Nonlinear Speed Control of PM Synchronous Motor with Extended Kalman Filter Observer

Nga Thi-Thuy Vu* · Jin-Woo Jung**

Abstract

This paper proposes a nonlinear speed controller for a permanent magnet synchronous motor (PMSM). In this paper, the load torque is estimated by an extended Kalman filter (EKF) observer because the proposed controller needs its knowledge. To confirm the effectiveness of the proposed control scheme, simulations and experiments are performed under motor parameter variations with a prototype PMSM drive system.

Key Words: Extended Kalman filter (EKF), Nonlinear control, Permanent magnet synchronous motor (PMSM), Speed control

1. Introduction

Permanent magnet synchronous motors (PMSM) are widely used in industrial applications because of their high power density, high efficiency, and rugged construction. However, PMSMs are nonlinear multivariable systems. Moreover, the model parameters can be changed by temperature and current, and the load torque is also widely unknown. Thus, it is difficult to achieve high-performance speed or position control with linear control methods such as a PI control or an LQ regulator. Therefore, nonlinear control techniques can be a promising alternative to precisely control the PMSM.

Recently, many researchers presented various nonlinear control methods [1-9] to deal with the above problems by directly considering the nonlinear PMSM dynamics. In [3], a nonlinear backstepping control scheme has been proposed to control the speed of the PMSM. The most appealing point of that method is to use the virtual control variable to make the high-order system simple, and thus the final control outputs can be derived step-by-step through appropriate Lyapunov functions. However, this control scheme requires the exact model knowledge of the control system. The fuzzy techniques [4-6] and a sliding-mode control [7-9]
have also been reported for speed control of the PMSM. Although these control methods can obtain good performance, they are quite complex to be implemented. In [10-11] the authors have presented the feedback linearization techniques for designing the controller of nonlinear systems such as robot manipulators, induction motors, and PMSMs. In comparison with the others, the feedback linearization techniques are very useful methodologies for AC motor control. The controllers, which have been presented in [10-11], however, require full knowledge of the system parameters and load conditions with sufficient accuracy.

This paper proposes a nonlinear speed controller for PMSM. The controller consists of a nonlinear compensating term and a stabilizing term. The extended Kalman filter (EKF) observer is used for estimating the load torque which the proposed controller needs. To evaluate the performance of the proposed regulator, simulation and experimental results are presented in the presence of motor parameter variations with a prototype PMSM drive system.

2. Nonlinear Controller Design

2.1 PMSM model

Based on the d-q reference theory, the stator voltage equations of a three-phase surface-mounted PMSM can be expressed as (1), and Fig. 1 illustrates the equivalent circuit of the PMSM.

\[
\begin{align*}
V_{qs} &= R_s i_{qs} + L_s \frac{di_{qs}}{dt} + \omega L_s i_{ds} + \lambda_m \omega \\
V_{ds} &= R_s i_{ds} + L_s \frac{di_{ds}}{dt} - \omega L_s i_{qs}
\end{align*}
\]

(1)

where \( R_s \) is the stator resistance, \( L_s \) is the stator inductance, \( \omega \) is the electrical rotor angular speed, \( \lambda_m \) is the magnetic flux, \( i_{qs} \) is the \( q \)-axis current, \( i_{ds} \) is the \( d \)-axis current, \( V_{qs} \) is the \( q \)-axis voltage, and \( V_{ds} \) is the \( d \)-axis voltage, respectively.

![Fig. 1. PMSM equivalent circuit](image)

Also, the electromagnetic torque (\( T_e \)) can be given by

\[
T_e = \frac{3}{2} p \frac{J}{p} \frac{d\omega}{dt} + \frac{2}{p} B \omega + T_L
\]

(2)

where \( p \) is the number of poles, \( J \) is the rotor inertia, \( B \) is the viscous friction coefficient, and \( T_L \) is the load torque plus parameter imprecision, respectively.

From (1) and (2) the PMSM can be represented by the following differential equations:

\[
\begin{align*}
\dot{i}_{qs} &= k_4 i_{qs} - k_5 \omega - k_3 T_L \\
\dot{\lambda}_m &= -k_4 i_{qs} - k_5 \omega + k_5 V_{qs} - \omega i_{qs} \\
\dot{i}_{ds} &= -k_4 i_{ds} + k_5 V_{ds} + \omega i_{ds}
\end{align*}
\]

(3)

where \( k_i > 0, i = 1 \cdots 6 \) are the parameter values given by the nominal values of \( p, R_s, L_s, J, B, \) and \( \lambda_m \), respectively.
2.2 Speed Regulator Design and Stability Analysis

In this paper, the following assumptions, which are widely used in most papers, will be made to design a nonlinear speed regulator:

A1: $\theta$, $i_{qw}$, $i_{ds}$ are measurable.

