Fast Self-Similar Network Traffic Generation Based on FGN and Daubechies Wavelets

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ABSTRACT

Recent measurement studies of real teletraffic data in modern telecommunication networks have shown that self-similar (or fractal) processes may provide better models of teletraffic in modern telecommunication networks than Poisson processes. If this is not taken into account, it can lead to inaccurate conclusions about performance of telecommunication networks. Thus, an important requirement for conducting simulation studies of telecommunication networks is the ability to generate long synthetic stochastic self-similar sequences. A new generator of pseudo-random self-similar sequences, based on the fractional Gaussian noise and a wavelet transform, is proposed and analysed in this paper. Specifically, this generator uses Daubechies wavelets. The motivation behind this selection of wavelets is that Daubechies wavelets lead to more accurate results by better matching the self-similar structure of long range dependent processes, than other types of wavelets. The statistical accuracy and time required to produce sequences of a given (long) length are experimentally studied. This generator shows a high level of accuracy of the output data (in the sense of the Hurst parameter) and is fast. Its theoretical algorithmic complexity is O(n).

키워드: Self-similar 프로세스(Self-Similar Process), Teletraffic 생성기(Teletraffic Generator), Fractional Gaussian Noise, Daubechies Wavelet, Complexity, Hurst 변수(Hurst Parameter)

1. Introduction

The search for accurate mathematical models of data streams in modern telecommunication networks has attracted a considerable amount of interest in the last few years. Several recent teletraffic studies of local and wide area networks, including the World Wide Web, have shown that commonly used teletraffic models, based on Poisson or related processes, are not able to capture the self-similar (or fractal) nature of teletraffic [1-4], especially when these networks are engaged in such sophisticated services as variable-bit-rate video transmission [5-7]. The properties of teletraffic in such scenarios are very different from both the properties of conventional models of telephone traffic and the traditional models of data traffic generated by computers.

The use of traditional models of teletraffic can result in overly optimistic estimates of performance of telecommunication networks, insufficient allocation of communication
and data processing resources, and difficulties in ensuring the quality of service expected by network users [3, 8, 9]. On the other hand, if the strongly correlated character of teletraffic is explicitly taken into account, this can also lead to more efficient traffic control mechanisms.

Several methods for generating pseudo-random self-similar sequences have been proposed. They include methods based on fast fractional Gaussian noise [10], fractional ARIMA processes [11], the M/G/∞ queue model [12], autoregressive processes [12], spatial renewal processes [13], etc. Some of them generate asymptotically self-similar sequences and require large amounts of CPU time. For example, Hosking’s method [11], based on the F-ARIMA(0, d, 0) process, needs 1.5 hours to produce a self-similar sequence with 131,072 (2^17) numbers on a Pentium II [1, 14]. It requires O(n^2) computations to generate n numbers. Even though exact methods of generation of self-similar sequences exist (for example [10]), they are only fast enough for short sequences. They are usually inappropriate for generating long sequences because they require multiple passes along generated sequences. To overcome this, approximate methods for generation of self-similar sequences in simulation studies of telecommunication networks have been proposed [15, 16].

The evaluation of our generator based on Daubechies wavelets (DW) concentrates on two aspects. (i) how accurately a self-similar process can be generated; and (ii) how quickly the method generates long self-similar sequences. Our method, based on the fractional Gaussian noise (FGN) and Daubechies wavelets, will be called the FGN-DW method.

A summary of the basic properties of self-similar processes is given in Section 2. Section 3 describes the spectral density of FGN processes, while a discrete wavelet transform (DWT) for synthesising approximate FGN is presented in Section 4. In Section 5, a generator of pseudo-random self-similar sequences, based on FGN and DW, is described. Numerical results of analysis of sequences generated by this generator are discussed in Section 6.

2. Self-Similar Processes and Their Properties

2.1 Mathematical Definition of Self-Similarity

One can distinguish two types of stochastic self-similarity. A continuous-time stochastic process \( Y_t \) is distinctly self-similar with a self-similarity parameter \( H(1/2 < H < 1) \), if \( Y_t \) and \( c^HY_t \) (the rescaled process with time scale \( c^t \)) have identical finite-dimensional probability for any positive time stretching factor \( c \) [17-19]. This means that, for any sequence of time points \( t_1, t_2, \ldots, t_n \), and for any \( c > 0 \),

\[
(Y_{ct_1}, Y_{ct_2}, \ldots, Y_{ct_n}) \sim (c^HY_{t_1}, c^HY_{t_2}, \ldots, c^HY_{t_n}),
\]

where \( \sim \) denotes equivalence in distribution. This definition of the strictly self-similarity is in a sense of probability distribution (or narrow sense), quite different from that of the second-order self-similar process (or self-similar process in a broad sense). Self-similarity in the broad sense is observed at the mean, variance and auto-correlation level, whereas self-similarity in the narrow sense is observed at the probability distribution level.

