Torque Density Improvement of Five-Phase PMSM Drive for Electric Vehicles Applications

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Abstract

In order to enhance torque density of five-phase permanent magnetic synchronous motor with third harmonic injection for electric vehicles (EVs) applications, optimum seeking method for injection ratio of third harmonic was proposed adopting theoretical derivation and finite element analysis method, under the constraint of same amplitude for current and air-gap flux. By five-dimension space vector decomposition, the mathematic model in two orthogonal space plane, \( d_1 - q_1 \) and \( d_3 - q_3 \), was deduced. And the corresponding dual-plane vector control method was accomplished to independently control fundamental and third harmonic currents in each vector plane. A five-phase PMSM prototype with quasi-trapezoidal flux pattern and its five-phase voltage source inverter were designed. Also, the dual-plane vector control was digitized in a single XC3S1200E FPGA. Simulation and experimental results prove that using the proposed optimum seeking method, the torque density of five-phase PMSM is enhanced by 20%, without any increase of power converter capacity, machine size and iron core saturation.

Key Words: Dual-plane vector control, Finite element analysis, Five-phase PMSM, FPGA, Harmonic control, Multi-phase systems

I. INTRODUCTION

In recent decades, the development of next generation vehicles which are more efficient and have less air pollution is being carried out actively throughout the world. EV is a road vehicle which involves an electric propulsion system. With this broad definition in mind, EVs include battery electric vehicles (BEVs), hybrid electric vehicles (HEVs) and fuel–cell electric vehicles (FCVs) [1]–[3]. The electric motor propulsion system is the heart of the EVs, which consists of the electric motor, its power converter, and an electronic controller. The requirements for the EV motor drive include: large torque, high speed, high power density, quick response and good dependability [4].

Selection of right electric motor is of primary importance to EVs designer. Nowadays, three-phase induction motors and permanent magnet machines are more appropriate solutions due to their lower cost and higher reliability. However, multi-phase machines offer advantages when compared to the three-phase counterpart [5], [6]. These advantages are especially interesting for propulsion applications, like more-electric aircraft [7], EVs [8]–[11], or ship propulsion [12], [13]. Compared to the three-phase counterparts, multi-phase machines offer additional degrees of freedom which can be used for fault-tolerant operation [14]–[16], multi-motor series/parallel-connected drive [17], [18] or torque density enhancement [19]–[22]. The ability of fault-tolerant operation is very important in life-dependent applications, such as EVs and HEVs. Notice also that the low inverter DC link voltage provided by the battery imposes high-phase currents in the electric drive, making multi-phase drives especially suitable in the EV propulsion systems by means of the lower per-phase current. And torque density enhancement ability has shown good prospects for the integration of EV propulsion system. It is based on the principle that the interaction of the spatial and electrical harmonics of the same order can generate a component rotating at fundamental frequency. This component is contributes to positive torque, also providing a flattened magnetic motive force (MMF) shape which is useful to avoid saturation and improve iron utilization.

One of the most interesting multi-phase machines for these applications is the five-phase machine. For five-phase permanent magnetic synchronous motor (FPMSM), this enhancement benefits from the third harmonic air-gap flux, which effectively increases the magnitude of the fundamental flux density, without saturating the machine iron, and the third harmonic component also contributes to positive torque. The quasi-trapezoidal air-gap flux density due to the combination of the two fluxes is essential for torque density enhancement, assuming the same peak air-gap flux density and phase current amplitude. To get this aim, the stator should be wound such that the induced back EMF is quasi-trapezoidal and is supplied by combined sinusoidal and third harmonic current. So this type of FPMSM is called third harmonic injection FPMSM (THI-FPMSM), which benefits from the controllability of PMSM and high torque density of BLDC [23].

