Variable Coefficient Inductance Model-Based Four-Quadrant Sensorless Control of SRM

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Abstract

The phase inductance of a switch reluctance motor (SRM) is significantly nonlinear. With different saturation conditions, the phase inductance shape is clearly changed. This study focuses on the relationship between coefficient and current in an inductance model with ignored harmonics above the order of 3. A position estimation method based on the variable coefficient inductance model is proposed in this paper. A four-quadrant sensorless control system of the SRM drive is constructed based on the relationship between variable coefficient inductance and rotor position. The proposed algorithms are implemented in an experimental SRM test setup. Experimental results show that the proposed method estimates position accurately in operating two/four-quadrants. The entire system also has good static and dynamic performance.

Key words: Four-quadrant, Rotor position estimation, Switch reluctance motor, Variable coefficient inductance model

I. INTRODUCTION

Switched reluctance motors (SRMs) have the advantages of simple and firm structure, high efficiency, wide speed range, and high starting torque [1]. However, high-performance control of SRM needs to detect the rotor position with a position sensor attached to the motor shaft. The use of position sensors increases cost and machine size as well as decreases reliability. Therefore, the sensorless drive technique has high research value [2], [3].

SRMs without position sensor technology have aroused widespread concern in recent years. Many scholars have proposed a series of control schemes for position estimation. Most of them infer the instantaneous position of the rotor by measuring and monitoring one phase or several phases of the windings current and flux linkage. The flux-current method takes advantage of the relationship between the rotor position, flux linkage, and phase current [4]-[6]. However, the method is generally achieved by look-up table, which consumes a large amount of memory. Indirect position sensing is implemented by comparing the estimated flux linkage and pre-stored flux linkage of reference commutation position [7].

The proposed method greatly decreases the need for microcontroller memory and capacity. However, continuous position estimation is impossible with this method. Achieving high-grade performance and dynamic torque control continuous position estimation is a requirement. State observer methods are used to estimate rotor position and velocity [8]-[11]. The main disadvantages of these methods are real-time implementation of complex algorithms that require high-speed DSP and a significant amount of stored data. Based on the limitation of the traditional methods above, many scholars have proposed several new methods. The rotor position-estimated algorithms are implemented based on ANN [12]-[15]. Although the position estimator based on ANN above does not need an accurate SRM model and can theoretically provide good nonlinear mapping between input data and output data sets, it is a computation-intensive algorithm. Offline training of the synopsis weights is a rigorous procedure. Phase inductance expressed by the Fourier series, relation between the rotor electrical angle, and inductance can be derived [16], [17]. A position estimation method of calculating the rotor position in real time by the measured inductance is then proposed. The method is relatively easy to implement, simple, and reliable. However, the authors use pulse injection method in a non-energized phase to detect the phase inductance, which inevitably decreases the torque and efficiency of SRM and is unsuited for higher speed applications. The inductance of the
energized phase is calculated by dividing the flux linkage by phase current. This method overcomes the defects in [16]-[17], but the authors do not analyze the change trends of model coefficients with inductance saturation. However, this part has an important significance on the accuracy of rotor position estimation. The regional comparison of the three-phase inductance method is proposed [19]. The method is based on the logic relationship of phase inductance that varies regionally with the change in rotor position and is easy to implement. However, the regional comparison method suffers from the influence of inductance saturation and can introduce position estimation errors in dynamic operation. The phase current slope difference based calculation method is utilized to identify the full-cycle inductance [20]. The position versus inductance characteristics is modeled to estimate the rotor position based on the inductance characteristics. However, the method can be applied in light load or no-load conditions only. When the motor is operating in the saturation region, the method has certain limitations.

This paper presents the relationship between coefficient and current in the inductance model. A rotor position estimation method based on the variable coefficient inductance model is then presented. Using this method, we construct the SRM control system and verify the proposed method. Experimental results show that the proposed method is feasible.

II. SIMPLIFIED INDUCTANCE MODEL

A. SRM Structure and its Characteristics

The structure of a three-phase SRM is shown in Fig. 1. The measured flux-linkage versus current characteristics of the three-phase SRM at different rotor positions are shown on the left side of Fig. 2. Fig. 2 shows that the machine saturates at approximately 12 A.

