A Novel Phase Extraction for the Detection of Time Parameters in Signal

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Abstract: A unique technique to extract the phase in time domain is proposed in order to measure the time parameters such as speed and depth by transmitting sound and electric waves. In the signal analysis processing, the phase of pulse signal can be transformed and digitalized with local data in real time without the effect of direct current bias and Nyquist limits. This method is sensitive to base frequency of pulse signal with high spatial resolution and is effective to compare two signals which have different forms. It is expected that the phase analysis technique will be applied to the measurement of the speed and depth accurately by ultrasonic pulse signal in water.

Key words: Phase extraction, Ultrasonic pulse signal, Signal analysis, Time domain, Time parameter

1. Introduction

Ultrasonic pulse signal is widely used to measure the speed of moving targets and depth of water because it has time and distance resolution and can be well transmitted in waters(Jerk, 1987). In order to measure them, propagation time and Doppler frequency have to be detected in comparing transmitted signal with received signal. But receiving signal is different waveform with transmitted ultrasonic pulse signal which has several frequency elements. In signal processing and analysis of the particular application, are a group of time domain techniques which yield a description of a real signal in terms of its instantaneous envelope and phase. The instantaneous phase and its time derivative are powerful tools for measuring ship’s speed and depth of water in Doppler log with high accuracy since it corresponds to the time within a period(Lee et al., 1995).

The most important techniques used in the calculation of phase functions are Complex Demodulation, Hilbert Transformation, In-Phase and In-Quadrature Filtering, and Phase Locked Loop Demodulation(Penklis et al., 1980). By the way, they are unsuitable to the real time measurement of instantaneous information such as the accurate depth and the profile velocity of moving targets due to filter and integral operation with total signal.

So we propose a new phase extraction technique in order to analyze ultrasonic pulse signal with local data in time domain. In the analysis, the both parts of real and imaginary can be calculated to get complex signal and simple digital processing with local small data in real time.

In the proposed method, the fixed phase is independent on direct current bias and the magnitude of amplitude. It is useful for comparing signals which have different forms and analyzing Doppler pulse signal beyond the Nyquist limits(Baeck et al., 1989).

2. Signal analytic indication

2.1 Signal parameters

Any real signal \( f(t) \) can be expressed in the form:

\[
 f(t) = A(t) \cos[\phi(t)]
\]

and thus it may be regarded as originating from a modulation of the amplitude and phase of a cosine wave. The two function \( A(t) \) and \( \phi(t) \) are called, respectively, the envelope and the phase function of \( f(t) \). In more specific applications, as in the analysis of transient waveform, the envelope function helps to localize, with a sharp pictorial description, the time variations of the signal energy, pointing out the instant of maximum energy occurrence as well as the starting and ending times of the transient signal. On the other hand, the instantaneous phase and its time derivative are powerful tools for detecting apparent periodicities in the frequency content of a signal as a function of time(Kim et al., 2003).

2.2 Conception of phase

The phase is a variable to indicate the state that changes
continuously and regularly with periodicity according to time. The idea of phase is extensively used to describe the state of continuous sinusoidal signal. It is the same value of signal at the interval of period and indicates the detail time to be independent on the magnitude of signal within a period (Beranek, 1991). The continuous sinusoidal signal is written by

\[ f(t) = A \cos (wt + \theta) \]  

(2)

where \( A \), \( w \) are the amplitude, the angle frequency in radians per second and the initial phase \( t = 0 \) in radian. The phase at the time \( t \) is expressed as \( wt + \theta \), which is the accumulated rotation angle from the initial phase.

2.3 Derivation of time from phase

The time \( t_s \) in one period \( T \) can be derived in the form:

\[ t_s = \frac{2\pi}{T} \theta. \]  

(3)

The accumulated total time \( t \) which a signal takes to propagate from the starting point to the point can be generalized as follows:

\[ t = nT + t_s. \]  

(4)

Fig. 1 shows the relation between time and phase in continuous sinusoidal signal.

![Fig. 1 derivation of time from phase](image)

2.4 Display in complex coordinate

Continuous sinusoidal signal can be described in complex coordinate by two dimensional vectors in stead of real signal. This complex signal \( C(t) \) has the amplitude \( A(t) \) and phase \( \theta(t) \) in Fig. 2 (Athina, 1992).

This complex signal \( C(t) \) can be written by imaginary unit as

\[ C(t) = Re\{C(t)\} + j Im\{C(t)\}\]

\[ = \cos (wt + \theta) + j (\sin (wt + \theta)) \]  

(5)

where \( \Re\{C(t)\} \) and \( \Im\{C(t)\} \) are real part and imaginary part of \( C(t) \), respectively.