A2: The load torque $T_L$ is unknown and it changes very slowly, i.e., $\frac{d}{dt}T_L$ can be set as 0.

A3: The desired rotor angular speed, $\omega_d$, is twice differentiable, and $\omega_d$, $\dot{\omega}_d$, $\ddot{\omega}_d$ are bounded.

Next, let us define the rotor position error ($\bar{\theta}$), rotor speed error ($\eta$) and rotor angular acceleration error ($\bar{\eta}_d$) as

$$
\bar{\theta} = \int_0^t \omega_d d\tau = \int_0^t (\omega - \omega_d) d\tau
$$

$$
\omega = \omega_d + \eta
$$

$$
\eta = \dot{\omega} = k_1\eta + k_2\omega - k_3T_L
$$

$$
\bar{\eta}_d = \ddot{\omega} = \ddot{\omega}_d + \dot{\eta} = \dot{\bar{\theta}}
$$

Let the control inputs $V_{qw}$ and $V_{ds}$ be defined as

$$
V_{qw} = \frac{1}{k_1k_6}(u_{gq} + u_{fd}), \quad V_{ds} = \frac{1}{k_6}(u_{gd} + u_{fd})
$$

$$
u_{gq} = \dot{\bar{\theta}} + k_1\eta + k_2\omega + k_3\omega
$$

$$
u_{gd} = k_4\omega_d - \omega\dot{\omega}
$$

where $u_{gq}$ and $u_{gd}$ are the linearizing control terms used to compensate for the nonlinear characteristics of PMSM, and $u_{fd}$ is the feedback control term used to stabilize the error dynamics.

Then, the model (3) can be transformed into the following:

$$
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}, \quad B = \begin{bmatrix}
0 \\
0 \\
1 \\
0
\end{bmatrix}, \quad x = \begin{bmatrix}
\theta \\
\omega \\
\eta \\
\dot{\bar{\theta}}
\end{bmatrix}
$$

$$
u = \begin{bmatrix}
u_{gq} \\
u_{gd}
\end{bmatrix}.
$$

By defining the feedback control terms ($u_{hd}$ and $u_{hfd}$) as $u_b = [u_{gq}, u_{hd}] = Kx$, the closed-loop control system can be expressed by:

$$
\dot{x} = (A + BK)x
$$

where $K \in \mathbb{R}^{4d}$ denotes the controller gain matrix.

Since the pair $(A, B)$ can be stabilized, the standard results [12] imply the existence of a stabilizing gain, $K$ such that for some positive-definite matrix $P_c$

$$
P_c(A + BK) + (A + BK)^T P_c < 0
$$

Let us define the Lyapunov function as $V_c = x^TP_cx$. From the closed-loop system (8), its time derivative is written as

$$
\dot{V}_c = \frac{d}{dt}x^TP_cx = 2x^TP_c[A + BK]x \leq 0
$$
The above equation (10) implies that the origin $x = 0$ is exponentially stable. Therefore, the following Theorem 1 can be obtained.

**Theorem 1**: Consider the closed-loop control system of (3), (5), and $\dot{\theta}_b = [\dot{u}_b, \dot{\theta}_b] = Kx$. Assume that the gain matrix $K$ stabilizes $(A + BK)$. Then, $x$ exponentially converges to zero.

### 2.3 EKF-Based Load Torque Observer

The proposed nonlinear speed regulator needs the knowledge of the rotor angular acceleration ($\dot{\theta}$). As shown in (4), it can be easily obtained if the load torque is given. Accordingly, the control performance can be severely degraded if the term $T_L$ is unknown. In this paper, an observer based on Extended Kalman filter (EKF) [13] is used to estimate the load torque because it is well-known that the EKF-based observer is insensitive to motor parameters and load torque variations, and can also suppress the measurement errors caused from the sensors [14].

From (2), the load torque observer can be represented as the following continuous-time state space equation:

\[
\begin{align*}
\dot{x}_o &= A_o x_o + B_o u_o \\
y_o &= C_o x_o \\
\hat{T}_L &= C_T x_o
\end{align*}
\]

(11)

where $\hat{T}_L$ is an estimate of $T_L$, and

\[
\begin{align*}
x_o &= [\theta \ \dot{\theta} \ \hat{T}_L] \quad x_c = [\theta \ \omega \ \hat{T}_c], \quad u_o = T_c,
\end{align*}
\]

and

\[
A_o =  \begin{bmatrix}
0 & 1 & 0 \\
0 & -\frac{P}{2}k_2 & -k_3 \\
0 & 0 & 0
\end{bmatrix}, \quad B_o = \begin{bmatrix}
0 \\
k_3 \\
0
\end{bmatrix}.
\]