When the weakly continuous-time self-similar process \( Y_t \) has stationary increments, i.e., the finite-dimensional probability distributions of \( Y_{t_1}, \ldots, Y_{t_n} \) do not depend on \( t_0 \), we can construct a stationary incremental process \( X = (X_i = Y_{i+1} - Y_i : i = 0, 1, 2, \ldots) \). Namely, in the discrete-time case, let \( X \) be a (discrete-time) stationary incremental process with mean \( \mu = E[X] \), variance \( \sigma^2 = E[(X - \mu)^2] \), and (normalised) auto-correlation function (ACF) \( \rho_k, k = 0, 1, 2, \ldots \), where

\[
\rho_k = \frac{E[(X_i - \mu)(X_{i+k} - \mu)]}{\sigma^2}.
\]

(1)

\( X \) is strictly stationary if \( (X_{i_n}, X_{i_n+1}, \ldots, X_{i_{n+k}}) \) and \( (X_{i_n+k}, \ldots, X_{i_n+k+s}) \) possess the same joint distribution. However, we limit our attention to processes with a weaker form of stationarity, i.e., second-order stationarity (or weak, broad, or wide sense stationarity). Let \( X^{(m)} = (X^{(m)}_i, X^{(m)}_{i+1}, \ldots, X^{(m)}_{i+s}) \) be a sequence of batch means, that is,

\[
X^{(m)}_i = \frac{1}{m}(X_{i+m-n+1} + \ldots + X_{i+m}), \quad i \geq 1,
\]

(2)

and let \( \rho^{(m)}_k \) denote the ACF of \( X^{(m)} \). The process \( X \) is called exactly second-order self-similar with \( 1/2 < H < 1 \), if for all \( m \geq 1 \),

\[
\rho^{(m)}_k = \rho_k, \quad k \geq 0.
\]

(3)

In other words, the process \( X \) and the aggregated processes \( X^{(m)}, m \geq 1 \), have an identical correlation structure.
The process $X$ is asymptotically second-order self-similar with $1/2 < \theta < 1$, if for all $k$ large enough,

$$\rho_k^{(m)} \to \rho_0, \quad m \to \infty. \quad (4)$$

The most frequently studied models of self-similar traffic belong either to the class of fractional autoregressive integrated moving-average (F-ARIMA) processes to the class of fractional Gaussian noise processes; see [1, 11, 16]. F-ARIMA($p, d, q$) processes were introduced by Hosking [11] who showed that they are asymptotically self-similar with Hurst parameter $H = d + 1/2$, as long as $0 < d < 1/2$, where $p$ is the order of autoregression in the ARIMA process and $q$ is the order of the moving average in the ARIMA process. For the second class, the FGN process is the incremental process $(Y_t) = (X_t - X_{t-1})$, $k \geq 0$, where $(X_t)$ designates a fractional Brownian motion (FBM) random process. This process is a (discrete-time) stationary Gaussian process with mean $\mu$, variance $\sigma^2$ and ($\rho_p = (1/2) (k^H - 1/k^H)$), $k > 0$. An FBM process, which is the sum of FGN increments, is characterized by three properties [20]: (i) it is a continuous zero-mean Gaussian process $(X_t) = (X_s; s \geq 0$ and $0 < H < 1$) with ACF given by $\rho_{s,t} = 1/2 (s^H + t^H - |s - t|^H)$ where $s$ is time lag and $t$ is time; (ii) its increments $(X_t - X_{t-1})$ form a stationary random process; (iii) it is self-similar with Hurst parameter $H$, that is, for all $c > 0$, $(X_{ct}) \approx (c^H X_t)$, in the sense that, if time is changed by the ratio $c$, then $(X_t)$ is changed by $c^H$.

2.2 Properties of Long-Range Dependent Self-Similar Processes

Main properties of self-similar processes include (1, 17, 21).

- **Long-range dependence**: A process $(X_t)$ is called a stationary process with long-range dependence (LRD) if its ACF $\rho_k$ is non-summable, i.e., $\sum_{k=0}^{\infty} \rho_k = \infty$. The speed of decay of auto-correlations is more hyperbolic than exponential. Another definition of LRD is given by

$$\rho_k \sim L(t) k^{-1 - 2H}, \quad k \to \infty, \quad (5)$$

where $1/2 < H < 1$ and $L(\cdot)$ slowly varies at infinity, i.e., $\lim_{x \to \infty} \frac{L(x)}{L(t)} = 1$, for all $x > 0$; see [1]. The Hurst parameter $H$ characterises the relation in (5), which specifies the form of the tail of the ACF.

