Spatial Multiplexing Receivers in UWB MIMO Systems based on Prerake Combining

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Abstract—In this paper, various ultra-wideband (UWB) spatial multiplexing (SM) multiple input multiple output (MIMO) receivers based on a prerake diversity combining scheme are discussed and their performance is analyzed. Several UWB MIMO detection approaches such as zero forcing (ZF), minimum mean square error (MMSE), ordered successive interference cancellation (OSIC), sorted QR decomposition (SQRD), and maximum likelihood (ML) are considered in order to cope with inter-channel interference. The UWB SM systems based on transmitter-side multipath preprocessing and receiver-side MIMO detection can either boost the transmission data rate or offer significant diversity gain and improved BER performance. The error performance and complexity of linear and nonlinear detection algorithms are comparatively studied on a log-normal multipath fading channel.

Index Terms— Ultra-WideBand (UWB), Spatial Multiplexing, Multiple Input Multiple Output (MIMO), Zero Forcing (ZF)

I. INTRODUCTION

ULTRA-wideband (UWB) radio transmission technology has received great interest in high speed indoor radio communications. UWB communication systems can be combined with multiple input multiple output (MIMO) systems for certain applications such as high definition video transmission [1]. Such a UWB MIMO scheme can improve the data rate through spatial multiplexing (SM) without boosting transmit power. Based on multiple transmit and receive antennas and appropriate detection, the UWB SM system effectively creates parallel MIMO subchannels to transmit independent data streams. In [2], the performance of UWB MIMO systems over indoor wireless multipath channels has been analytically investigated on the basis of zero forcing (ZF) detectors followed by rake combiners. It has been shown to offer an \( L(N_r - N_g + 1) \) -order path diversity for \( N_r \) transmit antennas, \( N_g \) receive antennas, and \( L \) resolvable multipath components, i.e., \( (N_r, N_g, L) \) system. In [3], the BER performance of a UWB SM MIMO receiver consisting of rake combiners followed by a ZF detector, called a rake-ZF in this work, has been analyzed. It has been found that the rake-ZF architecture provides an \( (LN_r - N_g + 1) \) -order path diversity for the \( (N_r, N_g, L) \) system and thus improves the UWB SM MIMO system of [2] based on the ZF followed by rake combining.

Pulsed UWB communications can resolve individual multipath components and a rake receiver can be used to gain path diversity and combine multipath signals. The number of rake fingers has a strong influence on the receiver complexity as well as performance. Thus, effective capturing of multipath energy at the receiver is a challenging task in UWB communications. In order to effectively collect the energy with a simple receiver structure, the prerake diversity combining technique has been investigated for UWB communications [4]. In [5], a time-reversed UWB MIMO system has been discussed to increase the transmission data rate. In [6], the UWB SM MIMO system consisting of the prerake diversity combiners in the transmitter and the ZF detector in the receiver has been analytically examined on a log-normal fading channel. This architecture is called a prerake-ZF. It has been demonstrated that the prerake-ZF scheme achieves an \( (LN_r - N_g + 1) \) -order path diversity for the \( (N_r, N_g, L) \) system. It has been shown that the BER performance of prerake-ZF for UWB SM MIMO \( (N_r, N_g, L) = (N, M, L) \) system is equivalent to that of rake-ZF \( (N_r, N_g, L) = (M, N, L) \) systems of [3] with lower computational complexity in the receiver. In the case of \( N_r > N_g \), the prerake system outperforms the rake system. As the number \( N_g \) of multiplexed data streams with fixed \( N_r \) increases, the performance of prerake system is degraded. Although the prerake-ZF scheme in [6] established the new architecture for designing simple UWB receivers, it did not show the performance of other linear and nonlinear receivers except for ZF. In [7], an ordered successive interference cancellation (OSIC) algorithm based on minimum mean square error (MMSE) criterion has been evaluated in the prerake system.

