PAPR Reduction Improvement for WHT-based OFDM System using Data Grouping Technique

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ABSTRACT

The conventional OFDM (Orthogonal Frequency Division Multiplexing) modulation can be combined with WHT (Walsh-Hadamard Transform) to reduce PAPR (Peak-to-Average Power Ratio) and improve BER (Bit Error Rate) performance. However, this WHT-based OFDM system still suffers a relatively high PAPR. Therefore, we suggest a new technique, called DGT (Data Grouping Technique) and design an OFDM system employing it and WHT to further decrease PAPR without the BER performance degradation. A salient property of DGT is the independence of the side information which is inherently a principal drawback of the well-known PAPR reduction techniques for OFDM system as well as WHT-based OFDM. The simulation programs have been also performed to verify the validity of the proposed system.

Key Word : OFDM, WH Transform, DGT, PAPR

1. Introduction

Communication channel makes signal pulses broadened in time as they travel through the channel (multi-path effect), leading to Inter-Symbol Interference (ISI). The pulse spread restricts the speed at which adjacent data pulses can be sent without overlap, thus limiting the maximum information rate of the wireless system. One technique to avoid the detrimental effect of multi-path, without sacrific-
meet a drastically increasing demand of information, communication and entertainment services such as voice, data and video, etc which can be accessed anywhere at anytime. However, OFDM adversely suffers some severe problems.

First, for the frequency-selective fading channels, error bits occur as their subcarriers experience deep fades in the frequency spectrum. Although this obstacle can be solved in part by using forward error-correction (FEC) codes such as convolutional code or block code, the frequency utilization efficiency is decreased. An alternative method is to perform WHT before OFDM modulation [2]-[4]. The nature of WH transform is to send one data symbol on all subcarriers simultaneously to make the frequency diversity benefit at most. As a consequence, in addition to achieving high performance and avoiding the frequency selectivity of the channel, the WHT-based OFDM systems can decrease PAPR because the orthogonality of spreading codes guarantees the probability that all subcarriers combine constructively is very small. Moreover, [5] proposed the solution to adapting the situations where the number of subcarriers is not equal to $2^n$ by making use of WH matrices with different sizes. In addition, the iterative detection can be utilized to further increase BER performance at the expense of the implementation complexity [6].

Second, the conventional OFDM system (C-OFDM) has a high PAPR that tends to lessen the power efficiency of the RF amplifier. Many PAPR reducing techniques have been suggested [1] such as signal distortion techniques, typically clipping, peak windowing and peak cancellation, coding techniques, SLM (Selected Mapping Method) and PTS (Partial Transmit Sequence). Moreover, the above mentioned WHT as well as the selection of the sets of spreading sequences between Walsh-Hadamard and complementary sequences [4] is an effective way to deal with PAPR problem. However, similar to SLM and PTS, the technique in [4] suffers the severe bit error performance degradation due to the erroneous side information (SI) which contains the information of selected spreading codes at the transmitter. In general, applying these PAPR reduction techniques is a cost of increasing transmission bandwidth, deteriorating system’s performance and making system more complicated.

As we see, WHT-based OFDM system is better than C-OFDM system in many aspects such as high performance and low PAPR. However, PAPR of WHT-based OFDM system is still high. Therefore, we propose DGT whose function is to choose a proper set of phase control gains for groups to attain a lowest PAPR before transmitting the signal over the channel. Moreover, the receiver doesn’t require SI of phase set which was embedded into transmitted data to decode the original data and thus, the system performance is independent of SI. This is the proposed technique’s prominent advantage over SLM, PTS [1] and the sequence selection [4] whose SI has an adverse effect on system performance if it is not correctly recovered.

The rest of this paper is organized as follows. Section 2 discusses details of the proposed system. The simulation results are presented in section 3 and part 4 is conclusion of the paper.

2. Proposed System Structure (P-OFDM)

2.1 Transmitter model

The suggested system block diagram is shown in (Fig. 1). First of all, the $l^{th}$ data block of modulated symbols with symbol duration $T_S$ is serial-to-parallel converted into $I$ branches which is subdivided into $M$ groups of length $L-1$, except the first group of length $L$, namely $B_m$ where $m=1,\cdots, M$ as in (Fig. 2).

The relation of $M$, $I$ and $L$ is given by

$$I = M(L-1) + 1$$

The groups with indices from 2 to $M$ are added one more symbol $b_m (m=1,\cdots, M-1)$ taken from the optimizer to produce a lowest PAPR among its all possible combinations of vector $b\{b_1, b_2, \cdots, b_M\}$.