By introducing an Euler approximation method, the approximate discrete-time model of the PMSM can be expressed as

\[
x_o(k + 1) = A_{od} x_o(k) + B_{od} u_o(k) \\
y_o(k) = C_o x_o(k) \\
\hat{T}_L(k) = C_T x_o(k)
\]

(12)

where

\[
A_{od} = \begin{bmatrix}
T & T & 0 \\
0 & T(1 - \frac{P}{2}k_2) & -Tk_3 \\
0 & 0 & T
\end{bmatrix}, \quad B_{od} = \begin{bmatrix}
0 \\
Tk_3 \\
0
\end{bmatrix},
\]

$T$ is the sampling time, and $x_o(k), u_o(k), y_o(k)$ are the values of $x_0, u_0, y_0$ at the sampling instant $k$.

Fig. 2 depicts an algorithm of the EKF-based load torque observer [13] used in this paper. It is noted that $P$ is the error covariance matrix, $L$ is the Kalman gain, $Q$ and $R$ are the noise covariance matrices, $\bar{x}_o$ is the prediction of $x_o$, $P$ is the prediction of $P$, and $x_o(0)$ and $P(0)$ are the initial values of $x_o$ and $P$, respectively.

As shown in Fig. 2, the EKF algorithm to estimate the load torque consists of two steps. The
first step performs the prediction of two quantities \( \tilde{x}_o(k), \tilde{P}(k) \) based on the previous estimates. That is, the predicted value \( \tilde{x}_o(k) \) is calculated from the previous state \( x_o(k-1) \) and previous input \( u_o(k-1) \), and the predicted value \( \tilde{P}(k) \) is derived from the previous error covariance matrix \( P(k-1) \) and noise covariance matrix \( Q \). In the second step, the Kalman gain \( L(k) \) is computed by using \( \tilde{P}(k) \) and \( R \), and then the state \( x_o(k) \) is estimated with \( L(k) \) and \( \tilde{x}_o(k) \). Also, \( P(k) \) is updated for the next calculation using \( \tilde{P}(k) \) and \( L(k) \). Finally, the estimated load torque \( \tilde{T}_L \) can be obtained from \( x_o \) and \( C_T \).

3. Simulations and Experiments

To confirm the effectiveness of the proposed nonlinear speed controller, simulations and experiments are performed. Table 1 shows the nominal parameters of a prototype PMSM considered in this paper. According to the parameters shown in Table 1, the dynamic equations can be rewritten as follows:

\[
\begin{align*}
\dot{s} &= 3539.6i_{qs} - 0.24844\omega - 4968.87T_L \\
\dot{\omega}_q &= -170.1i_{qs} - 13.6\omega + 171.8V_q - \omega\dot{d}_s \\
\dot{\omega}_s &= -170.1i_{ds} + 171.8V_q + 6\omega q_s
\end{align*}
\] (13)

<table>
<thead>
<tr>
<th>Table 1. Specifications of PMSM</th>
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<tbody>
<tr>
<td>Number of poles ((p))</td>
</tr>
<tr>
<td>Stator resistance ((R_s))</td>
</tr>
<tr>
<td>Stator inductance ((L_s))</td>
</tr>
<tr>
<td>Magnetic flux ((\lambda_o))</td>
</tr>
<tr>
<td>Equivalent inertia ((J))</td>
</tr>
<tr>
<td>Viscous friction coefficient ((B))</td>
</tr>
</tbody>
</table>

To design the EKF-based load torque observer, the following matrices are chosen:

\[
A_0 = \begin{bmatrix} T & T & 0 \\ 0 & -0.49T & -4968.87 \\ 0 & 0 & T \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad C_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\]

\[
x_o(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad P(0) = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & -0.01 \\ 0 & 0 & 2 \end{bmatrix}
\]

\[
Q = 5 \times 10^{-6} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R = 0.001.
\]

From the EKF algorithm shown in Fig. 2, the Kalman gain \( L \) is below calculated with respect to time.

\[
L(k) = P(k)C_0^T \left( C_0P(k)C_0^T + R \right)^{-1}
\]

Next, the observer-based feedback control terms can be expressed as

\[
u_{fb} = \begin{bmatrix} u_{fbq} \\ u_{fbd} \end{bmatrix} = K\hat{\omega}
\]

where \( \hat{\omega} = [\theta, \omega, \dot{\theta}, \dot{\omega}]^T \), \( \hat{\eta} = \hat{\theta} = k_1i_{qs} - k_2\omega - k_3\hat{T}_L \), \( \dot{\eta}_x = \dot{\hat{\theta}} = \dot{\omega} \).