- **Slowly decaying variance**: The variance of the sample mean decreases more slowly than the reciprocal of the non-overlapping batch size $m$, i.e., $\text{Var}[(X^m_t)] \to \frac{c_1 m^{-\beta}}{m \to \infty}$, where $c_1$ is a constant and $0 < \beta < 1$.

- **Hurst effect**: Historically, the importance of self-similar processes lies in the fact that they provide an elegant explanation and interpretation of strong correlations in some empirical data. Namely, for a given sequence of random variables $X = (X_1, X_2, \ldots, X_n)$, one can consider the so-called rescaled adjusted range $R(t, m)/S(t, m)$ (or R/S-statistic), with

$$R(t, m) = \max \left[ \frac{N_{t+1} - N_t - \frac{i}{m} (N_{t+1} - N_t), 0 \leq i \leq m} \right],$$

$$S(t, m) = \sqrt{\frac{1}{m} \sum_{i=1}^{m} \left( \frac{X_i - \bar{X}_{1,m}}{m} \right)^2}, \quad (6)$$

where $0 \leq t \leq n$, $m$ is the batch size and $N_t = \sum_{i=1}^{t} X_i$; and

$$R(t, m) \sim \sqrt{m} \frac{c_2 m^{\beta}}{m \to \infty}, \quad (7)$$

where $\bar{X}_{1,m} = \frac{1}{m} \sum_{i=1}^{m} X_i$.

Hurst found empirically that for many time series observed in nature, the expected value of $R(t, m)/S(t, m)$ asymptotically satisfies the power-law relation:

$$E\left[\frac{R(t, m)}{S(t, m)}\right] \to c_2 m^\beta, \quad m \to \infty, \quad 1/2 < H < 1,$$

where $c_2$ is a finite positive constant [17]. This empirical finding was in contradiction to previously known results for Markovian and related processes. For a stationary process with SRD, $E(R(t, m)/S(t, m))$ behaves asymptotically like a constant times $m^{\beta}$. Therefore, for large values of $m$, the R/S-statistic plot is randomly scattered around a straight line with slope $1/2$. Hurst's finding that for the Nile River data, and for many other hydrological, geophysical, and climatological data, $R(t, m)/S(t, m)$ is randomly scattered around a straight line with slope $H > 1/2$, is known as the Hurst effect, and $H$ is known as the Hurst parameter (or self-similarity parameter). Mandelbrot and Wallis [20] showed that the Hurst effect can be modelled by FGN with the self-sim-
ilarity parameter $1/2 < H < 1$.

- $1/f$-noise: The spectral density $f(\lambda; H)$ obeys a power law near the origin, i.e., $f(\lambda; H) \rightarrow c_2 \lambda^{1-2H}$, as $\lambda \rightarrow 0$, where $c_2$ is a finite positive constant and $1/2 < H < 1$.

We will use these properties to investigate characteristics of generated self-similar sequences.

3. Spectral Density of FGN Processes

In our generator, numbers representing the spectral density function of FGN are obtained by applying appropriate transformations to originally uniformly distributed pseudo-random numbers. The spectral density $f(\lambda; H)$ of an FGN process is given by

$$f(\lambda; H) = 2c_2(1 - \cos(\lambda)) \sum_{k=-\infty}^{\infty} |2\pi k + \lambda|^{-2H-1}$$  \hspace{1cm} (8)

for $0 < H < 1$ and $-\pi \leq \lambda \leq \pi$, where $c_2 = \sigma^2(2\pi)^{-1} \sin((\pi H)/2H+1)$ and $\sigma^2 = \text{Var}[X_1]$; see [17].

The main difficulty with using Equation (8) to compute the spectral density is the vexing infinite summation. The approximation of the above $f(\lambda; H)$ is given in [17] as

$$f(\lambda; H) = c_2 |\lambda|^{1-2H} + O(|\lambda|^{\min(3-2H,2)})$$  \hspace{1cm} (9)

where $O(\cdot)$ represents the residual error.

A generator of self-similar sequences based on FGN was also proposed by Paxson [16], but his method was based on a more complicated approximation of $f(\lambda; H)$ than this one given by Equation (9).

4. Discrete Wavelet Transform

Our method for generating synthetic self-similar FGN sequences in a time domain is based on the discrete wavelet transform (DWT). It has been shown that wavelets can provide compact representations for a class of FGN processes [22, 23]. This is because the structure of wavelets naturally matches the self-similar structure of the long range dependent processes [24]. Wavelets are complete orthonormal bases which can be used to represent a random time series in two domains: time and frequency. In Hilbert space $L^2(R)$, scaled and shifted functions $\psi_{j,m}(k)$ of wavelets can be represented as $\psi_{j,m}(k) = 2^{-j/2} \phi_k(2^{-j} k - m)$ where $j$ and $m$ are positive integers [25]. Since such wavelets are obtained by scaling and shifting a single function, $\phi_0(k), \phi_k(k)$ is called the mother wavelet. Moreover, all base functions $\psi_{j,m}(k)$ have the same shape as the mother wavelet and therefore are self-similar with each other.