(Fig. 1) Proposed transmitter model

(Fig. 2) Structure of $M$ groups

$a_i$: data symbols, $b_m$: phase control factors
The symbols \( b_m \) have no meaning at the receiver because the purpose of inserting \( b_m \) into useful data is only to control the phase of transmitted signal. Therefore, PAPR reduction is paid only for the decrease in spectral efficiency. Compared to the system without using the PAPR reducing technique, that is, all groups consisting of \( L \) data symbols, the bandwidth loss of this system can be calculated as follows

\[
\text{bandwidth loss} = 1 - \frac{M(L-1)+1}{ML} = \frac{M-1}{ML}
\]  

(2)

Here \( ML \) is chosen to be equal to the number of subcarriers \( N \). If \( ML \) is large, then the above loss is negligible.

We denote the input vector and output vector of WH transform block by \( a \) and \( w \), respectively

\[
a = [a_1 \ a_2 \ldots \ a_B]^T
\]

(3)

\[
w = d \ a
\]

(4)

where \( d \) is Walsh-Hadamard matrix, each column of which is considered as the spreading code of the symbol \( a_k \) in frequency domain.

\[
d = \begin{bmatrix}
d_{1,1} & \cdots & d_{1,N} \\
\vdots & \ddots & \vdots \\
d_{B,1} & \cdots & d_{B,N}
\end{bmatrix}
\]

(5)

The condition for an OFDM system.

\[
wh_{rX} = \sum_{k=1}^{B} \sum_{n=1}^{N} d_{n,k} a_k \cos(2\pi n k \Delta f)
\]

(6)

Subsequently, the signals from all parallel branches after spread in frequency domain by \( d_k \) are summed together which results in a composite signal as follows

\[
\text{cs}(t) = \sum_{k=1}^{B} \sum_{n=1}^{N} A_{a} \cos(2\pi n k \Delta f) p(t - nNT_i)
\]

(7)

where \( p(t) \) is a unit-amplitude rectangular pulse over the interval [0,NTi]. 

In practice, multi-carrier modulation block can be implemented efficiently by IFFT (Inverse Fast Fourier Transform). Therefore, the whole block diagram of (Fig. 3) can be replaced by a WH transform followed by IFFT as in (Fig. 1).

As usual, N-point IFFT is used to perform multi-carrier modulation which is equivalent to the sampling the continuous signal \( \text{cs}(t) \) with the sampling period \( 1/N\Delta f \). However, this sampling period is not short enough to capture the peaks of the signal \( \text{cs}(t) \) and thus leading to the large error in computing PAPR compared to the true PAPR value. Therefore, to increase the degree of accuracy of calculated PAPR, we follow [7] that ensures the computed PAPR to be almost approximate with the continuous PAPR value by applying oversampling with a factor of greater than four. In this paper, the signal is oversampled by a factor of 8. Therefore, the sampling duration is given by
$$T_{\text{Sampling}} = \frac{T_s}{8}$$  \hspace{1cm} (9)

To implement this intention by IFIT, we pad 7N zero samples at the tail of vector \( w \) and take 8N-point IFFT on the resulting vector to have the signal \( cs(mT_{\text{Sampling}}) \) in the discrete form which is used to calculate the PAPR defined as

$$\text{PAPR} = \frac{\max_{0 \leq m < 8N} |cs(mT_{\text{Sampling}})|^2}{\frac{1}{8N} \sum_{m=0}^{8N-1} |cs(mT_{\text{Sampling}})|^2}.$$  \hspace{1cm} (10)

where \( cs(mT_{\text{Sampling}}) \) is the transmitted signal during one OFDM symbol. For the C-OFDM, PAPR can reach the maximum value of \( N \) in the worst case as all subcarriers combine coherently.

It is easy to illustrate that there exists the data symbol combinations as an input of WH transform section which produce a low PAPR. \(<\text{Table 1}>\) giving an example of tested all possible combinations for case of \( N=4 \) shows that there are only 4 combinations of the vector \( a \) with the generated large PAPR. These combinations are easily avoided by the optimizer. Therefore, the operation mechanism of the optimizer is to compute PAPR for all possible combinations of vector \( b \) and store in its buffer. When the PAPR computation is finished, the optimizer selects an optimal combination of \( b \) corresponding to the lowest PAPR and turns the switch on to send \( cs(t) \) to the next block.

\(<\text{Table 1}>\) PAPR for different data symbol combinations for \( N=4 \)

<table>
<thead>
<tr>
<th>Input data combinations</th>
<th>Average PAPR</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0 1 1 1], [1 0 1 1], [0 1 0 0], [1 0 0 0]</td>
<td>1.3082</td>
</tr>
<tr>
<td>Others</td>
<td>0.9444</td>
</tr>
</tbody>
</table>

One of OFDM system’s outstanding advantages is capability of suppressing completely ISI (Inter-Symbol Interference) caused by multi-path dispersion channel. This is validated by inserting a guard interval \( d \) with length greater than the maximum delay spread of the channel. Although this interval reduces the spectral efficiency, the complexity of receiver is significantly decreased. Finally, the integrated signal \( cs(t) \) is transmitted after RF-up conversion.