The phase and envelope is expressed as the function of time by (Athina, 1992)

\[ \theta(t) = \arg (\Re \{C(t)\} + j \Im \{C(t)\}), \]  

(6)

\[ A(t) = \sqrt{\Re \{(C(t))^2 + \Im \{C(t)\}^2}. \]  

(7)

With the phase function, integrated time derivation is calculated with equation (4) as,

\[ t = t_1 + t_2 \]

\[ = nT + \frac{2\pi}{T} \theta(t) \]  

(8)

where \( t_1, t_2 \) are the times of period and the detail time \( \Delta t \) within a period.

2.5 Calculation of phase

In order to calculate the phase at the time \( t \), it is necessary to estimate imaginary part against real part in terms of given data. Let signal \( f(t) \) be sampled with the period \( T_s \), and sampling phase angle \( \theta_s \) is given by

\[ \theta_s = \frac{2\pi}{T_s} \]  

(9)

where \( T \) is the period of sinusoidal signal. The sample values can be expressed generally by the set \( s(n) \) such that:

\[ s(n) = Ae^{jn\theta + \theta}. \]  

(10)

For \( n = k \), \( s(k) \) is given by

\[ s(k) = Ae^{j(n\theta + \theta)} = U. \]  

(11)
Here, the rectangular component $V$ corresponds to $U$ can be written by

$$V = Ae^{j(\omega t + \frac{\pi}{2})}$$

(12)

Since Re$U$, $V$ is $u$, $v$ respectively, the amplitudes $(u, v)$ of two rectangular components are obtained as follows:

$$u = A\cos(k\theta_s + \theta)$$
$$v = -A\sin(k\theta_s + \theta).$$

(13)

Using the relation between $u$ and $v$ in Eq.(13), the initial phase $\theta$ becomes as in Fig. 3

$$\theta = (-\frac{v \cos k\theta_s + u \sin k\theta_s}{u \cos k\theta_s - v \sin k\theta_s}).$$

(14)

Fig. 3 Estimation of phase using rectangular components

While $u$, $\sin k\theta_s$, $\cos k\theta_s$ are determined by selecting sample point in Eq.(12), $v$ has to be estimated by interpolating in terms of given sample data. Fig. 4 shows how to interpolate to get $v$. In this figure, $u$ is the sampling value and $v$ is the rectangular component against $u$ that has value between sampling value $m_1$ and $m_2$. If the value $u$ can be set as

$$u = A\cos \theta$$

(15)

Thus, $m_1$, $m_2$ can be expressed as

$$m_1 = A\sin (\theta - \alpha)$$
$$m_2 = A\sin (\theta + \theta_s - \alpha),$$

(16)

where $\theta$, is sampling phase angle, $\alpha$ is the angle between point $v$ and point $m_1$ written by

$$\alpha = \frac{\pi}{2} - \theta_s \lfloor \frac{\pi}{2\theta_s} \rfloor$$

(17)

where $\lfloor \rfloor$ is Gaussian symbol.

Then, in order to get the rectangular value $v$ against the value $u$, the values of interpolation $p_1$, $p_2$ are obtained by

$$p_1 = -A\sin \theta = \frac{m_1 - u \sin \alpha}{\cos \alpha}$$
$$p_2 = -A\sin \theta = \frac{m_2 - u \sin (\theta_s - \alpha)}{\cos (\theta + \alpha)}.$$  

(18)  
(19)

In the same way, the values of interpolation $p_3$, $p_4$ for minus rectangular components can be determined. So the rectangular component $v$ against $u$ is estimated as the average of interpolation values $p_1$, $p_2$, $p_3$ and $p_4$ for considering noise influence.

3. New phase extraction in time domain

3.1 Modulation of phase

Accumulated phase can be divided into two parts. One of them is the wave number that is the numbers of rotation, the other is the remainder that is smaller than 360°. Continuous sinusoidal signal $f(t)$ can be expressed as follows:

$$f(t) = A\sin(\omega t + \theta),$$
$$= A\sin(2\pi n(t) + \theta(t)), $$
$$= A\sin \theta(t),$$

(20)

where $n(t)$ is defined as

$$n(t) = \lfloor \frac{\omega t + \theta}{2\pi} + \frac{1}{2} \rfloor.$$  

(21)

$n(t)$ has the largest integer value which is not larger that the value in square bracket. And $\theta(t)$ is written by

$$\theta(t) = (\omega t + \theta + \pi) \bmod (2\pi) - \pi.$$  

(22)

$\theta(t)$ has the value in the range from $-180^\circ (-\pi)$ to $180^\circ$ ($\pi$) as the function of time. Here let $\theta(t)$ be called modulated phase.
3.2 Calculation of modulation phase

In the first place, it is convenient to recall the function value \( f(t + \tau) \) of continuous sinusoidal signal written by

\[
  f(t + \tau) = A \sin(\omega t + \theta + \omega \tau),
\]

where \( \tau \) is any time interval from \( t \).