$L_a$, is the aligned position inductance, $L_u$ is the unaligned position inductance, and $L_m$ is the inductance midway from the aligned position. $L_a$ and $L_m$ are related to the phase currents by functions that can be easily calculated by dividing the flux linkage by the phase current.

$$L_a(i) = \frac{\Phi_a(i)}{i} \quad (1)$$

$$L_u(i) = \frac{\Phi_u(i)}{i} \quad (2)$$

$$L_m(i) = \frac{\Phi_m(i)}{i} \quad (3)$$

B. Coefficients of the Simplified Inductance Model for Different Currents

The solid line in Fig. 3 represents inductance as a function of position for different currents obtained by the magnetization curve measured from the locked-rotor test. 0 elec. deg. is defined as the position where the rotor and stator poles of B-phase are unaligned. The measured phase inductance can be represented by the Fourier series as follows:

$$L = \sum_{n=0}^{\infty} L_n \cos(nN_s \alpha + \phi_n) \quad (4)$$

where $N_s$ is the number of rotor poles, $L_n$ is the coefficients of the Fourier series, and $\phi_n$ is the initial phases of the Fourier series. Considering the first three components ($n = 0, 1, 2$) of the Fourier series in Eq. (4), each phase inductance is described by the following equations [17]:

$$L_A = L_0 + L_1 \cos(\theta - \theta_{elec}) + \frac{2}{3} \pi \cos(\theta - \theta_{elec} + \frac{2}{3} \pi) \quad (5)$$

$$L_B = L_0 + L_1 \cos(\theta - \theta_{elec}) + L_2 \cos(\theta - \theta_{elec} + \frac{2}{3} \pi) \quad (6)$$

$$L_C = L_0 + L_1 \cos(\theta - \theta_{elec} + \frac{4}{3} \pi) + L_2 \cos(\theta - \theta_{elec} + \frac{4}{3} \pi) \quad (7)$$

$L_0$, $L_1$, and $L_2$ can be derived as a function of the maximum inductance $L_a$, minimum inductance $L_u$, and middle inductance $L_m$ from Eqs. (8), (9), and (10).
where the phase currents by polynomial functions given by

\[ I_p(t) = A_0 + A_1 t + A_2 t^2 + A_3 t^3 + A_4 t^4 + A_5 t^5 \]

The obtained numerical result for every point is shown in Fig. 4 from 5A and (10) for every point is shown in Fig. 4.

**TABLE I**

<table>
<thead>
<tr>
<th>( L_a )</th>
<th>( A_0 )</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( A_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_0 )</td>
<td>0.044 2</td>
<td>0.001 2</td>
<td>-1.25e-4</td>
<td>3.28e-6</td>
<td>-3.48e-8</td>
</tr>
<tr>
<td>( L_1 )</td>
<td>0.935 1</td>
<td>0.002 8</td>
<td>-2.8e-4</td>
<td>8.4e-6</td>
<td>-1.23e-7</td>
</tr>
<tr>
<td>( L_2 )</td>
<td>0.005 2</td>
<td>1.415e-4</td>
<td>-2.667e-5</td>
<td>9.19e-7</td>
<td>-1.3e-8</td>
</tr>
</tbody>
</table>

Thus, the phase inductance is obtained. Fig. 4 shows the different conduction sequences of a phase as well as the active phase is detected by the sensor. After the phase inductance is obtained, the relationship between the phase inductance and rotor position can be derived. Considering the B-phase, the following equation is derived from Eq. (6):

\[ 2 L_2 \cos^2(\pi - \theta_{\text{elec}}) + L_0 \cos(\pi - \theta_{\text{elec}}) + L_0 - L_2 = 0 \]

Thus, the expression of the electrical angle \( \theta_{\text{elec}} \) can be expressed as the following expression:

\[ \theta_{\text{elec}} = \pi - \cos^{-1}\left(-\frac{L_1 + \sqrt{L_1^2 - 8 L_2 (L_0 - L_2)}}{4 L_2}\right) \]
TABLE II
RELATIONSHIP BETWEEN THE ESTIMATED AND REAL POSITIONS