In this paper, we comparatively investigate the performance and complexity of several candidate detection techniques in UWB SM MIMO communication systems employing the prerake-ZF structure. It is
anticipated that other advanced linear and nonlinear MIMO detection algorithms can enhance the performance of prerake combining-based ZF receiver. Linear receivers are simpler to implement than an optimum maximum likelihood (ML) receiver, but their performance fails to reach that of the ML receiver. A number of other suboptimum detectors, originally proposed for narrowband MIMO systems, provide much smaller complexity. This work is particularly interested in various MIMO detectors that can exploit the potential of spatial diversity offered by the UWB MIMO channel. In this paper, several MIMO detection methods such as ZF, MMSE, OSIC, sorted QR decomposition (SQRD), and baseline ML are considered. The UWB SM configuration employing multipath preprocessing block in the transmitter side and MIMO detector in the receiver side as in the prerake-ZF scheme can acquire high data rates as well as extra diversity gain. Thus, we evaluate the BER performance of a number of MIMO detectors in prerake systems via Monte-Carlo simulations on a log-normal multipath fading channel.

II. PRERAKE DIVERSITY COMBINING SYSTEM FOR UWB MIMO DETECTION

In this work, the prerake diversity combining system for UWB SM MIMO detection presented in [6], which is composed of the multipath preprocessing block at the transmitter and MIMO detector in the receiver, is briefly introduced. The detailed prerake system model including log-normal multipath fading channels is described in [6]. Consider a multiple antenna UWB communication system with \( N_t \) transmit and \( N_r \) receive antennas. The input data are serial-to-polar converted into independent \( N_r \) data streams with the binary pulse-amplitude of a short-duration UWB pulse, which are preprocessed by prefilters used at the transmitter. It is assumed that the transmitter exploits only \( L \) resolvable paths among the total number of resolvable multipath components. Eventually, the preraked signals are simultaneously transmitted on \( N_t \) transmit antennas. The pulse repetition interval is assumed to be sufficiently large compared with the channel delay spread in order to avoid severe inter-symbol interference (ISI) caused by the multipath channel. In the absence assumption of inter-path interference (IPI) when time intervals between all adjacent paths are larger than the UWB pulse duration, the discrete-time received signal vector, \( \mathbf{h}^T = [h_{t1}^T, h_{t2}^T, ..., h_{tN_{sk}}^T] \), at the UWB pulse matched filter output of the receive antennas is given by [6]

\[
\mathbf{h}^T = [\mathbf{H}(0), \mathbf{H}(1), ..., \mathbf{H}(L-1)]
\]

(2)

\[
\mathbf{H}(l) = [h_{t1}(l)^T, h_{t2}(l)^T, ..., h_{tN_{sk}}(l)^T]^T
\]

(3)

\[
\mathbf{h}_{n}(l) = [h_{n1}(l), h_{n2}(l), ..., h_{N_{sk}n}(l)]
\]

(4)

where \( h_{n}(l) \) is the channel fading coefficient of the \( l \)th path for the signal to the \( m \)th receive antenna sent from the \( n \)th transmit antenna and modeled as \( h_{n}(l) = \alpha_{mn}(l)\beta_{mn}(l) \) where \( \beta_{mn}(l) \) is the log-normal fading amplitude and \( \alpha_{mn}(l) \in \{\pm 1\} \) represents the phase inversion with equal probability. The information symbol vector is represented by \( \mathbf{b} = [b_1, b_2, ..., b_{N_{sk}}] \) with the information bit, \( b_{m} \in \{\pm 1\} \), from the \( m \)th data stream. The noise signal vector is denoted by \( \mathbf{w} = [w_1, w_2, ..., w_{N_{sk}}] \), where the zero-mean noise component \( w_{m} \) has a variance of \( \sigma^2 = N_0/2 \). Most of the notations and parameters related to the expression (1) are defined following [6].

