Approximate Equivalent-Circuit Modeling and Analysis of Type-II Resonant Immittance Converters

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Abstract
Resonant immittance converter (RIC) topologies can transform a current source into a voltage source (Type-I RICs) and vice versa (Type-II RICs), thereby making them suitable for many power electronics applications. RICs are operated at a fixed frequency where the resonant immittance network (RIN) exhibits immittance conversion characteristics. It is observed that the low-frequency response of Type-II RINs is relatively flat and that the state variables associated with Type-II RINs affect the response only at the high frequencies in the vicinity of the switching frequency. The overall response of a Type-II RIC is thus dominated by the filter response, which is particularly important for the controller design. Therefore, an approximate equivalent circuit model and a small-signal model of Type-II RICs are proposed in this paper, neglecting the high-frequency response of Type-II RINs. While the proposed models greatly simplify and speed-up the analysis, it adequately predicts the open-loop transient and small-signal ac behavior of Type-II RICs. The validity of the proposed models is confirmed by comparisons of their results with those obtained from a cycle-by-cycle simulation and with an experimental prototype.

Key words: Current supplies, Equivalent Circuit, Immittance Converters, Resonant power conversion, Soft-switching

I. INTRODUCTION
Resonant converters (RCs) have been a potential candidate in many power electronics applications involving high-frequency power processing such as dc-dc converters [1], high voltage power supplies [2], power factor correction [3], electronic lamp ballasts [4], induction heating [5], etc. The merits of RCs include soft switching, high frequency operation, high efficiency, small size, gainful utilization of the circuit parasitic components, etc. Some RCs also inherently have unique and useful characteristics that may not be exhibited by the other classes of power electronics converters.

An immittance converter (IC), an abbreviation of impedance-admittance converter, is a two-port network, in which the input impedance is proportional to the output admittance connected across the output terminals [6]. In an IC, the output current is proportional to the input voltage and the output voltage is proportional to the input current. This characteristic feature of an IC enables the conversion of a constant voltage (CV) source to constant current (CC) source and vice versa, which is useful in many power electronics applications.

The immittance conversion characteristics (ICC) of a quarter-wave distributed constant line has been explored in the past for lamp ballasts [7], induction heating [8], and corona and plasma discharge applications [9]-[11] operating in the megahertz-range. Since the length of the distributed constant line becomes prohibitively long for power converters operating in kilohertz-range, some lumped-element IC topologies based on transmission line approximation, emulated using discrete inductors and capacitors, have been studied and reported [6]. However, only two of the reported IC topologies are suitable for power electronics applications, wherein the exciting voltage is commonly a square-wave, which is conveniently obtained by operating power semiconductor switches at a high frequency.

Therefore, a separate class of RCs, referred to as resonant immittance converters (RICs), has been recently reported in [12], in which the resonant network (RN) exhibits ICC. In all, 24 RICs have been identified with three and four reactive elements, out of which 9 topologies (termed Type-I RICs) are suitable for CC to CV conversion and 15 topologies (termed Type-II RICs) are suitable for CV to CC conversion.

LCL-T RC is the simplest of the Type-II RICs. A detailed analysis and design of the LCL-T RC as a constant-current (CC) power supply is reported in [13]. The addition of clamp diodes on the primary side of the converter results in an in-built constant-current – constant-voltage (CC-CV)
characteristic [14] making the converter suitable for capacitor charging applications [15]. The presence of a transformer winding capacitance changes the LCL-T RC into a fourth-order LC-LC topology, which has been shown to behave as a Type-II RIC [16]. The various modes of operation and issues in the design of a LCL-T RC with asymmetrical duty-cycle control are described in [17]. LCL-T RC topologies are also reported for various applications such as radio frequency inductive discharges [18], inverters for photovoltaic systems [19] and non-contact energy transmission systems [20]. The application of a Type-I RIC (CLC-π network) in a high-voltage dc transmission link is reported in [21]. The characteristics of a five-element RIC topology are studied in [22]. The analysis and design of a five-element T-type Type-II RIC topology has been reported in [23].

