in Fig. 3.

Fig. 2. Circuit diagram of Multiple Feedback band-pass filter (here, \( C_1 = C_2 = C \))

Output \( i_{1z}(t) \) obtained from ZCD is further passed through a clipper circuit to get a unipolar signal.
$i_{p}(t)$ as illustrated in Fig. 4.

![Clipper circuit with input output characteristics](image)

Fig. 4. Clipper circuit with input output characteristics

Pulse wave $i_{p}(t)$ is connected to the clock input of a D flip-flop through an inverter buffer. This inversion compensates the $180^\circ$ phase shift that occurred previously by the band-pass filter.

On the other hand, system voltage, considered to be containing no harmonic components [20], is sensed by a potential transformer (PT) (Fig. 1) and passed through similar ZCD and clipper section as discussed above. Output of clipper $v_{r}(t)$ is fed to a D flip-flop as D input through a buffer.

The flip-flop is clocked by current pulse while D input is fed by voltage pulse which results in latching the voltage polarity at the current zero crossing. Output of D flip-flop $D_{Q}(t)$ can be interpreted as follows.

$$D_{Q}(t) = \begin{cases} 1; & \text{PF lagging} \\ 0; & \text{PF leading} \end{cases}$$

$v_{r}(t)$ and $i_{p}(t)$ are then applied to an AND gate, and pulse wave $A(t)$ is obtained as an output whose width is varied with respect to PF. The relationship between this pulse width modulated (PWM) wave and PF is demonstrated in Fig. 5.

![Generation of PWM wave](image)

Fig. 5. Generation of PWM wave, (a) at $\phi = 0^\circ$ (PF=1), (b) at $\phi = 45^\circ$ (PF=0.7), (c) at $\phi = 90^\circ$ (PF=0)

The two signals, $D_{Q}(t)$ and $A(t)$, are then fed to a microcontroller port for further processing and decision. Inside the microcontroller $D_{Q}(t)$ is used for deciding whether PF is leading or lagging, whereas pulse duration $t$, as shown in Fig. 5, is calculated in seconds to determine DPF expressed below.

$$\text{DPF} = \cos \left\{ 2\pi f \left( \frac{1}{120} - t \right) \right\}$$  \hspace{1cm} (19)

Here, $f$ represents frequency of power system which is considered to be 60[Hz] in the calculation.

Two RMS to DC converters IC are employed in this design to get $I_{s}$ and $I_{max}$ which can be expressed by (20) and (21) respectively.

$$I_{s} = \frac{I_{m}}{\sqrt{2}}; \text{ where } I_{m} \text{ is the peak value of } i_{p}(t)$$  \hspace{1cm} (20)

$$I_{rms} = \sqrt{I_{0}^{2} + I_{1}^{2} + I_{2}^{2} + \ldots + I_{n}^{2}}$$  \hspace{1cm} (21)

The ripple contained in the output of RMS to the DC converter can be reduced by further passing it through a $2^{nd}$ order active-RC filter shown in Fig. 6 [28].
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Fig. 6. RMS to DC Converter with post filter

Filtered outputs $I_1$ and $I_{rms}$ are then applied to the ADC channels of a microcontroller for A/D conversion. The digital value that results is used afterwards for determining PF value according to expression (9).

A LCD, driven by microcontroller, can display the real time value of PF as well as its lead/lag nature.

4. Algorithm for PF Measurement

Simplified flowcharts of the implemented algorithm are described in Fig. 7. The PWM wave is treated as an external interrupt signal for the microcontroller. At the rising edge of the PWM wave an interrupt is generated and the main program branches to the interrupt service routine.

Fig. 7. Flow charts for programs. (a) main program; (b) interrupt service routine

Fig. 8. Simulation environment in Proteus ISIS

Fig. 9. Frequency response of Multiple Feedback band-pass filter. (X-axis: frequency in [Hz], left Y-axis: gain in [dB], right Y-axis: phase in degree)
(ISR). The interrupt service routine (Fig. 7b) loads the Timer to start measurement of pulse duration. On the falling edge of the PWM wave the Timer is stopped and the timer register is read. This timing data along with ADC channels’ readings is used for calculating a true power factor. Lead/lag discrimination is obtained by the input state of $D_{q}(t)$.

