Analysis and Design of a PFC AC–DC Converter with Electrical Isolation

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Abstract

This study presents a single-phase power factor correction AC–DC converter that operates in discontinuous conduction mode. This converter uses the pulse-width modulation technique to achieve almost unity power factor and low total harmonic distortion of input current for universal input voltage (90 V<sub>in</sub> to 264 V<sub>in</sub>) applications. The converter has a simple structure and electrical isolation. The magnetizing-inductor energy of the transformer can be recycled to the output without an additional third winding. The steady-state analysis of voltage gain and boundary operating conditions are discussed in detail. Finally, experimental results are shown to verify the performance of the proposed converter.

Key words: DCM, PFC, PWM, THD

I. INTRODUCTION

DC power sources are widely used in industrial products and consumer electronics, such as battery chargers, DC power supplies, uninterruptible power supplies, inverters, and instruments. Thus, AC–DC power conversion is an important consideration. Diode-bridge or thyristor rectifiers can realize AC–DC power conversion, but such rectifiers will result in power pollution, including pulsating input current, low power factor, and high total harmonic distortion of input current (THD). Several power factor correction (PFC) AC–DC converters have been investigated to address these issues. These converters possess non-isolated and isolated topologies. Non-isolated topologies include the boost [1]-[4], buck [5]-[7], buck-boost [8]-[10], Cuk [11], SEPIC [12], [13], and ZETA types [14]. These converters are operated in continuous conduction mode and discontinuous conduction mode (DCM) for different output-power applications. Isolated topologies include forward [15]-[17] and flyback types [18]-[20]. The forward types achieve low output voltage ripple and high THD. Nevertheless, this converter requires the third winding to recycle the magnetizing-inductor energy of the transformer. The flyback types are shown in Fig. 1(a). This converter achieves high power factor and low THD. Moreover, flyback types have a simple structure and lower cost but have low efficiency because of the transformer leakage inductor. The clamping method is presented in [21], [22] to recycle leakage inductor energy. However, this method makes the power circuit complicated.

We propose a single-phase PFC AC–DC converter, as shown in Fig. 1(b). The proposed converter circuit configuration is very simple. The configuration includes only a set of input filter L<sub>f</sub>-C<sub>f</sub>, a diode-bridge rectifier, a transformer T<sub>n</sub>, an inductor L<sub>n</sub>, an output diode D<sub>n</sub> and an output capacitor C<sub>n</sub>. The proposed converter does not have the transformer leakage inductor issue associated with the flyback converter. Moreover, transformer magnetizing-inductor energy can be recycled to the output without an additional third winding. This converter is operated in DCM by using the pulse–width modulation technique to achieve high power factor and low THD, for universal input-voltage applications.

II. OPERATING PRINCIPLES

The equivalent circuit of the proposed converter is shown in Fig. 2. The transformer is modeled as a magnetizing inductor L<sub>m</sub> and an ideal transformer. Some key waveforms in a half line-source period are shown in Fig. 3. The operating principle is analyzed as 0 < ωt < π, where ω is the line angular frequency, because of the symmetrical characteristics of the single-phase system. Mode 1, [kT<sub>n</sub> t<sub>l</sub>]; S<sub>1</sub> is switched on. The current- flow path is shown in Fig. 4(a). The line source energy is transferred to the magnetizing inductor L<sub>m</sub> of the transformer and the
The energy stored in output capacitor $C_o$ is released to empty during each switching period.

The magnetizing inductor $L_m$ becomes larger than inductor $L_1$ linearly. Thus, the currents $i_{sm}$ and $i_{s1}$ are increased linearly. Given that magnetizing inductor $L_m$ is significantly larger than inductor $L_1$, the magnetizing-inductor current $i_{sm}$ becomes lower than inductor current $i_{s1}$. The energy stored in magnetizing inductor $L_m$ is the residual magnetism of the transformer. The energy stored in output capacitor $C_o$ is discharged to load $R$. This mode ends when $S_1$ is switched off.