The ability of a Type-II RIC to convert a voltage source into a current source is very useful in a variety of applications wherein a CC source is either inherently required or can be advantageously applied. These application areas include capacitor charging power supplies, battery chargers, laser diode drivers, power supplies for electromagnets, power supplies for electro-chemical processes, high voltage power supplies, CC and CC-CV power supplies for electric arc welding, illumination systems, etc.

While a methodology for the steady-state analysis and design of RICs is well established and demonstrated [12] – [23], transient and small-signal analysis have not been reported. The dynamic response of a converter during transients must be considered while formulating its design procedure. Similarly, small-signal analysis is necessary to derive various transfer functions and to design the feedback controller.

Various methods have been reported in the literature [24]-[30] for the transient and small-signal analysis of RCs. These methods can also be applied to RICs since they are a special type of RCs. However, the objective of this paper is to propose a simpler and approximate equivalent circuit model of the Type-II RIC family that predicts the averaged response of the terminal voltages and currents under large-signal variations in the operating conditions. It can be seen that the overall response is dominated by the state variables associated with the output filter, which is therefore particularly important for the controller design. The effect of the state variables associated with the rest of the converter is neglected, which affects only the high frequency response of the converter [24]-[30]. A small-signal model is subsequently derived by applying the perturbation and linearization to the average model.

The derived models greatly simplify and speed-up the analysis either via simple analytical treatment or by using the models in circuit simulation tools. These models are validated by comparing their results with those obtained from cycle-by-cycle simulations and with an experimental prototype.

II. TYPE-II RICS

A block diagram of a two-port IC is shown in Fig. 1, in which the voltages and currents at the input and output ports (represented by $V_1$, $I_1$, $V_2$ and $I_2$, respectively) are related as given by the following expression:

$$
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix} = \begin{bmatrix}
0 & \pm jZ_n \\
\pm j(1/Z_n) & 0
\end{bmatrix} \begin{bmatrix}
V_2 \\
-I_2
\end{bmatrix}
$$

(1)

where $Z_n$ is the characteristic impedance of the circuit.

An electrical networks composed of lumped reactive elements (inductors and capacitors) exhibiting ICC and having either a low-pass or band-pass frequency response, (consistent with the definition of a RC) has been termed a resonant immittance network (RIN) [12]. When a RIN is used in place of an ordinary RN in a RC, the resulting power converter topology
is termed a RIC. A generic block diagram for a voltage-source or Type-II dc-dc RIC can be drawn as shown in Fig. 2. An inverter (full-bridge, half-bridge or push-pull) excites the Type-II RIN with a high frequency square-wave voltage waveform. Figure 3 summarizes the 15 Type-II RINs, with three and four reactive elements, reported in [12]. In the nomenclature, T stands for T-type RIN, P stands for π-type RIN and LA stands for ladder-type RIN. A transformer at the output of a RIC is used to step-up or down the voltage according to the requirements and to provide galvanic isolation. A rectifier and filter are used to get the dc output.

RICs exhibit ICC only if various reactances obey certain conditions, which are satisfied only at a particular frequency of operation and when the values of the reactive elements are suitably chosen [12]. These circuits have different properties at other frequencies. Therefore, as opposed to other RCs, RICs can not be controlled by varying the switching frequency or other frequencies. Therefore, as opposed to other RCs, RICs are suitably chosen [12]. These circuits have different properties at other frequencies. Therefore, as opposed to other RCs, RICs can not be controlled by varying the switching frequency or other frequencies. Therefore, as opposed to other RCs, RICs are suitably chosen [12]. These circuits have different properties at other frequencies.

III. APPROXIMATE EQUIVALENT CIRCUIT MODEL OF TYPE-II RICS

The use of equivalent circuits is an intuitive approach which allows the well-known techniques of circuit analysis to be employed. Averaging has been one of the most important tools for power converter design and analysis since it adequately describes the functional relationships between sources, outputs and control parameters, while ignoring ripple.

A. Approximate Equivalent Circuit Model of Type-II RINs

The RIN shown in the block diagram of Fig. 2 can be any of the three- and four-element networks, shown in Fig. 3, for a Type-II RIC.