<table>
<thead>
<tr>
<th>Work condition</th>
<th>Energized phase</th>
<th>Estimated position</th>
<th>Real position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motoring mode</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>(\theta_{\text{elec}} / N_r)</td>
<td>((\theta_{\text{elec}} + 2/3 \pi) / N_r)</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>(\theta_{\text{elec}} / N_r)</td>
<td>(\theta_{\text{elec}} / N_r)</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>(\theta_{\text{elec}} / N_r)</td>
<td>((\theta_{\text{elec}} + 4/3 \pi) / N_r)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Regenerating braking mode</th>
<th>Estimated position</th>
<th>Real position</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(\theta_{\text{elec}} / N_r)</td>
<td>((4/3 \pi - \theta_{\text{elec}}) / N_r)</td>
</tr>
<tr>
<td>B</td>
<td>(\theta_{\text{elec}} / N_r)</td>
<td>((2\pi - \theta_{\text{elec}}) / N_r)</td>
</tr>
<tr>
<td>C</td>
<td>(\theta_{\text{elec}} / N_r)</td>
<td>((2/3 \pi - \theta_{\text{elec}}) / N_r)</td>
</tr>
</tbody>
</table>

When the A phase or C phase is excited, \(\theta_{\text{elec}}\) is calculated using \(L_A\) or \(L_C\) instead of \(L_B\) in Eq. (18). The relationship between the estimated and real positions is shown in Table II.

B. Phase selection and estimator commutation

For a 12/8 SRM, a mechanical cycle is 45°. Figure 6 shows a 15 mech. deg. phase shift between \(L_A\), \(L_B\), and \(L_C\). Each of the three phases should provide 15 mech. deg. information for position estimation. The phase conduction angle must satisfy the following condition to satisfy the continuity of the angle estimation:

\[
\theta_{\text{cond}} \geq 15^\circ
\]  

(19)

The turn-on angle \(\theta_{\text{on}}\) and turn-off angle \(\theta_{\text{off}}\) must satisfy the following condition:

\[
\theta_{\text{on}}^{k+1} \leq \theta_{\text{off}}^k
\]  

(20)

The satisfied conditions of \(\theta_{\text{on}}, \theta_{\text{off}},\) and \(\theta_{\text{cond}}\) are shown in Figs. 6(a) and 6(b). The process of position estimation using the inductance method for the motoring mode with \(\theta_{\text{on}} = 0^\circ\) is also shown in Fig. 6(a). Only the 15 deg. region of each phase inductance is needed to estimate the position, and the variation of each phase inductance is relatively clear in the solid line region. Thus, the solid line part of the three-phase inductance is combined to form the three-phase synthesis inductance to estimate the rotor position. The region of estimation is 4° to 19° for the motoring mode, whereas the two-phase overlap conducted region is only 2°. Fig. 6(b) shows the process of rotor position estimation using three-phase synthesis to regenerate the braking mode with \(\theta_{\text{on}} = 22.5^\circ\). The position estimation procedure to regenerate the braking mode is similar to the procedure for motoring mode. However, the solid line part of the inductance falling region is selected to estimate the position for the regenerative braking mode. The change trend of the estimated result is modified from increasing to decreasing.

IV. SENSORLESS CONTROL STRATEGY OF SRM DRIVES

A. Position Estimation at Standstill

From Eqs. (14) and (16), we can obtain the following equation:
The pulse injection method operates at standstill to obtain the initial rotor position. The principle of the pulse injection method is shown in Fig. 7. At the start-up moment, the DC bus voltage is applied to three stator windings in a short period to obtain three corresponding response currents. The three phase currents can then be compared to determine the initial conduction phase. The DC voltage is approximately 514 V, whereas the pulse frequency is set at 4 kHz in this paper. The pulsing current should decay to zero before the next pulse injection starts, so the duty ratio of the pulse is set to 0.4. Thus, the pulse excitation duration \( \Delta t \) is 100 \( \mu s \). The initial phase selection is shown in Table III for forward command. Table III shows that the two phases are conducted simultaneously at certain rotor positions to ensure a larger starting torque.

\[
\begin{align*}
\text{TABLE III} & \quad \text{INITIAL PHASE SELECTION} \\
\text{Sector} & \quad \text{Response current} & \quad \text{Initial conduction phase} \\
5 & \quad I_a \geq I_b \geq I_c & \quad \text{A-phase and C-phase} \\
1 & \quad I_a \geq I_c \geq I_b & \quad \text{C-phase} \\
4 & \quad I_b \geq I_a \geq I_c & \quad \text{A-phase} \\
6 & \quad I_b \geq I_c \geq I_a & \quad \text{A-phase and B-phase} \\
3 & \quad I_c \geq I_a \geq I_b & \quad \text{B-phase and C-phase} \\
2 & \quad I_c \geq I_b \geq I_a & \quad \text{B-phase} \\
\end{align*}
\]

\[
u_k = R_k + L_k \frac{di_k}{dt} + i_k \frac{dL_k}{d\theta} \omega \tag{21}
\]

where \( i_k \frac{dL_k}{d\theta} \omega \) is the back-EMF term. At standstill, the back-EMF is negligible. The voltage drop across the winding resistance is also small compared with the voltage applied to the phase winding. As a result, the following equation is obtained from Eq. (21).