Although the principle how THI-FPMSM works has been
discussed in [24], little literature has been presented relating
to determination of windings, permanent magnet steel shapes
or third harmonic current injection ratio. This paper aims at
improving torque density. Theory basis that third harmonic
current generates positive and constant torque is given via
deduced mathematic model. The design method for a prototype
with quasi-rectangular back EMF and optimization method
for third harmonic injection are described. Simulation and
experimental results verify the torque density of THI-FPMSM
can be enhanced greatly by third harmonic injection.

II. MODELING OF THI-FPMSM

The drive system which consists of a five-phase voltage
source inverter (VSI) and THI-FPMSM is shown as Fig. 1.
The stator winding of THI-FPMSM is connected in star and
the neutral point is isolated.

According to amplitude invariant criterion and extended
symmetrical component method, the transformation from
phase-variable model under natural coordinate system to syn-
chronization rotating coordinate system can be deduced as:

\[
T(\theta) = \frac{2}{5} \begin{bmatrix}
    c(\theta_1) & c(\theta_2) & c(\theta_3) & c(\theta_4) & c(\theta_5) \\
    -s(\theta_1) & -s(\theta_2) & -s(\theta_3) & -s(\theta_4) & -s(\theta_5) \\
    c(3\theta_1) & c(3\theta_2) & c(3\theta_3) & c(3\theta_4) & c(3\theta_5) \\
    -s(3\theta_1) & -s(3\theta_2) & -s(3\theta_3) & -s(3\theta_4) & -s(3\theta_5) \\
    1/2 & 1/2 & 1/2 & 1/2 & 1/2
\end{bmatrix}
\]

where, \(c(\cdot)\) and \(s(\cdot)\) indicate cosine and sine function, respectively. \(\theta_1 = \theta - i\alpha\), \(\theta\) denotes angular displacement of rotor, and \(\alpha = 2\pi/5\).

Using the transformation of (1), the \((10n\pm1)\)th \((n=1, 2, 3, \ldots)\) harmonics and \((10n\pm3)\)th harmonics of five-phase
variables are mapped into two orthogonal subspaces which
are referred as \(d_1-q_1\) and \(d_3-q_3\) from now on. And the zero-
sequence component is restrained to zero for star-connected
stator winding with isolated neutral point. The \(d_1-q_1\) and \(d_3-q_3\) subspaces synchronously rotate at the frequency of \(\omega\) and \(3\omega\), respectively. So only the fundamental and third harmonics
of five-phase variables can be regarded as DC components,
which can contribute to the torque positively [5], [6], [22].
So the mathematic model of THI-FPMSM under orthogonal
rotating coordinate system is:

\[
\begin{bmatrix}
    u_{d1} \\
    u_{q1} \\
    u_{d3} \\
    u_{q3}
\end{bmatrix}
= \begin{bmatrix}
    i_{d1} \\
    i_{q1} \\
    i_{d3} \\
    i_{q3}
\end{bmatrix}
+ \begin{bmatrix}
    L_{d1} & 0 & L_{13} & 0 \\
    0 & L_{q1} & 0 & L_{13} \\
    L_{13} & 0 & L_{d3} & 0 \\
    0 & 0 & 0 & L_{q3}
\end{bmatrix}
\begin{bmatrix}
    i_{d1} \\
    i_{q1} \\
    i_{d3} \\
    i_{q3}
\end{bmatrix}
+ \omega \left[\begin{array}{c}
    -L_{d1}q_1 - L_{13}q_3 \\
    L_{d1}q_1 + L_{13}q_3 \\
    -3L_{13}q_3 \\
    3L_{13}q_3
\end{array}\right]
+ \omega \left[\begin{array}{c}
    0 \\
    0 \\
    -3\psi_{m2} \\
    3\psi_{m2}
\end{array}\right]
\]

where, \(u_{d1}(\cdot), u_{q1}(\cdot), i_{d1}(\cdot), i_{q1}(\cdot), L_{d1}(\cdot), L_{q1}(\cdot)\) are stator
voltage, current and inductance in \(d_1, q_1, d_3, q_3\) axes respectively. \(L_{13}\) is mutual inductance of fundamental and
third harmonic components induced by influence of salient-pole. \(r_s\) is stator resistance. \(\psi_{m1}\) and \(\psi_{m3}\) are fundamental
and third harmonic components of stator flux linkage due to the
permanent magnet. \(p\) is the differential operator. For surface-
mounted FPMSM, the air-gap can be considered as uniform,
so \(L_{d1} = L_{q1} = L_1, L_{d3} = L_{q3} = L_3\) and \(L_{13} = 0, d_1 - q_1\) and
\(d_3 - q_3\) subspaces decouple thoroughly.