For our generator, we chose Daubechies wavelets, which belong to the class of orthonormal wavelets, because they produce more accurate coefficients of wavelets than Haar wavelets (for more detailed discussions, see also literature in [26, 27]; and our results of the comparison in Section 6). They are defined as $\phi(k) = \sum_{i=-2^s}^{2^s-1} (-1)^i \rho_{i-1} \phi(2k-i)$, where $(\rho_i)$ is the two-scale sequence of $\phi(k)$ and $\phi(k) = \sum_{i=0}^{2^s} \rho_k \phi(2k-i)$. A discrete-time process $(X_n)$ can be represented through its inverse DWT $(X_k) = \sum_{j=1}^{2^{s+1}-1} \sum_{m=0}^{2^s} d_{j,m} \phi_{j,m}(k)$, where $0 \leq k \leq 2^s$; and $S$ is a positive integer which characterizes the limited resolution in time; and $d_{j,m}$ are wavelet coefficients which can be obtained through the DWT, since $d_{j,m} = \sum_{i=0}^{2^s-1} X_i \phi_{j,m}(k)$.

4.1 Daubechies Wavelets

Daubechies discovered one of the original wavelet families [26]. This family contains a single wavelet for each possible even valued filter length, beginning with four coefficients. As the filter length grows, the wavelets move from highly localized (due to the small number of non-zero filter coefficients) to highly smooth (for larger numbers of coefficients). The formation of the smallest of this family of wavelets, often referred to as Daub(4), which stands for the Daubechies wavelets with four coefficients, is calculated below [26].

The transformation matrix for a Daubechies wavelet of length four is given by the below matrix.

$$\begin{bmatrix}
c_0 & c_1 & c_2 & c_3 & 0 & 0 & 0 & \ldots \\
c_3 & -c_2 & -c_1 & -c_0 & 0 & 0 & 0 & \ldots \\
0 & c_0 & 0 & 0 & 0 & 0 & 0 & \ldots \\
0 & 0 & c_0 & -c_2 & -c_1 & -c_0 & 0 & \ldots \\
0 & 0 & 0 & c_0 & -c_2 & -c_1 & -c_0 & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots
\end{bmatrix}$$  \hspace{1cm} (10)

The odd rows of this matrix represent a convolution of
the data vector with the coefficients \([ c_0, c_1, c_2, c_3 ]\). These rows can be efficiently calculated due to its sparsity (requiring four multiplications and three additions per data value). The even rows of this matrix represent a convolution of the data vector with the coefficients \([ c_3, -c_2, c_1, -c_0 ]\). They are also efficiently calculated, requiring the same number of operations. The total effect of this operation is to convolve the data vector with two different four-coefficient filters and reduce the results by a factor of two.

The inverse (reconstruction) transform is the inverse of this matrix, and is equal to its transpose. It can be used to generate the following requirements for the values \([ c_0, c_1, c_2, c_3 ]\).

\[
\begin{align*}
  c_0^2 + c_1^2 + c_2^2 + c_3^2 &= 1, \\
  c_0 c_2 + c_1 c_3 &= 0.
\end{align*}
\]

For the values of these coefficients to form the desired high- and low-pass filters, we must also require:

\[
\begin{align*}
  c_3 - c_2 + c_1 - c_0 &= 0, \\
  0 \times c_3 - 1 \times c_2 + 2 \times c_1 - 3 \times c_0 &= 0.
\end{align*}
\]

The unique solution to these four equations is:

\[
\begin{align*}
  c_0 &= \frac{(1+\sqrt{3})/4 \times \sqrt{2}}{2}, \\
  c_1 &= \frac{(3+\sqrt{3})/4 \times \sqrt{2}}{2}, \\
  c_2 &= \frac{(3-\sqrt{3})/4 \times \sqrt{2}}{2}, \\
  c_3 &= \frac{(1-\sqrt{3})/4 \times \sqrt{2}}{2}.
\end{align*}
\]

This general method can be used to form Daubechies wavelet filter values for any even filter length greater than four.

The Daub(4) DWT of an input data vector is calculated as follows (this method assumes a data vector length that is a power of two; the use of other lengths requires special treatment).