5. Simulation Results and Performance Analysis

The design and simulation of the proposed method are performed by Proteus ISIS which uses the SPICE3f5 analogue simulator kernel with a fast event-driven digital simulator to provide seamless mixed-mode simulation [29]. It has the ability to simulate interaction between software running on a microcontroller and any analog or digital electronics connected to it. Fig. 8 shows the simulation environment in Proteus ISIS.

Frequency response (Fig. 9) of the multiple feedback band-pass filter confirms its effectiveness in eliminating all the harmonic components contained in the distorted current wave.

Waveforms obtained by simulation for a test case (THD=113.12[%] and $i(t)$ containing odd harmonics up to 33rd order) is illustrated in Fig. 10 where the signal labels correspond to that of Fig. 1. Here, Total Harmonic Distortion (THD) is defined as [7, 21, 22, 25]

$$\% \text{THD} = \frac{I_{d_{th}}}{I_{i}} \times 100 \quad (22)$$

where $I_i$ is the rms value of the fundamental component and $I_{d_{th}}$ is the rms value of the total harmonic component.

Simulation is performed for different values of THD and harmonic contents. The values of PF that were obtained are then compared to the theoretical ones and a percentage of errors is calculated. Simulation results, as shown in Table 1, justify the effectiveness of the proposed method for measuring PF and its nature under nonlinear load current

![Fig. 10. Simulated output for THD = 113.12(%) while $i(t)$ contained odd harmonics up to 33rd order. (a) System voltage $v(t)$, load current $i(t)$ and output of band-pass filter $v(t)$. (b) output of ZCD $i_{p}(t)$ and clipper $i_{c}(t)$. (c) $CLK$ and $D$ input of D flip-flop. (d) PWM wave $A(t)$ and lead/lag indicator signal $D_{q}(t)$](image_url)
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### Table 1: Comparison of theoretical and simulation results ($L_d=PF$ leading; $L_g=PF$ lagging, $T=theoretical$)

<table>
<thead>
<tr>
<th>THD ($%$)</th>
<th>$I_0$ (A)</th>
<th>Peak value of $i(t)$ (A)</th>
<th>Harmonics in $i(t)$</th>
<th>Example waveform</th>
<th>PF (Theoretical)</th>
<th>PF (Simulation)</th>
<th>%Error</th>
</tr>
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<td>6.40</td>
<td>0.95</td>
<td>10</td>
<td>$5^{th}$ $7^{th}$ $11^{th}$</td>
<td></td>
<td>0.9638$L_d$</td>
<td>0.9693$L_d$</td>
<td>0.58</td>
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<td>For PF = 0.706$L_d$(T)</td>
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<td>0.8683$L_d$</td>
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<td>0.7235$L_d$</td>
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<td>0.4990$L_d$</td>
<td>0.516$L_d$</td>
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<td>35.76</td>
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<td>$5^{th}$ $7^{th}$ $11^{th}$</td>
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<td>0.9413$L_d$</td>
<td>0.9473$L_d$</td>
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<td>For PF = 0.470$L_d$(T)</td>
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<td>0.4708$L_d$</td>
<td>0.4877$L_d$</td>
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<td>104</td>
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<td>$3^{rd}$ $5^{th}$ $7^{th}$</td>
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<td>0.669$L_d$</td>
<td>0.6853$L_d$</td>
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<td>For PF = 0.669$L_d$(T)</td>
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<td>For PF = 0.150$L_d$(T)</td>
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<td>$3^{rd}$ to $19^{th}$ (odd ones only)</td>
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<td>$3^{rd}$ to $33^{rd}$ (odd ones only)</td>
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<td>0.6598$L_d$</td>
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<td>For PF = 0.174$L_d$(T)</td>
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<td>0.1714$L_d$</td>
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<td>3.05</td>
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condition with less than 4[%] error.

6. Conclusion

The proposed method for measuring power factor (PF) and its nature under nonlinear load current situation appears to be efficient, simple, and cost effective. Its accuracy was confirmed by simulation. The method is competitive with methods followed by commercially available equipment for the same purpose.

This method has the ability to measure PF under both sinusoidal and non-sinusoidal situations. Hence, it can be utilized for monitoring and control of PF and other related power parameters in a large electrical system.

Our future research will focus on hardware implementation of the proposed design. We plan to evaluate its performance in situations where we have to deal with issues like current and potential transformer saturation, input saturation of RMS to DC converter IC, and accurate generations of current harmonics and precise implementations of band-pass filters.

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References


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