III. MIMO DETECTION METHODS

Now we present various candidate receiver designs for spatially multiplexed UWB systems based on the prerake combining scheme. To spatially demultiplex the transmitted bits multiplexed at the transmitter, the received signal vector of (1) is filtered by a single linear or nonlinear MIMO detector such as ZF, MMSE, OSIC, SQRD, or ML. Then the sufficient statistic can be expressed as \( \mathbf{v}' = \mathbf{G}\mathbf{\Lambda}' \) where \( \mathbf{G} \) is the filter matrix. Hence, the decision variable for any information bit of the \( m \)th transmitted data stream is written as \( z_{m}' = \text{Re}\{v_{m}'\} \) where \( v_{m}' \) is the \( m \)th element of the sufficient statistic vector \( \mathbf{v}' = [v_{1}', v_{2}', ..., v_{N_{sk}}'] \).

A. Zero Forcing

The ZF filter matrix is defined as [6]

\[
\mathbf{G}_{\text{pre rake-ZF}} = \Xi = \left( \mathbf{H}^H\mathbf{H} \right)^{-1/2}
\]

(5)

Then the sufficient statistic is given by

\[
\mathbf{v}' = \mathbf{G}_{\text{pre rake-ZF}} \mathbf{\Lambda}' = \sqrt{E_b} \mathbf{b} + \left( \mathbf{H}^H\mathbf{H} \right)^{1/2} \mathbf{w}'
\]

(6)

The inter-channel interference (ICI) is completely suppressed and thus the signal components in the ZF detector output are perfectly separated. The prerake UWB systems using the ZF detector called a prerake-ZF has been analytically presented in [6].

B. Minimum Mean Square Error

In the prerake-ZF scheme, the MMSE detector instead
of the ZF can be employed to spatially process the \( N_g \) transmitted data streams. It is called a prerake-MMSE. Here, the modeling approach used in [8] to deal with the MMSE detection problem is considered. The \((N_g + N_g) \times N_g\) extended channel matrix \( \Xi \) of the channel matrix \( \Xi' \) received at the receiver and \((N_g + N_g) \times 1\) extended receive vector \( \Delta' \) of the received signal vector \( \Delta' \), respectively, are defined as

\[
\Xi = \begin{bmatrix} \Xi' \\ \sigma^2 I_{N_g} \end{bmatrix}, \quad \text{and} \quad \Delta' = \begin{bmatrix} \Delta' \\ \theta_{N_g,1} \end{bmatrix}
\]  

(7)

The MMSE filter can be obtained by the matrix

\[
G_{\text{prerake-MMSE}} = \left( \Xi'^T \Xi + \frac{\sigma^2}{E_b} I_{N_g} \right)^{-1} \Xi'^T
\]  

(8)

The resulting MMSE filter output signal vector for the \( l \)th path of a particular bit is given by

\[
\mathbf{v}' = G_{\text{prerake-MMSE}} \Delta' = \Xi' \Delta'
\]  

(9)

The remaining parts of the prerake-MMSE scheme except the MMSE filtering block for spatial processing at the receiver are the same as the prerake-ZF. Recall that the performance of ZF detector approaches that of the MMSE scheme at high SNRs [8].

C. Ordered Successive Interference Cancellation

In an OSIC technique [7,8], the signal with the largest post detection signal-to-noise (SNR) ratio is first selected for nulling operation and then cancelled from the overall received signal vector. Note that the nulling process can be performed by using the ZF or MMSE criteria. In this work, we refer to these two schemes as prerake-ZF-OSIC and prerake-MMSE-OSIC, respectively. The ZF-OSIC algorithm for the prerake-ZF-OSIC is shown in Fig. 1. Here, \( g_{mn}^{(j)} \) and \( \mathcal{Q} (\cdot) \) denote the row \( j \) of \( G_{\text{prerake-ZF,OSIC}} \), where \( \Xi = \left[ \mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_{N_g} \right] \) and quantization operation, respectively, and \( \Xi_{m+1} = \Xi_{m} \) represents the nulling operation of column \( k_m \) of the channel matrix \( \Xi_m \) in ZF-OSIC step \( m \). After obtaining the zero-forced signal vector at the end of \( N_g \) ZF-OSIC steps, the elements of the zero-forced vector are permuted according to the original sequence of the transmitted data streams. Then we obtain the \( m \)th element, \( V_{m-1}' \), of the permuted sufficient statistic vector of the zero-forced output for a particular bit of the \( m \)th transmitted data stream. In order to apply an MMSE-OSIC algorithm in the prerake-MMSE-OSIC scheme, the channel matrix \( \Xi \) and receive vector \( \Delta'_m \) in Fig. 1 are replaced by the extended matrix \( \Xi \) and extended receive vector \( \Delta'_m \), respectively.