Given the data vector:

\[
[y_0 \ y_1 \ y_2 \ y_3 \ y_4 \ y_5 \ y_6 \ y_7 \ y_8 \ y_{10} \ y_{11} \ y_{12} \ y_{13} \ y_{14} \ y_{15},]
\]

transform it with (multiply by) the forward matrix (10) to give:

\[
[y_0 \ d_0 \ y_1 \ d_1 \ y_2 \ d_2 \ y_3 \ d_3 \ y_4 \ d_4 \ y_5 \ d_5 \ y_6 \ d_6 \ y_7 ; d_7].
\]

where \( s \) are smooth responses (low-pass) and \( d \) are detail responses (high-pass). This vector is then permuted to collect the smooth and detail areas as follows:

\[
[s_0 \ s_1 \ s_2 \ s_3 \ s_4 \ s_5 \ s_6 \ s_7 \ d_0 \ d_1 \ d_2 \ d_3 \ d_4 \ d_5 \ d_6 \ d_7].
\]

For a DWT, this process is then iteratively repeated on the smooth values to obtain the following:

\[
[s_0 \ D_0 \ s_1 \ D_1 \ s_2 \ D_2 \ s_3 \ D_3 \ s_4 \ D_4 \ s_5 \ D_5 \ s_6 \ D_6 \ s_7 \ D_7].
\]

and then permuted to:

\[
[s_0 \ s_1 \ s_2 \ s_3 \ D_0 \ D_1 \ D_2 \ D_3 \ s_4 \ D_4 \ s_5 \ D_5 \ s_6 \ D_6 \ s_7 \ D_7].
\]

In this example we must now stop, as the length of the next smooth sub-vector (length four) equals the length of the analysis filter (length four). Reconstruction is an exact reversal of this procedure, using the transpose (inverse) of the forward transform matrix.

For our generator, we chose Daubechies wavelets because they produce more accurate coefficients of wavelets than Haar wavelets (for more detailed discussions, see also [26, 27]); and our results of the comparison in Section 6.

5. A Fast Algorithm for Generating Self-Similar Teletraffic

We present a new generator of pseudo-random self-similar sequences based on fractional Gaussian noise (FGN) and Daubechies wavelets (DW), called the FGN–DW method [28]. A pseudo-random generator of self-similar teletraffic based on Haar wavelet transforms has been proposed in [29, 30] and [31]. We used Daubechies wavelets because the generator based on Daubechies wavelets produces more accurate self-similar sequences than one based on Haar wavelets. In other words, not only estimates of \( H \) obtained from the Daubechies wavelets are closer to the true values than those from the Haar wavelets, but also variances obtained from the Daubechies wavelets are lower. The reason behind is that the Daubechies wavelets produce smoother coefficients of wavelets that are used in the discrete wavelet transform than the Haar wavelets [26, 27, 32]. Haar wavelets are discontinuous, and they do not have good time-frequency localisation properties, since their Fourier transforms decay as \(|\lambda|^{-1}\), for \( \lambda \to \infty \), meaning that the resulting decomposition has a poor scale. Therefore, Daubechies wavelets produce more accurate coefficients than Haar wavelets: for a more detailed discussion, see [26, 27].

Our method for generating synthetic self-similar FGN
sequences in a time domain is based on a discrete wavelet transform (DWT). Wavelets can provide compact representations for a class of FGN processes [22, 23, 32], because the structure of wavelets naturally matches the self-similar structure of long-range dependent processes [24, 26, 27].

We claim that the FGN-DW method is sufficiently fast for the practical generation of synthetic self-similar sequences that can be used as simulation input data. The general strategy behind our method is similar to Paxson's, who used the Fourier transform [16]. Figure 1 graphically illustrates a discrete Fourier and a discrete wavelet transform. Wavelet analysis transforms a sequence onto a time-scale grid, where the term scale is used instead of frequency, because the mapping is not directly related to frequency as in the Fourier transform. The wavelet transform delivers good resolution in both time and scale, as compared to the Fourier transform, which provides only good frequency resolution. The algorithm consists of the following steps.

Step 1: Given $H$. Start for $i = 1$ and continue until $i = n$.

1. Calculate a sequence of values $\{f_1, f_2, \ldots, f_n\}$ using Equation (9), where $f_i = \tilde{f}(\pi i/n; H)$, corresponding to the spectral density of an FGN process for frequencies $f_i$ ranging between $\pi/n$ and $\pi$. The main difficulty with using Equation (8) when computing the spectral density is that it requires to execute the infinite summation. This formula was used in the generation of self-similar sequences proposed in this paper. Another generator of self-similar sequences based on FGN was also proposed by Paxson [16], but his method was based on a more complicated approximation of $f(\lambda, H)$ as shown in Equation (8). Equation (9) can be used to determine $f(\lambda, H)$ for $\lambda \rightarrow \infty$, or for $n \rightarrow \infty$ at $\lambda = \pi/n$. For a large value of $\lambda$, $f(\lambda, H)$ can be calculated by Equation (8).