Finally, the following controller gain is used:

\[
K = 10^4 \times \begin{bmatrix} -5.2006 & -0.0194 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\]

Fig. 3 shows a block diagram of the observer-based speed controller. Based on Fig. 3, simulations and experiments are executed to evaluate the performance of the proposed control.
algorithm. Fig. 4 shows an overall block diagram of the proposed nonlinear control system. In this paper, a space vector PWM (SVPWM) technique is used due to well-known benefits, and PWM frequency and sampling frequency \((1/T)\) are selected as 5 [kHz] considering switching loss and current ripple.

Fig. 4 shows an overall block diagram of the proposed nonlinear control system. In this paper, a space vector PWM (SVPWM) technique is used due to well-known benefits, and PWM frequency and sampling frequency \((1/T)\) are selected as 5 [kHz] considering switching loss and current ripple.

Fig. 4 shows an overall block diagram of the proposed nonlinear control system. In this paper, a space vector PWM (SVPWM) technique is used due to well-known benefits, and PWM frequency and sampling frequency \((1/T)\) are selected as 5 [kHz] considering switching loss and current ripple.

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Fig. 5 shows the simulation results \((\omega_d, \omega, \hat{\omega}, i_{qs}, V_a, i_a)\) under nominal parameters. On the other hand, Fig. 6 shows the simulation results under 200[\%] variations of some parameters \((R_s, L_s, J)\) to verify the robustness of the proposed control scheme. In Fig. 6, the proposed nonlinear speed controller shows good control performance under even model parameter variations.

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Fig. 7 illustrates the experimental test setup to implement the proposed control algorithm. As shown in Fig. 7, it includes a PMSM, a brake, a host PC, a three-phase PWM inverter with a TMS320F28335 DSP. Fig. 8 shows the experimental results under the same conditions as Fig. 5, whereas Fig. 9 shows the experimental results under the same condition as Fig. 6. Fig. 8 (a) and 9 (a) show the desired speed ($\omega_d$), measured speed ($\omega$), and speed error ($\omega_e$). Fig. 8 (b) and 9 (b) show the load torque ($T_L$) and estimated load torque ($\hat{T}_L$). Fig. 8

(a) $\omega_d$, $\omega$ and $\omega_e$

(b) $T_L$ and $\hat{T}_L$

(c) $i_{qs}$ and $i_{ds}$

(d) $V_m$ and $i_a$

![Experimental setup](image)

Fig. 7. Experimental setup

![Experimental results](image)

Fig. 8. Experimental results under nominal parameters
(c) and 9 (c) show the measured \( q \)-axis current \( (i_{qs}) \) and \( d \)-axis current \( (i_{ds}) \). Fig. 8 (d) and 9 (d) show the phase a voltage \( (V_a) \) and phase a current \( (i_a) \).

From simulation and experimental results, it is shown that actual motor speed reaches the desired speed within about 10[msec], and the steady-state speed error is almost zero. Therefore, it is clearly realized that the proposed control scheme can accurately and quickly follow the reference trajectory of a PMSM in the presence of motor parameter variations.

4. Conclusion

This paper presented a nonlinear control scheme for PMSM drive systems. In this paper, an extended Kalman filter (EKF) observer was employed to estimate the load torque. Through the simulation and experimental results, it was verified that the proposed control method can precisely and rapidly
track the desired speed even under the variations of motor parameters and unknown load torque conditions.

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References


Biography

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Nga Thi-Thuy Vu was born in Vietnam in 1982. She received B.S. and M.S. degrees in Electrical Engineering from Hanoi University of Technology, Hanoi, Vietnam in 2005 and 2008, respectively. She is currently pursuing a Ph.D. in the Division of Electronics and Electrical Engineering, Dongguk University, Seoul, Korea. Her research interests are in the field of DSP-based electric machine drives and control of distributed generation systems using renewable energy sources.

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Jin-Woo Jung was born in Korea in 1969. He received B.S. and M.S. degrees in Electrical Engineering from Hanyang University, Seoul, Korea in 1991 and 1997, respectively, and a Ph.D. in Electrical and Computer Engineering from Ohio State University, Columbus, Ohio, USA, in 2005. From 1997 to 2000, he was with the Digital Appliance Research Laboratory, LG Electronics Co., Ltd., Seoul, Korea. From 2005 to 2008, he worked at the R&D Center and PDP Development Team, Samsung SDI Co., Ltd., Korea, as a senior engineer. Since 2008, he has been an Assistant Professor with the Division of Electronics and Electrical Engineering, Dongguk University, Seoul, Korea. His current research interests are in the areas of DSP-based electric machine drives, control of distributed generation systems using renewable energy sources (wind turbines, fuel cells, solar cells), design and control of power converters, driving circuits and driving methods of ac plasma display panels (PDP).