\[
u_k = L_k \frac{\Delta I}{\Delta t} \tag{22}
\]

B. Transition between Two Position Sensorless Algorithms

The pulse injection was selected to be in effect only at standstill [21], and the inductance model was selected during normal operation. Transition between two position sensorless algorithms at startup time is shown in Fig. 8. The initial rotor position information can be obtained by the pulse injection method. After the initial phase is excited and phase current is established, the position sensorless algorithms switched to the inductance model method. Even in the speed command reversal operation, the motor speed lowered to zero again. Thus, the pulse injection is no longer needed.

C. Four-Quadrant Sensorless Control

The part of inductance used for position estimation can be either the increasing or decreasing one depending on the active quadrant, as shown in Fig. 9. The commutation point can be determined by estimated rotor position \( \hat{\theta} \) and active quadrant. Chopped current control (CCC) with changed turn-on angle and fixed conducted angle (\( \theta_{\text{cond}} = 19^\circ \)) is used for motoring mode. The turn-on angle that changed according to speed is also shown in Fig. 8. In a very low-speed case, the turn-on angle is fixed at \( \theta_{\text{on}} = 0^\circ \) when the speed estimation \( \hat{\omega} < \omega_1 \) to increase the output torque. The turn-on angle is fixed at \( \theta_{\text{on}} = 2^\circ \) when the speed estimation \( \omega_1 < \hat{\omega} \leq \omega_2 \), which is favorable to improve efficiency. However, when speed estimation \( \omega > \omega_2 \), the turn-on angle changes in range from 2\(^\circ\) to 0\(^\circ\) smoothly until the speed estimation \( \hat{\omega} = \omega_3 \). For the regenerative braking mode, CCC with a fixed turn-on angle and turn-off angle is used. The conduction pulse slides to a negative inductance slope with \( \theta_{\text{on}} = 22.5^\circ \) and \( \theta_{\text{off}} = 42^\circ \).

V. EXPERIMENTAL VERIFICATION

A. Experimental Hardware Conditions and Parameters

The 12/8-pole three-phase SRM was tested to demonstrate the validity and practicality of the proposed sensorless control method. Table IV shows the specifications of the tested motor. The experimental drive system is shown in Fig. 10. The digital signal processor (DSP) TMS320F2812 is used for real-time rotor position estimation, speed, and CCC control. A 12-bit ADC sampling chip ADS7864 is used for DC voltage and phase current sample in real time. The IGBT control signals are sent to the driver module after logic and integrated treatment by the EP1K30QC208 FPGA of ALTERA. The three-phase asymmetric half-bridge circuit
implemented with the FF150R12KE3G as the main switching device IGBT is used as the power converter. The load machine is a DC motor. The DC motor is controlled by a Siemens 6RA70 in the experiment, which runs synchronously with the SRM drive.

**B. Software Implementation**

Figure 11 shows that the software within the dashed frame is implemented by DSP and a complex programmable logic device (EP1K30) hybrid controller, which consists of the following parts:

1) A PI controller is designed to provide the chopping current reference $I_{ref}$ and CCC pulse generator module. Thus, the speed closed-loop can be implemented.
2) Through sampling of the AD module, the bus voltage \( u_{dc} \) signal and three-phase current signal are converted to a digital signal that can be recognized by DSP.

3) The phase voltage is determined by means of a logic using the levels of the gate switch impulses (PWMST) and the bus voltage \( u_{dc} \) by Eq. 15. Through the flux calculated module, the ohmic voltage drop above the phase resistance is deducted from the phase voltage. This voltage difference was integrated up to the flux linkage by Eq. 14.

4) The flux linkage \( \psi \) and phase current \( i_{ABC} \) are transmitted to the phase inductance estimated module. The phase inductance \( \hat{L} \) was calculated by using Eq. 17.

5) The three coefficients \( L_0, L_1, \) and \( L_2 \) estimated by Eq. 13. The phase inductance value \( \hat{L} \) is transmitted to the position-estimated module. Finally, the rotor position \( \hat{\theta} \) was calculated by Eq. 18.