Step 2: Multiply $\{f_i\}$ by realisations of an independent exponential random variable with a mean of one to obtain $\{\tilde{f}_i\}$, because the spectral density estimated for a given frequency is distributed asymptotically as an independent exponential random variable with mean $f(\lambda, H)$ [8].

Step 3: Generate a sequence $\{Y_1, Y_2, \ldots, Y_n\}$ of complex numbers such that $|Y_i| = \sqrt{\tilde{f}_i}$ and the phase of $Y_i$ is uniformly distributed between 0 and $2\pi$. This random phase technique, taken from Schiff [33], preserves the spectral density corresponding to $\{\tilde{f}_i\}$. It also makes the marginal distribution of the final sequence normal and produces the requirements for FGN.

Step 4: Calculate the two synthetic coefficients of orth-normal Daubechies wavelets that are used in the inverse DWT (IDWT) [34]; also see Appendix A. The output sequence $\{X_1, X_2, \ldots, X_n\}$ representing approximately self-similar FGN process (in time domain) is obtained by applying the IDWT operation to the sequence $\{Y_1, Y_2, \ldots, Y_n\}$.

Using the previous steps, the proposed FGN-DW method generates a fast and sufficiently accurate self-similar FGN process $\{X_1, X_2, \ldots, X_n\}$. (Appendix A provides a program.
written in Matlab for implementing this method using the
pyramidal algorithm of IDWT). It took 16 seconds to gen-
erate a sequence of 1,048,576 numbers on a Pentium II (233
MHz, 512 MB). Its theoretical algorithmic complexity is
$O(n)$. Moreover, the accuracy of Daubechies wavelets is
slightly better than Haar wavelets, but there is no differ-
ence in the time taken to obtain the same number of
coefficients.

6. Analysis of Self-Similar Sequences

The generator of self-similar sequences of self-similar
pseudo-random numbers described in Section 5 has been
implemented in Matlab on a Pentium II (233 MHz, 512 MB):
see Appendix A. The mean times required for generating
sequences of a given length were obtained by using the
Matlab clock command and averaged over 30 iterations,
having generated sequences of 32,768 ($2^{15}$), 131,072 ($2^{17}$),
262,144 ($2^{18}$), 524,288 ($2^{19}$) and 1,048,576 ($2^{20}$) numbers.

We have also analysed the efficiency of the method. For
each of $H = 0.6, 0.7, 0.8$ and 0.9, the method was used to
generate over 100 sample sequences of 32,768 ($2^{15}$) numbers
starting from different random seeds. Self-similarity and
marginal distributions of the generated sequences were
assessed by applying the best currently available estima-
tors: the wavelet-based $H$ estimator and Whittle’s MLE.

- **Whittle’s approximate maximum likelihood estimate
(MLE):** for a more refined data analysis, used to obtain
confidence intervals (CIs) for the Hurst parameter $H$
[17]. On the other hand, it examines the properties in
frequency domain, while the R/S statistic plot and variance-
time plot focus on the time domain. Suppose
$(x_1, x_2, \ldots, x_n)$ is a sequence of a self-similar process
$(X_t)$ for which all parameters are known except
$\text{Var}(X_t)$ and $H$. Let $f(\lambda; H)$ be the spectral density
of $(X_t)$ when normalised to have variance 1, and $I(\lambda)$
be the periodogram of $(X_t)$. Then to estimate $H$, find
$\hat{H}$ that minimises the following equation:
$g(\hat{H}) = \int_{-\infty}^{\infty} I(\lambda)/f(\lambda; \hat{H}) \, d\lambda$.

- **Wavelet-based $H$ estimator:** The original wavelet-
based $H$ estimator, proposed by Abry and Veitch in 1998
[24], is a computationally simple and fast estimator
based on a wavelet transform, with a fast pyramidal al-
gorithm for the wavelet transform, with its complexity
of order $O(n)$. However, as argued in [35] this estimator
suffers from a bias associated with its log-log regression
component. Later, Veitch and Abry [35] proposed a new
improved method of estimation of the $H$ parameter with-
in a so-called wavelet-based joint estimator, which al-
low us to estimate both $H$ and so-called power parame-
ter, an independent quantitative parameter with the unit
of variance; see [35] and [36] for details. The resulted
wavelet-based $H$ estimator, that we further simply call
the wavelet-based $H$ estimator, is asymptotically un-
based and (almost) the most efficient [35]. For detailed
discussions, see [37].

6.1 Comparison of Daubechies Wavelets and Haar Wavelets
for Generation of LRD Sequences

The estimates of the Hurst parameter obtained from the
wavelet-based $H$ estimator and Whittle’s MLE, have been
used to analyse the accuracy of the generator. The relative
inaccuracy $\Delta H$ is calculated using the formula:
$\Delta H = (\hat{H} - H)/H \times 100\%$, where $H$ is the required value of
the Hurst parameter and $\hat{H}$ is the empirical mean value over
a number of independently generated sequences. The pre-
sented numerical results are all averaged over 100
sequences.