To study the small-signal behavior of Type-II RINs, the envelop simulation method using the SPICE-compatible models developed in [29], [30] is used. The designed component values in the experimental prototypes of topology T1 in [13], [14], [17], topology LA2 in [16] and topology T3 in [12] are used to obtain the input voltage-to-output current transfer functions from the simulation. The component values used for the simulation are summarized in table 1. Since only the small-signal behavior of a RIN is being investigated, the transformer, rectifier, filter and load resistance connected at the output port of the RIN are replaced by an equivalent ac resistance \( R_{ac} \) whose value corresponds to the rated maximum load resistance in the respective prototype. This is also mentioned in table 1. Figure 5 shows the gain and the phase response of the input voltage-to-output current transfer functions in these circuits. It can clearly be seen from the plots of Fig. 5 that the low-frequency response of these converters is...
TABLE I

<table>
<thead>
<tr>
<th>Designed Values of Component in Various RINs</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 (μH)</td>
</tr>
<tr>
<td>L2 (μH)</td>
</tr>
<tr>
<td>C1 (μF)</td>
</tr>
<tr>
<td>C2 (μF)</td>
</tr>
<tr>
<td>Rneo (Ω)</td>
</tr>
<tr>
<td>f (kHz)</td>
</tr>
</tbody>
</table>

B. Approximate Equivalent Circuit Model of Type-II RICs

While a Type-II RIN, can be represented by the equivalent circuit in Fig. 6, the nonlinear elements of a RIC (the inverter and rectifier at the input and output side, respectively) do not directly allow a simpler equivalent circuit representation of a complete RIC. If the shaded portion of the block diagram in Fig. 2 can be represented using an equivalent circuit describing the terminal relationship in terms of the average values of \( v_d, i_d \) and \( v_o, i_o \), the task of the circuit analysis would greatly be simplified since the rest of the circuit elements are linear.

The average values of \( i_d \) and \( i_o \) are given by:

\[
\langle i_d(t) \rangle_{T_1} = \frac{2\sqrt{2}I_1}{\pi}, \quad \langle i_o(t) \rangle_{T_1} = \frac{2\sqrt{2}I_2}{n\pi}
\]

where \( \langle x(t) \rangle_{T_1} \) denotes the average of \( x(t) \) over an interval of one period \( T_1 \):

\[
\langle x(t) \rangle_{T_1} = \frac{1}{T_1} \int_{t}^{t+T_1} x(\tau) d\tau
\]

\( I_1 \) and \( I_2 \) are the rms values of \( i_1 \) and \( i_2 \), respectively. Similarly, the rms values of the fundamental components of \( V_1 \) and \( V_2 \), \( v_d \) and \( v_o \) respectively are given in terms of the dc input and output voltage \( V_d \) and \( V_o \), respectively as:

\[
V_1 = \frac{2\sqrt{2}}{\pi} \langle v_d(t) \rangle_{T_1}, \quad V_2 = \frac{2\sqrt{2}}{n\pi} \langle v_o(t) \rangle_{T_1}
\]

By substituting (2) and (4) into (1) and neglecting the phasor terms for the dc quantities, the following is obtained:

\[
\begin{bmatrix}
\langle v_d(t) \rangle_{T_1} \\
\langle i_d(t) \rangle_{T_1}
\end{bmatrix} = \begin{bmatrix}
0 & \rho \\
\rho^{-1} & 0
\end{bmatrix} \begin{bmatrix}
\langle v_o(t) \rangle_{T_1} \\
\langle i_o(t) \rangle_{T_1}
\end{bmatrix}
\]

where:

\[
\rho = \frac{\pi^2}{8} nZ_o
\]

Thus the shaded portion of Fig. 2, whose terminal voltages and currents are described by (5), can be represented by the equivalent circuit shown in Fig. 7(a). By connecting the source and load at the input and output terminals of the equivalent circuit, an equivalent circuit model of a Type-II RIC can be constructed, as shown in Fig. 7(b). This equivalent circuit model can be easily analyzed using classical circuit theory to determine the steady-state characteristics as well as the transient response to track averaged large-scale changes in the terminal voltages and currents as the source or load undergo changes. Alternatively, the equivalent circuit can be directly simulated with various circuit simulation tools to directly obtain the averaged terminal response.