On referring to Eq. (20), \( f(t + \tau) \) is formulated as

\[
  f(t + \tau) = A \sin(\theta(t) + \omega \tau),
\]

\[
= A \sin \theta(t) \cos \omega \tau + A \cos \theta(t) \sin \omega \tau.
\]

From above equation (24), we can get some clues to the solution for calculating modulated phase defined in eq. (19).

The equation (24) consists of orthogonal function such as \( \sin \omega t \), \( \cos \omega t \). The product of orthogonal functions and their integration during a period time become zero. The characteristic of orthogonal and trigonometric function are used to calculate the modulated phase \( \theta(t) \).

Here, both sides of eq. (24) are multiplied by \( \sin \omega t \) and then integrated during the period time of this function. The following equation \( R(t) \) can be defined by

\[
  R(t) = \int_{0}^{T} f(t + \tau) \sin \omega \tau d\tau.
\]

Thus, \( C(t) \) can be easily derived as follows:

\[
  R(t) = A \sin \theta(t) \int_{0}^{T} \cos \omega \tau \sin \omega \tau d\tau + A \cos \theta(t) \int_{0}^{T} \sin^2 \omega \tau d\tau,
\]

\[
= \frac{\pi}{\omega} A \cos \theta(t).
\]

Subsequently, both sides of eq. (24) are multiplied by \( \cos \omega t \) and integrated during a period time. In the same way, \( I(t) \) is defined as

\[
  I(t) = \int_{0}^{T} f(t + \tau) \cos \omega \tau d\tau.
\]

\( I(t) \) can be rewritten by

\[
R(t) = A \sin \theta(t) \int_{0}^{T} \cos \omega \tau d\tau
\]

\[
+ A \cos \theta(t) \int_{0}^{T} \sin \omega \tau \cos \omega \tau d\tau.
\]

\[
\begin{align*}
= \frac{\pi}{\omega} A \sin \theta(t).
\end{align*}
\]

(28)

Here, if the values of two integral equations \( R(t), I(t) \) are known, the modulated phase \( \theta(t) \) of continuous sinusoidal signal is calculated as follows:

\[
\theta(t) = \arctan \frac{R(t)}{I(t)} + \beta,
\]

(29)

where \( \beta \) is the value in radian depended on the sign of \( R(t), I(t) \) which is expressed as

\[
R(t) \cdot I(t) \geq 0 : \quad R(t) > 0, \beta = 0,
\]

\[
R(t) < 0, \beta = \pi,
\]

\[
R(t) \cdot I(t) < 0 : \quad I(t) > 0, \beta = 0,
\]

\[
I(t) < 0, \beta = -\pi.
\]

(30)

Thus \( \theta(t) \) can be expressed by employing the method(eq. 5) to illustrate argument in complex coordinate in the form:

\[
\theta(t) = \arg(R(t) + jI(t)).
\]

(31)

Its value ranges from \(-\pi\) to \(\pi\) and is regarded as indefinite when both \( R(t) \) and \( I(t) \) are zero.

3.3 Base frequency

In general, the signals which actually used in many systems are different from continuous sinusoidal one. It is necessary to extend the idea of phase in any signals for the wide scope application. Here, the representative frequency \( f_b \) of any signal \( f(t) \) is introduced. It is obtained from the periodicity by using auto correction as(Swingler, 1999).

\[
A(\tau) = \int_{-\infty}^{\infty} f(t)(f(t + \tau))dt.
\]

(32)

On the curve of auto correction, the base frequency can be determined from the time interval \( T_b \) between the center peak point and the adjacent one as follows:

\[
f_b = \frac{1}{T_b}.
\]

(33)

3.4 Phase generalization

The phase can be displayed with real part and imaginary part of signal in complex coordinate. Using the base frequency \( f_b \) defined formerly and eq.(29) the phase \( \theta(t) \) is generalized in the form:
\[ \theta(t) = \arg \left( \int_{\frac{-T_b}{2}}^{\frac{T_b}{2}} f(t+\tau) \sin \omega t \, d\tau \right) \]
\[ + j \int_{\frac{-T_b}{2}}^{\frac{T_b}{2}} f(t+\tau) \cos \omega t \, d\tau \]  (34)

where \( w_b = 2\pi f_b \) is the angular base frequency.