**Mode 2, [$i_{s1}$, $i_{s2}$]:** $S_1$ is switched off. The current-flow path is shown in Fig. 4(b). The energy stored in magnetizing inductor $L_m$ and inductor $L_1$ are released to output capacitor $C_o$ and load $R$. The currents $i_{sm}$ and $i_{s1}$ are decreased linearly. This mode ends when the currents $i_{sm}$ and $i_{s1}$ are equal to zero. Therefore, the transformer residual magnetism can be released to empty during each switching period.

**Mode 3, [$i_{s2}$, $(k+1)T_s$]:** $S_1$ remains switched off. The current-flow path is shown in Fig. 4(c). The energies stored in magnetizing inductor $L_m$ and inductor $L_1$ are empty at $t = i_{s2}$. The energy stored in output capacitor $C_o$ is discharged to load $R$. This mode ends when $S_1$ is switched on at the beginning of the next switching period.

**III. STEADY-STATE ANALYSIS**

Given that the single-phase system is symmetrical, the following analysis is discussed for $0 < \alpha t < \pi$. For simplicity, the effect of the input filter is neglected. The line voltage is given by

$$v(t) = V_m \sin \omega t = V_m \sin \alpha t$$  \hspace{1cm} (1)

where $V_m$ and $V_m$ are the root-mean-square value and line voltage amplitude, respectively. The line voltage is considered a piecewise constant during each switching period because switching frequency $f_s$ is larger than line frequency $f_l$. If $m$ is the switching number within $[0, \pi/\omega]$, then $m$ is equal to $f_s/2f_l$. The following analysis is considered during switching period $[kT_s, (k+1)T_s]$, where $k = 0, 1, \ldots, m-1$. The magnetizing inductor $L_m$ is ignored in the following analysis because it is significantly larger than inductor $L_1$.

When $S_1$ switched turned on, the voltage across inductor $L_1$ is obtained as

$$v_{l1} = n \times \left| f_s(t_1) \right|$$  \hspace{1cm} (2)

where $e(t)$ is the input-voltage level during switching the
Then, given when period \([kT_o, (k+1)T_o]\), and the turns ratio of transformer \(N_o/N_i\). Then,

\[
\frac{di_{i1}(t)}{dt} = \frac{n \times [e_i(t_c)]}{L_i}, \quad kT_o \leq t \leq t_{i1}
\]

The inductor current \(i_{i1}\) is given by

\[
i_{i1}(t) = \frac{n \times [e_i(t_c)]}{L_i}(t - kT_o), \quad kT_o \leq t \leq t_{i1}
\]

When \(t = t_{i1}\), the peak value of inductor current \(i_{i1}\) is

\[
i_{i1,p} = \frac{n \times [e_i(t_c)]}{L_i}[t_{i1}] = \frac{n \times [e_i(t_c)]}{L_i}dT_o
\]

where \(t_{i1} = t_{i1} - kT_o = dT_o\).

When \(S_o\) is switched off, the voltage across inductor \(L_i\) is given by

\[
v_{i1} = -v_o
\]

Then,

\[
\frac{di_{i2}(t)}{dt} = \frac{v_o}{L_i}
\]

By solving (7), we derive the inductor current \(i_{i2}\) as follows:

\[
i_{i2}(t) = \frac{v_o}{L_i}(t - t_{i1}) + i_{i1,p}, \quad t_{i1} \leq t \leq t_{i2}
\]

Given that \(i_{i1}(t_{i1}) = 0\), the peak value of inductor current \(i_{i1}\) is

\[
i_{i1,p} = \frac{v_o}{L_i}t_{i,k}
\]

where \(t_{i,k} = t_{i2} - t_{i1}\).