IV. SMALL-SIGNAL AC MODEL

To construct a small signal ac model at a quiescent operating point it is assumed that the input voltage is equal to some given quiescent value \( V_d \) plus some superimposed small
ac variation \( \tilde{v}_d(t) \). As a result, the following is obtained:

\[
\langle v_d(t) \rangle_{T_1} = V_d + \tilde{v}_d(t) \tag{7}
\]

In response to this input, after the transients have subsided, the averaged input current \( \langle i_d(t) \rangle_{T_1} \), the averaged rectified output current \( \langle i_r(t) \rangle_{T_1} \) and the averaged output voltage \( \langle v_o(t) \rangle_{T_1} \) will be equal to the corresponding quiescent values \( I_d \), \( I_r \) and \( V_o \) plus some superimposed small ac variations, \( \tilde{i}_d(t) \), \( \tilde{i}_r(t) \) and \( \tilde{v}_o(t) \), respectively. As a result, the following is obtained:

\[
\langle i_d(t) \rangle_{T_1} = I_d + \tilde{i}_d(t) \tag{8}
\]

\[
\langle i_r(t) \rangle_{T_1} = I_r + \tilde{i}_r(t) \tag{9}
\]

\[
\langle v_o(t) \rangle_{T_1} = V_o + \tilde{v}_o(t) \tag{10}
\]

Substituting (7) – (10) into (5) and simplifying, the following is obtained:

\[
\begin{bmatrix}
\tilde{v}_d(t) \\
\tilde{i}_d(t) 
\end{bmatrix} =
\begin{bmatrix}
0 & \rho^{-1} \\
\rho & 0
\end{bmatrix}
\begin{bmatrix}
\tilde{v}_d(t) \\
\tilde{i}_d(t)
\end{bmatrix}
\tag{11}
\]

This relationship in the small-signal quantities, being similar to (5), can be represented by a similar equivalent circuit, as shown in Fig. 8. Subsequently, the line-to-output transfer functions and the input impedance of a Type-II RIC can be readily derived as:

\[
G_1(s) = \frac{\tilde{i}_d(s)}{\tilde{v}_d(s)} = \frac{1}{\rho^2} \frac{1}{\rho^2 + sR_LC_f} \tag{12}
\]

\[
G_2(s) = \frac{\tilde{v}_d(s)}{\tilde{i}_d(s)} = \frac{R_L}{\rho} \frac{1}{1 + sR_LC_f} \tag{13}
\]

\[
Z_m(s) = \frac{\tilde{v}_d(s)}{\tilde{i}_d(s)} = \frac{\rho^2}{R_L} \left(1 + sR_LC_f\right) \tag{14}
\]

V. SIMULATION AND EXPERIMENTAL RESULTS

The equivalent circuit models in Fig. 7 and Fig. 8 can be easily analyzed to determine the steady-state characteristics, the transient response to track the averaged large-scale changes in the terminal voltages and currents as the source or load undergoes a change, the derivation of the small-signal transfer function and the visualization of the frequency response. Alternatively, the equivalent circuit can be directly simulated with various circuit simulation tools.

To validate the models, they are implemented in OrCad PSpice and the results are compared with those obtained from a cycle-by-cycle simulation as well as an experimental prototype of the newly identified RIC topology T3 [12]. Fig. 9 shows the circuit diagram of a half-bridge T3 RIC. The converter operates with 220 V of dc input and provides 1 A of dc output current with a maximum load resistance of 250 Ω. The values of the reactive components of the RIN are already listed in table 1.
Fig. 11. Response of Topology T3 to step change in input dc voltage [(a), (b), (c)] and step change in $R_L$. [(d), (e), (f)]. (a), (d): Predicted average response from the equivalent circuit model. [Top trace: $v_{Tdv}$, dotted trace: $i_{Tri}$ and continuous trace: $i_{Toi}$]. (b), (e): Results of cycle-by-cycle simulation. (c), (f): Experimental results. In parts (b) and (c), the waveform at the top shows $v_d(t)$, the envelop shows $i_r(t)$ and the trace at the bottom shows $i_o(t)$.