\[
\text{for } m = 1, \ldots, N_g
\]

\[
G_{\text{prerake-ZF,OSIC}} = \mathbf{g}_{mn}^{(j)}
\]

\[
k_m = \arg \min_j \left\| g_{mn}^{(j)} \right\|
\]

\[
\mathbf{v}_{km} = g_{mn}^{(j)} \Delta'_m
\]

\[
\mathbf{b}_{km} = \mathcal{Q} [ \mathbf{v}_{km} ]
\]

\[
\Delta'_{m+1} = \Delta'_{m} - \mathbf{b}_{km} e_{km}
\]

\[
\Xi_{m+1} = \Xi_{m}^c
\]

end

Permute the elements of \( \mathbf{v}' \) according to the original sequence \( (1, 2, \ldots, N_g) \).

Fig. 1. ZF-OSIC algorithm in prerake-ZF-OSIC

D. Sorted QR Decomposition

Since the prerake-ZF-OSIC and prerake-MMSE-OSIC need multiple calculations of the pseudo-inverse of the channel matrix \( \Xi \) and \( \Xi' \), the main weakness of an OSIC algorithm consists of computational complexity. It has been shown in [8] that the OSIC can be efficiently restarted by performing the sorted QR decomposition (SQRD) of the channel matrix over frequency nonselective fading channels. Now the SQRD algorithm developed for narrowband MIMO systems is applied to the prerake scheme of UWB systems on frequency selective fading channels, which is called a prerake-ZF-SQRD. The QR decomposition of the matrix is represented by \( \Xi = \mathbf{Q} \mathbf{R} \), where the \( N_g \times N_g \) matrix \( \mathbf{Q} \) has orthogonal columns, \( q_m, \ m = 1, \ldots, N_g \), with unit norm and the \( N_g \times N_g \) matrix \( \mathbf{R} \) is upper triangular. The SQRD algorithm is shown in Fig. 2. After computing the orthogonal matrix \( \mathbf{Q} \), the detection process is made as in Fig. 3 and thus the interference-cancelled signal \( c_m \) is obtained. An MMSE criterion in SQRD algorithm is performed in prerake UWB systems, which is called a prerake-MMSE-SQRD. The MMSE-SQRD algorithm with slight modification refers to [9].

E. Maximum Likelihood

The optimum ML receiver (called a prerake-ML) searches for the nearest signal between the received signal and the various possible transmitted signals. The ML estimated signal \( \mathbf{b} \) can be obtained as

\[
\mathbf{b} = \arg \min \left\| \Delta' - \sqrt{E_b} \Xi \mathbf{b} \right\|^2
\]  

(10)

The ML receiver has a drawback that the increase in the number of receive antennas leads to the increase in the computational complexity. It is used as a benchmark for linear and suboptimum nonlinear receiver performance in prerake UWB systems.
TABLE I

<table>
<thead>
<tr>
<th>MIMO detection algorithm</th>
<th>Approximated computational complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prerake-ZF</td>
<td>$O(2N_P^2) + LN_P N_X^2 + N_X^2$</td>
</tr>
<tr>
<td>Prerake-MMSE</td>
<td>$O(2N_P^2) + LN_P N_X^2 + 2N_X^2 + N_X^2$</td>
</tr>
<tr>
<td>Prerake-ZF-OSIC</td>
<td>$O(N_X^2) + LN_P N_X^2 + N_X^2 \left( O(N_P^2) + 2N_X^2 + 2N_X \right)$</td>
</tr>
<tr>
<td>Prerake-MMSE-OSIC</td>
<td>$O(N_X^2) + LN_P N_X^2 + N_X^2 \left( O(N_P^2) + 4N_X^2 + 4N_X \right)$</td>
</tr>
<tr>
<td>Prerake-ZF-SQRD</td>
<td>$O(N_X^2) + LN_P N_X^2 + 0.5N_X^2 (N_X + 3)$</td>
</tr>
<tr>
<td>Prerake-MMSE-SQRD</td>
<td>$O(N_X^2) + LN_P N_X^2 + (N_X + 3)$</td>
</tr>
</tbody>
</table>