Using (5) and (9), time duration \(t_{i,k}\) can be given by

\[
t_{i,k} = \frac{n \times [e_i(t_c)]}{v_o}dT_o
\]

A. Power Factor Correction

As shown in Fig. 3, the average value of unfiltered input current \(i_{i1}\) in one switching period \(T_o\) can be computed as

\[
i_{i1,avg}(t) = \frac{t_{i1}n_i t_{i1,p}}{2T_o}
\]

where \(i_{i1,p}\) is the inductor current peak value for each switching period. Substituting (1) and (5) into (11), we derive the following equation:

\[
i_{i1,avg}(t) = \frac{t_{i1}n_i t_{i1,p}}{2T_o} = \frac{n^2 \pi^2 T_o v_o}{2L_i} \sin \omega t - \frac{v_o}{R}
\]

The average value of unfiltered input current \(i_{i1}\) is sinusoidal and in phase with the input voltage. Moreover, the harmonic components of current \(i_{i1}\) are distributed over the switching frequency multiples. The harmonic components are easily filtered out by using input filter \(L_i - C_i\). The input filter cutoff frequency is significantly lower than the switching frequency.

B. Voltage Gain

From Fig. 3, the average value of the output-capacitor current \(i_{o}\) during \([kT_o, (k+1)T_o]\) can be obtained as

\[
i_{o,avg} = \frac{1}{T_o} \int_{t_{i1}}^{t_{i1} + t_{i1,p}} i_{i1} dt
\]

Substituting (1), (5), and (10) into (13) yields

\[
i_{o,avg} = \frac{n^2 \pi^2 T_o^2 v_o}{2L_i} \sin \omega t - \frac{v_o}{R}
\]

The average value of output–capacitor current \(i_{o}\) during a half line-source period \([0, \pi/\omega]\) is written as follows:

\[
i_{o,avg} = \frac{\omega}{\pi} \sum_{k=0}^{\infty} \int_{kT_o}^{(k+1)T_o} i_{o,avg} \sin \omega t \sin \omega t dt = \frac{\omega}{\pi} \sum_{k=0}^{\infty} \frac{n^2 \pi^2 T_o^2 v_o}{2L_i} \sin \omega t - \frac{v_o}{R}
\]

Given that \(m\) is larger than 1, equation (15) is approximated as:

\[
i_{o,avg} = \frac{\omega}{\pi} \int_{0}^{\pi/\omega} \frac{n^2 \pi^2 T_o^2 v_o}{2L_i} \sin \omega t - \frac{v_o}{R} dt = \frac{n^2 \pi^2 T_o^2 v_o}{4L_i v_o} \frac{v_o}{R}
\]

The output voltage differential equation is given by
\[
\frac{d}{dt} V_r = \frac{1}{C_o} \left( n^2 d V_r^2 \tau_1 - V_r \right) \quad (17)
\]

The DC model equation is written as
\[
\frac{n^2 D^2 V_r^2}{4 L_s f_s V_o} = \frac{n^2 D^2 V_m^2}{4 L_s f_s V_o} = \frac{V_o}{R} \quad \text{(18)}
\]

where \( V_o \) and \( D \) are the DC quantities of \( v_o \) and \( d \), respectively.

The normalized inductor time constant is then defined as
\[
\tau_{1L} = \frac{L_s f_s}{R} \quad (19)
\]

Substituting (19) into (18), the voltage gain is derived as
\[
M = \frac{V_o}{V_m} = \frac{nD}{\sqrt{2\tau_{1L}}} \quad (20)
\]

C. Boundary Condition

The current \( i_{o1} \) must be zero in each switching period to ensure that the proposed converter is operated in DCM. From Fig. 3, the time duration \( t_s \) is obtained as
\[
t_s = t_{o1} + t_{o2} = \frac{DT(V_o + n|e_f|)}{V_o} \quad (21)
\]

When the maximum value of \( t_s \) is equal to \( T_s \) and \( |e_f| \) is equal to \( V_o \), the proposed converter is operated in boundary conduction mode. Therefore, substituting \( t_s = T_s \) and \( |e_f| = V_o \) into (21) can determine the boundary voltage gain as
\[
M_b = \frac{V_o}{V_m} = \frac{nD}{1-D} \quad (22)
\]

Using (20) and (22), the curves of voltage gain and boundary voltage gain are shown in Fig. 5. When the voltage gain \( M \) is equal to its boundary voltage gain \( M_b \), the boundary normalized inductor time constant \( \tau_{1Lb} \) is given by
\[
\tau_{1Lb} = \frac{(1-D)^2}{4} \quad (23)
\]

\( \tau_{1Lb} \) is plotted in Fig. 6. We can observe that the proposed converter is operated in DCM when \( \tau_{1L} < \tau_{1Lb} \).