Comparison results of sequences produced by generators
based on Haar and Daubechies wavelets with several co-
efficients are shown in Tables 1 and 2. The relative error
associated with each wavelet were also compared. The re-
results indicate that Daubechies wavelets with sixteen co-
efficients produce the most accurate results and are slightly
more accurate than Haar wavelets. In addition, <Table 3>
and <Table 4> show that the variances obtained from
Daubechies wavelets are smaller than those obtained from
Haar wavelets. However, the two wavelets theoretically re-
quire the same $O(n)$ operations to transform $n$ coeffi-
cients of their wavelets, and there is no difference in the
time required to obtain a given number of their coefficients.
For more detailed discussions of Haar wavelets, see [36, 27].

6.2 Analysis of Hurst parameters

The results for the wavelet-based $H$ estimator and
Whittle’s MLE of $H$ with the corresponding 95% CIs
$\hat{H} \pm 1.96 \frac{\sigma}{\sqrt{n}}$, see <Table 1> and <Table 2> for Daub(16),
show that for all input $H$ values, the FGN-DW method pro-
duces sequences with negatively biased $H$ values, except
$H = 0.6$, for the wavelet-based $H$ estimator. The FGN-DW method demonstrated a high level of accuracy and was fast.

Table 1: Comparison of Daubechies wavelets and Haar wavelets: mean values of estimated $H$ obtained using the wavelet-based $H$ estimator for $H = 0.6, 0.7, 0.8$ and $0.9$. Daub(#) stands for the Daubechies wavelets with # coefficients.

<table>
<thead>
<tr>
<th>Methods</th>
<th>$H$</th>
<th>Mean Values of Estimated $H$ and $\Delta H$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\bar{H}$</td>
<td>$\delta H$ (%$\bar{H}$)</td>
</tr>
<tr>
<td>Haar</td>
<td>6.073</td>
<td>+1.230</td>
</tr>
<tr>
<td>Daub(2)</td>
<td>6.019</td>
<td>+0.323</td>
</tr>
<tr>
<td>Daub(4)</td>
<td>6.065</td>
<td>+0.433</td>
</tr>
<tr>
<td>Daub(8)</td>
<td>6.036</td>
<td>+0.480</td>
</tr>
<tr>
<td>Daub(16)</td>
<td>6.013</td>
<td>+0.214</td>
</tr>
</tbody>
</table>

Table 2: Comparison of Daubechies wavelets and Haar wavelets: mean values of estimated $H$ obtained using Whittle’s MLE for $H = 0.6, 0.7, 0.8$ and $0.9$. Daub(#) stands for the Daubechies wavelets with # coefficients.

<table>
<thead>
<tr>
<th>Methods</th>
<th>$H$</th>
<th>Mean Values of Estimated $H$ and $\Delta H$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\bar{H}$</td>
<td>$\delta H$ (%$\bar{H}$)</td>
</tr>
<tr>
<td>Daub(2)</td>
<td>5.014</td>
<td>-3.106</td>
</tr>
<tr>
<td>Daub(4)</td>
<td>5.036</td>
<td>-2.736</td>
</tr>
</tbody>
</table>

Table 3: Variances of estimated $H$ obtained using the wavelet-based $H$ estimator for Daubechies wavelets and Haar wavelets for $H = 0.6, 0.7, 0.8$ and $0.9$. Daub(#) stands for the Daubechies wavelets with # coefficients.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Variances of Estimated $H$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma^2$</td>
</tr>
<tr>
<td>Haar</td>
<td>2.070e-04</td>
</tr>
<tr>
<td>Daub(2)</td>
<td>1.949e-04</td>
</tr>
<tr>
<td>Daub(4)</td>
<td>2.509e-04</td>
</tr>
<tr>
<td>Daub(8)</td>
<td>2.440e-04</td>
</tr>
<tr>
<td>Daub(16)</td>
<td>2.160e-04</td>
</tr>
</tbody>
</table>

6.3 Autocorrelation and Sequence Plot for the FGN-DW Method

ACFs characterise the correlation structure and are used to investigate the behaviour of self-similar sequences. For $H = 0.6, 0.7, 0.8$ and $0.9$, (Figure 2) shows the ACF of sequences obtained by the FGN-DW method and the theoretical ACFs from Equation (5). As the $H$ value increased, the ACF curves decayed hyperbolically and LRD was observed. For $H = 0.6, 0.7, 0.8$ and $0.9$, relative inaccuracy $\Delta ACF$ estimated from the ACF was $-0.0208\%$, $-0.0061\%$, $-0.0076\%$ and $-0.0218\%$, respectively.