$\mathbf{R} = \mathbf{0}$, $\mathbf{Q} = \Xi$, $S = \{1, 2, \ldots, N_X\}$.

for $m = 1, \ldots, N_X$

$$k_m = \arg \min_{j=m-1, \ldots, N_X} \| \mathbf{q}_j \|^2$$

exchange col. $m$ and $k_m$ in $\mathbf{Q}$, $\mathbf{R}$, $S$

$$r_{m,m} = \left\| \mathbf{q}_m \right\|$$

$$\mathbf{q}_m = \mathbf{q}_m / r_{m,m}$$

for $j = m+1, \ldots, N_X$

$$r_{m,j} = \mathbf{q}_m^\top \mathbf{q}_j$$

$$\mathbf{q}_j = \mathbf{q}_j - r_{m,j} \mathbf{q}_m$$

end

end

Fig. 2. ZF-SQRD algorithm in prerake-ZF-SQRD

$$\mathbf{u} = \mathbf{Q}^\dagger \mathbf{A}^\top$$

for $m = N_X, \ldots, 1$

$$\hat{a}_m = \sum_{j=m+1}^{N_X} r_{m,j}^\dagger \hat{b}_j$$

$$c_m = u_m - \hat{a}_m$$

$$\hat{b}_m = \mathbf{Q} \left[ \frac{c_m}{r_{m,m}^\dagger} \right]$$

end

Permute $\hat{\mathbf{b}}$ according to $S$

Fig. 3. Signal detection part in prerake-ZF-SQRD

**F. Computational Complexity**

It is shown in [6] that the prerake-ZF receiver has lower computational complexity than the rake combining-based ZF detector, especially when the number of transmit antennas is larger than the number of receive antennas. Here by taking only the multiplicative operation into account in the multiplication of a matrix by a matrix, a matrix by a vector, and a vector by a vector (other computation is ignored), the approximated computational complexity of the prerake-ZF, prerake-MMSE, prerake-ZF-OSIC, prerake-MMSE-OSIC, prerake-ZF-SQRD, and prerake-MMSE-SQRD can be calculated, as shown in the Table I. We see that the prerake-ZF-SQRD and prerake-MMSE-SQRD are computationally efficient.

**IV. SIMULATION RESULTS**

The statistical channel model adopted in the simulations follows [2] and [6]. The log-normal fading amplitude $\beta(l)$ of the $l$th path is expressed as $\beta(l) = e^{\rho(l)}$ where $\psi(l)$ is a Gaussian random variable with mean $\mu_{\psi(l)}$ and variance $\sigma_{\psi}^2$ (independent of $l$). Satisfying $E\{\beta(l)^2\} = e^{2\rho}$ requires $\mu_{\psi(l)} = -\sigma_{\psi}^2 - \rho l / 2$, where $\sigma_{\psi} = 5 / (20 \log_{10} e)$. Here, it is assumed that the standard deviation of $20 \log_{10} \beta(l) = \psi(l)(20 \log_{10} e)$ is 5-dB and the power decay factor is $\rho = 0$. We assume that both the transmitter and receiver in the prerake-ZF, prerake-MMSE, prerake-ZF-OSIC, prerake-MMSE-OSIC, prerake-ZF-SQRD, prerake-MMSE-SQRD, and prerake-ML scheme have perfect knowledge of the channel fading coefficients of $L$ resolvable paths.