For \(i_{d1} = i_{d3} = 0\), the electromagnetic torque can be written as

\[
T = \frac{5P}{2} (\psi_{m1}q_1 + 3\psi_{m3}q_3) = K_{T1}q_1 + K_{T3}q_3
\]

where, \(P\) denotes the number of pole pairs, \(K_T = 5\psi_{m1}/2\) and
\(K_{T3} = 15\psi_{m3}/2\) are torque coefficients of fundamental
and third harmonic current respectively.

From (3), it can be seen distinctly, third harmonic compo-
nent can contribute to the torque positively. That is intrinsic
argument for enhancing torque density of THI-FPMSM.

III. OPTIMAL DESIGN FOR THI-FPMSM DRIVE

The induced back electromotive force (EMF) of the motor
is a function of the stator winding distribution and air-gap
flux, where air-gap flux distribution depends on the magnet
dimensions and stator structure. Therefore, it is of significant
importance that the proper number of stator slots and winding
distribution is chosen.

A. Prototype machine design

This paper designs an eight-pole FPMSM with five identical
quasi-concentrated windings as sketched in Fig. 2, only wind-
ings of phase A and B are given here for succinctness. The
term of quasi-concentrated is used since each phase winding
consists of six fractional slot concentrated winding cells, which
are in serial connection. The magnetic steels of rotor are
beveled to inject low-order harmonic and constrain high-order
harmonic in the air-gap magnetic field.

Winding function of Phase A is shown in Fig. 3, by Fourier
analysis, the low order harmonic components can be expressed
as:

\[ n_A(\phi) = N \sum_{i=1}^{5} k_{wi} \cos(i\phi) \]  (4)

where, \( N \) denotes the number of turns per phase per slot, \( k_{wi} \) is the winding factor of \( i^{th} \) harmonic, and \( k_{w1} = 1.24, k_{w2} = -0.12, k_{w3} = -0.33, k_{w4} = 0.09, k_{w5} = 0.11 \). \( \phi \) represents the angle counterclockwise from the positive A-phase magnetic axis. The even-order components are introduced by the form of fractional slots, but don’t effect energy conversion when air-gap magnetic field is symmetrically distributed.

The air-gap flux density function due to the permanent magnet of rotor can be approximated as follows:

\[ b_m = B_1 \cos(\phi - \theta) + B_3 \cos(3(\phi - \theta)) + B_5 \cos(5(\phi - \theta)) \]  (5)

where, \( B_1, B_3 \) and \( B_5 \) are fundamental, third and fifth harmonic components of flux density, and for square wave air-gap flux density whose amplitude is \( B \), \( B_3 = -B/3 \), \( B_5 = B/5 \) and \( B_1 = 4B/\pi \).

From (4) and (5) the flux linkage of the A-phase winding due to the permanent magnet can be obtained as:

\[ \lambda_{Am} = rl \int_{-\pi/2}^{\pi/2} (N_A(\phi) b_m) d\phi \]

\[ = 2rlN \sum_{i=1}^{5} k_{wi} B_i \cos(i\theta) \quad i = 1, 3, 5 \]  (6)

where, \( r \) is the air-gap radius, and \( l \) is stator length, respectively.