Table 4: Variances of estimated $H$ obtained using Whittle’s MLE for Daubechies wavelets and Haar wavelets for $H = 0.6, 0.7, 0.8$ and $0.9$. Daub(#) stands for the Daubechies wavelets with # coefficients.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Variances of Estimated $H$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma^2$</td>
</tr>
<tr>
<td>Haar</td>
<td>1.311e-05</td>
</tr>
<tr>
<td>Daub(2)</td>
<td>1.191e-05</td>
</tr>
<tr>
<td>Daub(4)</td>
<td>1.653e-05</td>
</tr>
<tr>
<td>Daub(8)</td>
<td>1.100e-05</td>
</tr>
<tr>
<td>Daub(16)</td>
<td>1.094e-05</td>
</tr>
</tbody>
</table>
Sequence plots show stronger data correlation as the $H$ value increased; see (Figure 3) for the FGN-DW method. In other words, generated sequences demonstrated evidence of LRD properties.

Our results show that the generator produces approximately self-similar FGN sequences, with the relative inaccuracy, $\Delta H$, increasing with the increase of $H$, but always staying below 8%. Apparently there is a problem with more detailed studies of such a generator, since different methods of analysis of the Hurst parameter can give different results for the bias of $R$ in the same output sequences. More reliable methods for assessment of self-similarity in
pseudo-random sequences are needed.

6.4 Computational Complexity

The results of our experimental analysis of mean times needed by the generator for generating pseudo-random self-similar sequences of a given length are shown in Table 5. The main conclusion is that the FGN-DW method is fast. Table 5 shows that 2 seconds were needed to generate a sequence of 32,768 (2^15) numbers, while generation of a sequence with 1,048,576 (2^20) numbers took 51 seconds. The theoretical algorithmic complexity of forming spectral density, and constructing normally distributed complex numbers, is O(n), while the inverse DWT is O(n) [16, 27]. Thus, the time complexity of FGN-DW is also O(n).

In summary, our results show that a generator of pseudo-random self-similar sequences based on FGN and DW is sufficiently fast to make it applicable in practical computer simulation studies, when long self-similar sequences of numbers are needed.

<table>
<thead>
<tr>
<th>Complexity</th>
<th>Sequence of Numbers</th>
<th>Mean running time (minute : second)</th>
</tr>
</thead>
<tbody>
<tr>
<td>O(n)</td>
<td>0 : 2</td>
<td>0 : 13</td>
</tr>
<tr>
<td></td>
<td>0 : 25</td>
<td>0 : 51</td>
</tr>
</tbody>
</table>

7. Conclusions

In this paper we have proposed a generator of (long) pseudo-random self-similar sequences, based on the FGN and DW transform. It appears that this generator produces approximately self-similar sequences, with the relative inaccuracy of the resulted H below 8%, if 0.6 ≤ H ≤ 0.9. On the other hand, the analysis of mean times needed for generating sequences of a given length shows that this generator should be recommended for practical simulation studies of telecommunication networks, since it is very fast and accurate.

Acknowledgments

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References


Appendix A : A Program for Generating Self-Similar Sequences

Here is a set of Matlab functions for implementing the method, based on FGN and Daubechies wavelets, described in this paper.
% This function returns a self-similar sequence with n numbers and the Hurst parameter $H$.
function SS = FGNDW(n, $H$, Scale, VanishingMoment)

% Create n frequencies, then calculate fast, approximately power spectrum.
lambda = ((1:n)*pi)/n;
$F = FGNDWSpectrum(lambda, H) ;$

% Adjust for estimating power spectrum via periodogram.
$r = randperm('Exponential', 1, 1, n);
$f_r = f.*r ;$

% Construct corresponding complex numbers with random phase.
$re = sqrt(f_r) ;$
$im = randperm('Uniform', 0, 2*pi, 1, n) ;$

% Calculate complex spectral density and real part/imaginary part form.
real_part = re.*cos(im) ;
imag_part = re.*sin(im) ;
$z = real_part + imag_part*i ;$

% Calculate filter values using I. Daubechies’ algorithm.
$[h, g, rh, rg] = daub(VanishingMoment) ;$

% Calculate a sequence in time domain using inverse discrete wavelet transform (IDWT).
$SS = real(iwt(z, rh, rg, Scale)) ;$

% Returns an approximation of the power spectrum for Fractional Gaussian Noise at the given
% frequencies lambda and the given Hurst parameter $H$.
% function FGNS = FGNDWSpectrum(lambda, $H$)
$cf = (1/(2*pi))*var(lambda)*sin(pi*H)*gamma(2*H+1) ;
FGNS = cf*(abs(lambda)).*(1-2*H) ;$

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