6) Sensorless commutation control of the switched reluctance motor was accomplished based on the estimated result \( \hat{\theta} \) by commutation module. The rotor speed \( \hat{\omega} \) can be obtained according to the interval of commutation.

The time budget of the major software functions for the SRM controller is shown in Fig. 12.

C. Sensorless Starting

The short period pulse is injected to three phases of the 12/8 SRM simultaneously. Fig. 13(a) shows the current pulse depicted by the experiment, \( I_c > I_d \geq I_b \) , whereas the B-phase and C-phase are the initial phases. Fig. 13(b) shows the estimated position at the starting moment. The estimated position is continuous and smooth between 0° and 15° mechanical degrees for each cycle. This finding verifies that the initial position can be obtained precisely.

D. Position Estimation in Light Load Conditions at Different Speeds

Figs. 14 to 17 show the position estimation results at 20% of the rated load conditions at 100, 500, 1000, and 1350 rpm, respectively. The traces show the phase current pulses, synthetic inductance, actual position, and estimated rotor position from top to bottom. Fig. 14(a) shows the third trace, whereas the marked 1, 2, 3, and 4 regions of estimated position constitute one complete electrical cycle and exactly corresponds to one mechanical cycle of the actual position (0°–45°). Figs. 14 to 17 show that the proposed estimation method has good accuracy from low speed (100 r/min) to the maximum speed (1350 r/min). As mentioned in Part C of Section IV, the turn-on angle is changed according to speed and decreased to nearly 0° at 1350 r/min when speed...
Fig. 14. Experimental result at 100 r/min.

Fig. 15. Experimental results at 500 r/min.

Fig. 16. Experimental result at 1000 r/min.

Fig. 17. Experimental results at 1350 r/min.
estimation $\omega \geq 500$ r/min. Table V shows the estimation error under a light load at different speeds. As the speed increases, the estimation error decreases.

E. Position Estimation in Rated Conditions

Fig. 18 shows the position estimation results in rated load conditions at 1350 rpm. Figs. 18(a) and 18(b) show the traces from top to bottom, which depict the phase current pulses, synthetic inductance, actual position, and estimated position. The current shape in Fig. 18 is different from that in Fig. 17 because when the phase current pulses increase to a rated current, the back EMF is near the DC–bus voltage at 1350 rpm. Therefore, the current control is lost because of high back EMF in rated conditions and the current chopping phenomenon disappeared. Fig. 18(a) shows that the estimated position is still parallel with the actual position, and the commutation control is normal in rated load conditions.

Fig. 19 shows the prototype efficiency at different speeds in rated load conditions. The efficiency is increased with the increase in speed. When the speed is more than 700 r/min, the efficiency can reach above 90%.

F. Dynamic Performance in Speed Reversal Test

The speed reversal test was also performed to verify the practicality of the proposed system. During this test, the SRM drive in 50% of the rated load conditions operating as a motor starts in a clockwise direction and, after a certain number of revolutions, the speed command was changed from 1200 r/min to -1200 r/min. It would then stop and start in the opposite direction.

Fig. 20(a) shows the process in the speed reversal test. The reversal process is as follows: the motor first starts braking and decreases its speed. After a short transition at zero speed, the motor phase begins to pick up speed in the opposite direction. The figure shows that the speed reversal process takes approximately 1.2 s only.

Fig. 20(b) shows the detailed process in the speed reversal test. The estimated position based on the inductance model is almost unchanged at nearly zero speed. Thus, the algorithm is suitable at very low speeds.

VI. CONCLUSIONS

A new sensorless control method for SRM drives is presented in this paper. This approach can be used to estimate...
the position based on the variable coefficients inductance model. The new algorithm for position estimation combines pulse injection at a standstill, which can drive the SRM from standstill to high-speed operation smoothly. Phase selection and estimator commutation for different modes is also designed and implemented. The sensorless method works at a series of speed transients both in acceleration and deceleration. Results show that the four-quadrant sensorless control of the SRM drive is a feasible technique that can be considered ready for application. This sensorless method requires no additional hardware, which makes it relatively easy to implement. These features make the proposed method practical, reliable, and cost effective as well as acceptable in many variable-speed applications. Experimental results fully verify the proposed sensorless scheme and demonstrate its advantages.

REFERENCES


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