For the star-connected winding, fifth harmonics eliminate each other, so (6) can be simplified as:

\[ \lambda_{Am} = 2rlN (k_{w1} B_1 \cos(\theta) + k_{w3} B_3 \cos(3\theta)) \]  (7)

From (3),

\[ K_{T1} = \frac{5P}{2} \lambda_{m1} = 5P k_{w1} l N B_1 \]

\[ K_{T3} = \frac{15P}{2} \lambda_{m3} = 15P k_{w3} l N B_3 \]  (8)

For the prototype of this paper, \( r = 49 \text{mm}, l = 105 \text{mm}, B = 0.9 \text{T and } N = 94 \),

\[ K_{T1} = 13.7; \quad K_{T3} = 3.66; \]  (9)

**B. Third harmonic current injection ratio optimization**

Considering the phase current amplitude is restrained by the power capability of inverter, the third harmonic current injection ratio \( k_3 \) must be optimized to maximize the torque density of THI-FPMSM, according to the values of \( K_{T1} \) and \( K_{T3} \). The phase current of phase A can be given as

\[ i_{A1} = I_1 \sin(\omega t + \phi_1) \]  (10)

\[ i_{A3} = k_3 I_1 \sin(3\omega t + \phi_3) \]  (11)

where, \( I_1 \) and \( I_3 = k_3 I_1 \) are the amplitude values and \( \phi_1 \) and \( \phi_3 \) are the phase angles of the fundamental and third harmonic components respectively.

Assuming the peak value of phase A is \( I_A = 1 \), (10) and (11) must satisfy:

\[ \max(i_{A1} + i_{A3}) \leq I_A \]  (12)

According to the theoretical values of \( K_{T1} = 13.7 \) and \( K_{T3} = 3.66 \), optimal result can be solved by numerical method. The maximum output torque (16.5746 N-m) will be derived when \( \phi_1 = \phi_3 \) and \( k_3 = 0.1928 \) as shown in Fig. 4. It’s obvious that electromagnetic torque is enhanced by 21%, after injecting third harmonic current with optimum ratio.

**IV. DUAL-PLANE VECTOR CONTROL METHOD AND DIGITAL IMPLEMENTATION**

**A. Dual-plane vector control method**

From THI-FPMSM modeling of (2) and (3), it is found that \( d_1-q_1 \) and \( d_3-q_3 \) subspaces behave as two independent fictitious two-phase machines, sharing the same magnetic structure and mechanical coupling. To obtain the optimum torque density, the optimal design method is proposed in this paper. What’s more, fundamental and third harmonic current should be controlled to have the same initial phase and proper ratio of amplitude.

In vector control system of three-phase machines, the current loop is formed of two separate current controllers with PI behavior for the field-forming component \( i_d \) (comparable with the field current of the DC motor) and the torque-forming component \( i_q \) (comparable with the armature current of the DC motor). Similarly, the dual-plane vector control system for THI-FPMSM consists of two pairs of current

**Fig. 3. Normalized winding function and its harmonic synthesis.**

**Fig. 4. Optimal current injection ratio for third harmonic injection ratio.**
controllers with PI behavior also, each of which operate in the synchronous reference frames determined with the frequency of themselves respectively as shown in Fig. 5. The addition of the second current controller pair enables operation of the drive with expected combination of fundamental and third harmonic currents. The magnitude and rotating speed of each harmonic component are independently controlled. The current reference comes form outer speed loop, and is distributed to two inner current loops in a certain proportion determined by injection ratio $k_3$.

B. Digital implement based on FPGA

Dual-plane vector control is much more complex than traditional vector control of three-phase machine, and demands more computing resource. To overcome degradation of system performance caused by time delay of software processing, the implementation of the proposed controller has been accomplished using FPGA technology. In recent years, FPGA-based hardware implementation technology has been used to motor control systems due to the advantages of their programmable hard-wired feature, fast time-to-market and reusable IP (Intellectual Property) cores. Besides, execution time can be dramatically reduced by designing dedicated parallel architectures by means of hardware mode, allowing FPGA-based controllers to reach the level of performance of their analog counterparts without their drawbacks (parameter drifts, lack of flexibility) [25]–[27].