IV. SELECTIONS OF INDUCTOR AND CAPACITOR

A. Selection of Inductor \( L_1 \)

The appropriate \( \tau_{1Lb} \) is selected under the required voltage gain to ensure that the proposed converter is operated in DCM. The inductor \( L_1 \) needs to satisfy the following inequality:
\[
L_s < \frac{R}{f_s} \tau_{1Lb} \quad (24)
\]

B. Selection of Output Capacitor \( C_o \)

Using (14), the average value of output–capacitor current \( i_{o2} \) during one switching period is simplified as follows:
\[
i_{o2} = \frac{n^2 D^2 V_m^2}{4 L_s f_s V_o} - \frac{V_o}{R} \quad (25)
\]

Substituting (18) into (25), output–capacitor current \( i_{o2} \) is expressed as
\[
i_{o2} = \frac{-n^2 D^2 V_m^2 \cos 2\omega t}{4 L_s f_s V_o} \quad (26)
\]

Therefore, the output voltage ripple in one switching period is given by
\[
\Delta V_o = \frac{-n^2 D^2 V_m^2 \cos 2\omega t}{4 L_s f_s V_o} \quad (27)
\]

Then, the output voltage ripple function during time interval \([0, \pi/\omega]\) is obtained as
\[
\Delta V_o(t) = \int_{0}^{\pi/\omega} \left( \frac{-n^2 D^2 V_m^2}{4 L_s f_s V_o} \cos 2\omega t \right) dt = \frac{-n^2 D^2 V_m^2}{4 L_s f_s V_o} \frac{\sin 2\omega t}{2\omega} \quad (28)
\]

Using (28), the output voltage ripple during time interval \([0, \pi/\omega]\) is derived as
\[
V_{o, \text{ripple}} = \frac{2}{\pi} \left| \Delta V_o(t) \right|_{\text{peak}} = \frac{n^2 D^2 V_m^2}{4 L_s f_s V_o} \quad (29)
\]

Thus,
\[
V_{o, \text{ripple}} = \frac{n^2 D^2 V_m^2}{4 L_s f_s V_o} = \frac{n^2 D^2}{4 L_s f_s M^2} \quad (30)
\]

Output capacitor \( C_o \) must satisfy the following inequality to meet the following output voltage ripple percentage specification:
Fig. 7. Operating range of the prototype circuit.

Fig. 8. Control circuit of the proposed converter.

\[ C_v \geq \frac{n^2D^2}{4aL_1f_LM} \times \frac{V_o}{V_{ripple}} \]  

V. EXPERIMENTAL RESULTS

The prototype circuit is applied in the laboratory to demonstrate the performance of the proposed converter. Electrical specifications and circuit components are set as follows:

- Input voltage \( V_{text} = 90 \) V to 264 V (\( V_{text} = 127 \) V to 373 V)
- Output voltage \( V_o = 100 \) V
- Line frequency \( f_l = 60 \) Hz
- Switching frequency \( f_s = 50 \) kHz
- Output power \( P_o = 20 \) W to 100 W (\( R = 100 \) Ω to 500 Ω)
- Transformer \( T_r \): \( n = 0.5 \) (60 turns:30 turns), core ETD - 49, \( L_o = 850 \) μH
- Input filter \( L_f = 3.6 \) mH and \( C_f = 330 \) nF
- Switch \( S_1 \): IXFR34N80
- Diode \( D_2 \): DSEC30 – 04A