3.5 Phase extraction

Fig. 5 shows how the phase at the time \( t \) is extracted in time domain. The phase \( \theta(t) \) can be determined by calculating both real part \( R(t) \) and imaginary part \( I(t) \). In order to get the real part \( R(t) \), the local data \( \Delta f(t) \) which range from \( t - \frac{T_b}{2} \) to \( t + \frac{T_b}{2} \) in the \( f(t) \) and \( \cos w_b t \) are used. \( R(t) \) is determined by integrating \( \Delta f(t) \cos w_b t \) in the range from \( t - \frac{T_b}{2} \) to \( t + \frac{T_b}{2} \). In the same way, \( I(t) \) can be obtained by integrating \( \Delta f(t) \sin w_b t \) from \( t - \frac{T_b}{2} \) to \( t + \frac{T_b}{2} \).

![Fig. 5 The phase extraction method in time domain](image)

4.2 No influence on magnitude

Two signals \((f_1(t), f_2(t))\) in the scale of different amplitude is considered for the phase transformation. Then, the amplitude can be expressed by

\[ A_2(t) = a A_1(t), \]  (35)

where \( a \) is constant. And \( f_1(t), f_2(t) \) can be written by

\[ f_1(t) = A_1(t) f(t), \]
\[ f_2(t) = A_2(t) f(t), \]  (36)

\[ = a f_1(t). \]

According to eq.(34), the phase function \( \theta(t) \) of \( f_1(t) \), \( f_2(t) \) is independent on magnitude \( a \). Therefore, This method is useful for analyzing the tiny signals and comparing the signals with different magnitude in view of time.

4.3 No influence on D.C. bias

Output signals of systems often include the component of D.C. bias for some reason. It is necessary to eliminate the component. Here, let signal with D.C. component be considered as

\[ f_d = d + A \sin (\omega t), \]  (37)

where \( d \) means the magnitude of D.C. bias. According to the method defined in eq.(34), it can be seen that the phase function \( \theta(t) \) of \( f_d \) does not influence on D.C. bias.

4.4 No influence on Nyquist limits

The mean frequency aliasing problem originating from the pulse repetition frequency is one of major limitation in ultrasound pulse Doppler system. However, in this technique, Doppler frequency can be calculated from the difference of accumulated phase defined in time domain without Nyquist limits(Brekhovskikh et al., 2003).
4.5 The simple digital processing

In recent years, the high speed digital processing with computer sand A/D converters has been developed, which allows us to analyze signal digitally in real time. In this method, phase can be extracted with five sample data in digital processing. From the real part $R(t)$ and the imaginary part $I(t)$, the phase function $\theta(t)$ can be expressed as follows:

$$R(t) = \frac{1}{2}(a(t) \cos \theta - b(t) \sin \theta)$$
$$I(t) = \frac{1}{2}(a(t) \sin \theta + b(t) \cos \theta)$$

$$\theta(t) = \arg(a(t) + j(b(t) + c))$$

4.6 Comparison of other techniques

It has been shown thus far that the methods of Complex Demodulation, Hilbert Transformation and In-Phase and In-Quadrature Filtering lead to identical results, since all of them are methods to obtain the analytic signal associated with the given real signal (Zhu, 1989; Al-Nash, 1993).

Nevertheless, the computational procedure to obtain the real part and the imaginary part for analytic signal are different. Compared with other methods, the proposed technique in this paper has some features which are the high sensitivity in time information, which are to transform in time domain with simple digital processing and which is the way without the influence of D.C. bias and Nyquist limits.

5. Conclusion

The signal analysis in frequency domain is suitable for the design and evaluation of systems. In the case for estimating the information related to time such as speed and depth, it is desirable to analyze signal in time domain. In this paper, we proposed the unique technique to extract the phase in time domain in order to estimate the time parameters by transmitting sound and electric wave. With the method, the phase function of any signals can be extracted without influence of D.C. bias and Nyquist limits by processing the simple digital. It is expected that the phase analysis technique will be applied to the measurement of the speed and depth accurately by ultrasonic pulse signal in water.

References


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