The transformer turns ratio is 1:2.77 (a 9-turn primary and a 25-turn secondary on an EE 42.21.20 ferrite core). Fig. 10 shows a photograph of the experimental setup which is operated in the open loop to check the open-loop transient response and the small-signal transfer function.

The filter capacitor $C_f$ is 47 $\mu$F. When $Z_n \equiv \sqrt{L_1/C_1} = 32.09$ and $n = 2.77$, $\rho$ is calculated from (6) to be 109.66. Fig. 11(a), (b) and (c) shows the response of the converter operating with $R_L = 94$ $\Omega$, when the input dc voltage ($2V_d$) undergoes a step change from 80 V to 120 V and vice versa. Similarly, Fig. 11(d), (e) and (f) shows the response of the converter with a step change in $R_L$ from 47 $\Omega$ to 94 $\Omega$ and vice versa. The following observations are made:

1. The results of the equivalent circuit model are in excellent agreement with those obtained from the cycle-by-cycle simulation and the experiment, thereby confirming the validity of the proposed model.

2. In Fig. 11(a), (b) and (c), the waveform of $i_r(t)$ (or, $\overline{i_r}_{T_3}$) instantaneously follows the applied step variation in $v_d(t)$ (or, $\overline{v_d}_{T_3}$). However, the response of $i_o(t)$ (or, $\overline{i_o}_{T_3}$) is governed by the output filter. Similarly, in Fig. 11(d), (e) and (f), $i_o(t)$ (or, $\overline{i_o}_{T_3}$) is constant under the transient condition following the step change in $R_L$. The waveform of $i_d(t)$ (or, $\overline{i_d}_{T_3}$) shows the transient undershoot and overshoot due to an additional current absorbed or delivered by $C_f$ to maintain the charge balance with varying $v_o(t)$ (or, $\overline{v_o}_{T_3}$) as $R_L$ varies.

The proposed equivalent circuit model simplifies and speeds up the large signal transient analysis of the converter. Simulation of the model takes a few seconds, which is a huge improvement over the 10s of minutes required for a cycle-by-cycle simulation.

The results in Fig. 12 show that perturbations in $i_r(t)$ (or, $\overline{i_r}_{T_3}$) are in phase with the sinusoidal perturbations in $v_d(t)$ (or, $\overline{v_d}_{T_3}$) over a wide range of frequencies (1 Hz – 1000 Hz). However, perturbations in $i_o(t)$ (or, $\overline{i_o}_{T_3}$) are significantly attenuated and shifted in phase at higher frequencies due to the output filter capacitor.

Finally, the converter’s line-to-output small-signal transfer function given by (12), with $C_f = 47$ $\mu$F, $R_L = 94$ $\Omega$ and $\rho = 109.66$, is found to be in excellent agreement with the experimental observations shown in Fig. 13.

VI. CONCLUSIONS
RICs are operated at a fixed frequency where the RIN exhibits ICC. Following the envelop simulation method, the small-signal behavior of Type-II RINs is studied and it is observed that the low-frequency response is relatively flat and that the state variables associated with these RINs affect the response only at the high frequencies in the vicinity of the switching frequency. Therefore, an approximate equivalent circuit model and a small-signal model of Type-II RICs, neglecting the high-frequency effect of the state variables associated with RINs, is proposed in this paper. While the proposed models greatly simplify and speed-up the analysis, the open-loop transient and small-signal ac behavior of Type-II RICs is also adequately predicted. The validity of the proposed models is confirmed by comparing their results with those obtained from a cycle-by-cycle simulation and with those of an experimental prototype. It is shown that the low-frequency open-loop transient and small-signal ac behavior of Type-II RICs is governed by only the filter while the converter along with the RIN do not contribute to the low-frequency dynamics.

REFERENCES


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