The voltage gain \( M \) is varied from 0.27 to 0.79 according to the electrical specifications. Substituting \( M = 0.79 \) and \( n = \)

0.5 into (22), the maximum duty ratio \( D_{max} \) is derived as 0.61. Substituting \( D_{max} = 0.61 \) into (23), \( r_{L1} \) is obtained as 0.038. Using (24), the inductor \( L_1 \) is given by

\[ L_1 < \frac{R}{f_s} \tau_{L1} = \frac{100}{50k} \times 0.038 = 76 \] μH
Fig. 10. Experimental waveforms at 230 – V line voltage: (a) \( e_s \) and \( i_n \), (b) \( i_{N1} \) and \( i_{N2} \), (c) \( i_L1 \) and \( i_{Do} \), (d) \( v_{S1} \)

Fig. 11. Measured efficiency for the proposed converter and conventional flyback converter.

Fig. 12. Measured results: (a) power factor and (b) THD

The inductor \( L_1 \) is 60 \( \mu \)H, and the core is EI-40. Thus, \( \tau_{L1} \) is equal to 0.03 at full load \( R = 100 \) \( \Omega \) and is equal to 0.006 at light load \( R = 500 \) \( \Omega \). Substituting the two values of \( \tau_{L1} \) and \( n = 0.5 \) into (20), the operating area of the experimental prototype is shown in Fig. 7. The proposed converter is operating in DCM.

Under the operating conditions \( V_{rms} = 90 \) V and \( R = 100 \) \( \Omega \), \( M \) and \( \tau_{L1} \) are derived as 0.79 and 0.03, respectively. Substituting \( M = 0.79 \), \( n = 0.5 \), and \( \tau_{L1} = 0.03 \) into (20), duty ratio \( D \) is obtained as 0.55. The ripple percentage of \( V_o \) is selected as 5%. From (31), the output capacitor inequality is given by
Thus, output capacitor \( C_o \) is selected as 600 \( \mu \)F.

The control circuit is shown in Fig. 8. Figs. 9 and 10 show the experimental waveforms under \( V_{in} = 115 \text{ V}, V_o = 100 \text{ V}, P_o = 100 \text{ W} \) and \( V_{rms} = 230 \text{ V}, V_o = 100 \text{ V}, P_o = 100 \text{ W} \), respectively. In Figs. 9(a) and 10(a), we observe that input current is sinusoidal and is in phase with input voltage. The current waveforms of the transformer primary and secondary sides \( i_{N1} \) and \( i_{N2} \) are shown in Figs. 9(b) and 10(b), respectively. The waveforms are taken at the peak value of input voltage. The current \( i_{N2} \) drops to zero during each switching period, which indicates that the transformer residual magnetism is released to empty during each switching period. The currents \( i_{L1} \) and \( i_{L0} \) are shown in Fig. 9(c) and 10(c), respectively. We observe that the proposed converter is operated in DCM. The waveform \( V_o \) across the switch drain source is shown in Figs. 9(d) and 10(d). The measured efficiencies of the proposed converter and flyback converter are compared in Fig. 11. We observe that efficiency is improved in the proposed converter. The measured power factor and THD are shown in Fig. 12. The measured power factor is higher than 0.96, whereas the measured THD, is lower than 5.8%.

VI. CONCLUSIONS

The forward and flyback PFC AC–DC converters are efficient choices for electrical isolation because of their simple structure. However, the forward AC–DC converter cannot achieve high power factor and low THD. Additionally, this converter requires a third winding to recycle transformer magnetizing inductor energy. The flyback AC–DC converter can achieve high power factor and low THD. Nevertheless, the transformer leakage inductor results in low efficiency. Therefore, we present a single-phase AC–DC converter that has a simple structure and is operated in DCM to achieve high power factor and low THD. A steady-state analysis is conducted. We implement a hardware circuit with simple control logic in the laboratory. The experimental results reveal the performance of the converters. The measured efficiencies reveal that the proposed converter exhibited higher efficiency than the conventional flyback converter.

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