Fig. 6 shows the internal architecture of the FPGA-based dual-plane vector control IC. The developed control IC consists of four major submodules: a speed controller, two current controllers for fundamental and third harmonic respectively and a five-phase PWM modulator. The data transfers between these elementary operators are managed by a global time sequence controller, which is a finite-state machine (FSM) and synchronized with the system clock signal. The control unit of a module is always activated via a StartPulse signal. When the computation time process is over, an EndPulse element signal indicates to the global controller that the data outputs of the module are ready to be used.

As the execution time of submodules is knowable, it is feasible to control the starting time of sampling circuits, such as ADC and RDC, to make data more immediate. Besides, some particular modules can be triggered in parallel according to the direction and setup time of data, to shorten the total computation time. The timing sequence of current control loop is given in Fig. 7 as the core of digital controller in Fig. 6. Clark, Cordic and PI submodules are triggered in parallel respectively, and the start time of RDC submodule is set up properly to guarantee that the rotor position information is addressable precisely at the moment Park transformation triggered.

All the submodules are described in Verilog HDL and synthesized using the Xilinx project navigator and support tools. The synthesized Verilog HDL source code is placed, routed and mapped. Finally, a bit file is created. This file is downloaded into the Xilinx XC3S1200E FPGA to interface with peripheral devices. The resources utilization and execu-
TABLE I
RESOURCES AND EXECUTION TIME OF SUBMODULES

<table>
<thead>
<tr>
<th>Submodules</th>
<th>Multiplier</th>
<th>Slices</th>
<th>LUTs</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADC</td>
<td>0</td>
<td>105</td>
<td>42</td>
<td>5.2us</td>
</tr>
<tr>
<td>5/4 Trans.</td>
<td>1</td>
<td>153</td>
<td>175</td>
<td>0.6us</td>
</tr>
<tr>
<td>Cordic Trans.</td>
<td>0</td>
<td>205</td>
<td>404</td>
<td>0.6us</td>
</tr>
<tr>
<td>4/5 Trans.</td>
<td>1</td>
<td>183</td>
<td>245</td>
<td>0.48us</td>
</tr>
<tr>
<td>PI Regulator</td>
<td>2</td>
<td>85</td>
<td>162</td>
<td>0.4us</td>
</tr>
<tr>
<td>RDC</td>
<td>1</td>
<td>76</td>
<td>92</td>
<td>4us</td>
</tr>
</tbody>
</table>

TABLE II
PARAMETERS OF THI-FPMSM PROTOTYPE

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_s$ (Ω)</td>
<td>17.5</td>
<td>$P$</td>
<td>4</td>
</tr>
<tr>
<td>$L_{s1}$ (mH)</td>
<td>44</td>
<td>$K_{T1}$ (N·m/A)</td>
<td>12.7/14.1/14.16</td>
</tr>
<tr>
<td>$L_{s3}$ (mH)</td>
<td>15</td>
<td>$K_{T3}$ (N·m/A)</td>
<td>3.66/3.43/3.48</td>
</tr>
</tbody>
</table>

The execution time of submodules are shown in TABLE I. The system clock is 25MHz. Execution time is shortened dramatically by adopting single-clock embedded hardware multiplier.

V. RESULTS

A. Two-dimension finite element analysis

Finite element method is employed to analyze the magnetic field distribution of the proposed THI-FPMSM. Fig. 8 shows the cross section and flux density plots only with the permanent magnet excitation. The flux density is approximately equal under each pole and distribute as quasi-trapezoid. That means the saturation level is identical in most of the iron core. This characteristic is quite helpful for boost the utilization of iron core.

The values of $K_{T1}$ and $K_{T3}$ obtained by infinite element method are listed in TABLE II. There is some error between simulation results and theoretical calculated values in (9). This is caused by the assumption of square wave air-gap flux density in (5), while the actual flux density is trapezoidal as shown in Fig. 8.