Figs. 4 and 5 show the performance results of various MIMO detectors as a function of $E_b / N_0$ in decibel for prerake diversity combining-based UWB SM $(N_T, N_P, L) = (4, 3, 2)$ and $(N_T, N_P, L) = (2, 3, 2)$ systems, respectively. It can be seen that ZF-OSIC and MMSE-OSIC receivers, respectively, outperform the linear receivers of ZF and MMSE. The BER performance of MMSE-OSIC is close to that of the ML receiver. The ZF-SQRD and MMSE-SQRD with lower computational complexity have quite similar performance to the ZF-OSIC and MMSE-OSIC. By comparing the $(N_T, N_P, L) = (4, 3, 2)$ system with the $(N_T, N_P, L) = (2, 3, 2)$ case, the use of larger number of transmit antennas increases a
higher spatial diversity gain and thus improves the BER performance. In other words, decreasing the number of transmit antennas boosts the ICI and thus degrades the BER performance. It is also observed that more transmit antennas make the BER performance of ZF-OSIC detector to approach to that of the MMSE-OSIC.

Fig. 6 shows the BER results as a function of the number of receive antennas $N_{r}$ (i.e. the number of multiplexed data streams), for prerake systems with $E_s/N_0 = 3\, \text{dB}$, $N_{f} = 4$ transmit antennas and $L = 2$ resolvable multipath components. As the number of receive antennas is larger, the performance is worse. This is due to the larger ICI. In contrast, decreasing the number of receive antennas boosts the diversity gain. Meanwhile, increasing the number of receive antennas increases the transmission data rate and thus obtain multiplexing gain. The MMSE-OSIC and MMSE-SQRD give significantly enhanced performance even for larger $N_{r}$. The MMSE-OSIC and MMSE-SQRD achieve approximately 2-fold data rate compared to the ZF and MMSE for the BER value of $10^{-3}$.

In Fig. 7, the BER performances of prerake systems versus the SNR per bit for different number of resolvable paths with $N_{f} = 2$ and $N_{r} = 4$ are shown, where the SNR per bit in decibel is defined as $\eta_b = E_s/N_0 (\text{dB}) + 10 \log_{10} L$. The increase of the number of resolvable paths boosts the diversity order of prerake-ZF, prerake-ZF-SQRD, prerake-MMSE-SQRD, and ML detectors. Even if the number of multiplexed data streams in prerake systems is larger than the number of transmit antennas, its BER performance can be enhanced by the augmentation of resolvable paths combined. When using the nonlinear and ML receivers, the degree of performance improvement (i.e. spatial diversity gain) gets smaller as
the number of resolvable paths increases.

It has been shown in [6] that the performance of prerake-ZF UWB system with \((N_f, N_k, L) = (N, M, L)\) agrees with that of rake-ZF with \((N_f, N_k, L) = (M, N, L)\) presented in [3]. Various MIMO detection algorithms used in prerake UWB systems can be applied to rake combining-based UWB systems of [3], which consist of rake combiners followed by a MIMO detector in the receiver. Even if the BER performance results of the rake combining-based systems are not presented here, it can be expected that they match well with those of the prerake systems.

V. CONCLUSIONS

In this paper, various narrowband MIMO SM schemes have been extended to the prerake diversity combining-based UWB systems over indoor log-normal multipath fading channels. The UWB SM MIMO system employs the prerake diversity combiners that captures \(L\) resolvable paths in the transmitter and then an MIMO detector in the receiver to spatially separate the \(N_k\) multiplexed data streams. It has been found that suboptimum nonlinear detection techniques significantly outperform linear receivers. It has been observed that when the number of receive antennas (i.e. the number of multiplexed data streams) is larger than the number of transmit antennas, the prerake-MMSE-OSIC and prerake-MMSE-SQRD offers much higher data rate than the prerake-ZF receiver with the same BER performance. The results say that with an appropriate detection scheme, an increasing number of receive antennas yield a higher data rate and also using larger number of transmit antennas produces a significantly improved diversity order. Especially, the SQRD receiver is significantly less complex than OSIC while achieving similar performance.

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REFERENCES


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