The third harmonic content is smaller, and the optimum injection ratio is a little different, to be $k_3 = 0.1895$. Maintaining the amplitude of phase current to be 1A and phase difference between fundamental and third harmonic current to be zero, the output torque of prototype under different injection ratio is given in Fig. 9. Only when the third harmonic injection ratio is set to the optimum ratio, the output torque will reach its maximum value.

B. Experiment results

To verify the proposed torque density optimum algorithm for THI-FPMSM, a VSI-driven power conversion system was set up (Fig. 10) and tested. The parameters of the prototype motor are listed in TABLE II, where calculated, emulational and experimental values of $K_{T1}$ and $K_{T3}$ are given successively.

The switching module used in the five-phase PWM inverter was a 600V, 20A rated IGBT manufactured by Infineon Co. During the test, the dead time was set at 2us to prevent arm shorts. The phase currents were measured by a LA55-P LEM Co. current transducer. The measured phase currents were converted to digital signals, by a 16bit A/D converter, and were sent up to a laptop by RS232 communication port of FPGA controller for further analysis.

The measured back-EMF of prototype is illustrated in Fig. 11.

In Fig. 11(a), the quasi-trapezoidal back-EMF is the phase voltage between terminal and neutral point as shown in Fig. 1. It consists of third and fifth harmonic mainly. In Fig. 11(b), the fifth harmonic is eliminated by a star-connected 10 kΩ resistance load. It can be calculated that the third harmonic component is 24.3% of fundamental component.

Fig. 12 shows the currents of phase A and B and loci of current vector under $\alpha_1$, $\beta_1$ and $\alpha_3$, $\beta_3$ stationary coordinates when $k_3 = 0$ and $k_3 = 0.1895$.

Under this two situations, the amplitude of currents is identical, 1A, nevertheless the output torque increases from 14.04 N·m to 16.9 N·m, about 20.4%. In Fig. 12(a), there
Fig. 9. Relationship of third harmonic injection ratio with electromagnetic torque.

Fig. 10. Photograph of the experimental setup.

Fig. 11. Measured back-EMF of THI-FPMSM prototype.

Fig. 12. Waveforms and loci of current under $\alpha_1-\beta_1$ and $\alpha_3-\beta_3$ stationary coordinates when (a) $k_3 = 0$ (b) $k_3 = 0.1895$.

is no injection of third harmonic current, phase currents are sinusoidal. Fundamental current maps into $\alpha_1-\beta_1$ stationary coordinate as a circular locus whose amplitude is 1A, while the third harmonic current vector degenerates to a point. In Fig. 12(b), third harmonic current is injected at the ratio of $k_3 = 0.1895$, phase current appears as a saddle-shaped. The amplitude of fundamental current vector increases to 1.16A, and 0.22A for third harmonic current vector. Obviously, torque density can be enhanced by injecting the third harmonic current by proper ratio and phase without increasing current amplitude. The enhancement of torque density benefits from the increase of fundamental current and addition of third harmonic current.

VI. CONCLUSION

In this paper, the design method for a prototype with quasi-rectangular back EMF and optimization method for third harmonic are described to enhance the torque density of FPMSM. On the base of multi-dimension characteristic, the interaction of third harmonic field and current can produce constant and positive torque, which is added to the torque generated by fundamental component. By injecting third harmonic field into the sinusoidal rotor field, the peak of fundamental field is clipped. The obtained air-gap flux distributes as quasi-trapezoid and is quite helpful for boost the utilization of iron core. Adopting the presented dual-plane vector control method, fundamental and third harmonic components can be controlled effectively and independently to have the same initial phase and proper ratio of amplitude. Under the constrain of identical amplitude and injection of third harmonic current with optimum ratio, the amplitude of fundamental current can be increased by 15%, and so it is to the corresponding torque component. Furthermore, the third harmonic component also produces 5% increase of torque. So the fundamental and third harmonic components enhance torque density by 20 in together, comparing with sinusoidal situation.
REFERENCES