2.1.2. Channel Model

The complex equivalent low-pass time-variant impulse response of \( p \)-path frequency selective Rayleigh fading channel is given by \( [8] \)

$$h(t, \tau) = \sum_{i=1}^{p} g_i(t) \delta(t - \tau_i)$$

where \( g(t), \tau_i \) are gain and delay of the \( i \)-th path in the power delay profile of channel. This paper assumes the channel to be WSSUS (Wide Sense Stationary Uncorrelated Scattering), and thus, \( g(t) \) is a mutually independent complex Gaussian random process with zero mean, variance \( \sigma_g^2 \) (Jakes-like algorithm for \( g(t) \) coefficients generation found in \( [8] \)) and the autocorrelation function \( [8] \):

$$R_g(\Delta t) = \sigma_g^2 J_0(2 \pi f_d r_{\text{dop}} \Delta t)$$

in which \( J_0(.) \) and \( f_{\text{dop}} \) are the zero-order Bessel function of the first kind and maximum Doppler frequency, respectively.

If the condition \( B_t < B_t < B_W \) (\( B_t \): coherent bandwidth of channel, \( B_W \): total bandwidth of system) is satisfied, then each subcarrier only undergoes a flat fading. Therefore, the frequency-selective Rayleigh fading channel’s transfer function \( H_n \) can be modeled as

$$H_n = \alpha_n e^{j \theta_n}$$  \hspace{1cm} (11)

where \( \alpha_n \) is the amplitude and \( \theta_n \) the phase in the \( n \)-th subchannel or the \( n \)-th subcarrier due to fade; \( \alpha_n \) are statistically independent Rayleigh random variables and \( \theta_n \) are uniform random variables over the interval \([0, 2\pi]\). In this paper, \( \alpha_n \) and \( \theta_n \) are assumed to be unchanged during each symbol interval but fluctuate over longer periods of time.

2.1.3 Receiver model

The received signal after frequency down-converted, guard-interval removed and analog-to-digital transformed is processed sequentially as shown in (Fig. 4). First, the discrete signal \( r(m) \) given by

$$r(m) = \sum_{k=1}^{N} A_k e^{j \phi_k} e^{j 2 \pi f_d m \Delta t} a_d s_k + v(m)$$  \hspace{1cm} (12)

is N-point fast Fourier transformed for multi-carrier demodulation, where \( a_d \) and \( \phi_k \) are in (11); \( v(m) \) is a AWGN component with zero mean and variance \( \sigma_v^2 \); \( T \) sampling period.
The resultant signal $r$ is of the following form with an assumption that the frequency synchronization is perfect:

$$
\begin{bmatrix}
    r_1 \\
    \vdots \\
    r_N
\end{bmatrix} = A \begin{bmatrix}
    a_1 \\
    \vdots \\
    a_N
\end{bmatrix} \oplus \begin{bmatrix}
    H_{11} \\
    \vdots \\
    H_{1N}
\end{bmatrix} \begin{bmatrix}
    V_1 \\
    \vdots \\
    V_N
\end{bmatrix} + \begin{bmatrix}
    V_o \\
    \vdots \\
    V_o
\end{bmatrix}, \quad n=1,\ldots,N
$$  

(13)

where

$$
r_x = \sum_{k=1}^{N} A \alpha_k e^{j\theta_k} a_k d_{nk} + V_o, \quad n=1,\ldots,N
$$  

(14)

and noise vector $V=[V_1\ V_2\ \cdots\ V_N]^T$ is FFT of $v=[v(1)\ v(2)\ \cdots\ v(N)]^T$. $\oplus$: element-by-element product; $H$: matrix of channel coefficients with each element given by (11).

Eq. (13) shows that there is interference among data symbols, called IDSI (InterData-Symbol Interference), which degrades the performance of the system.

Before the $a_k$ is decoded, $r$ is despread by its own codes $d_k = [d_{nk}\ d_{nk-1}\ \cdots\ d_{nk-(N-1)}]^T$ to produce a vector:

$$
y_k = [y_{1k}\ y_{2k}\ \cdots\ y_{nk}]^T = r \oplus d_k
$$  

(15)

Then we apply the MMSE (Minimum Mean Square Error) condition [9] on $y_k$ to recover $a_k$ as follows

$$
a'_k = \sum_{\alpha} e_{\alpha} y_{\alpha,k}
$$  

(16)

where $e_{\alpha}$ are combination coefficients given by [10]:

$$
e_{\alpha} = \frac{H^*_{\alpha}}{|H_{\alpha}|^2 + 1/\text{SNR}}
$$  

(17)

in which SNR is signal-to-noise power ratio.

It is straightforward to infer from (17) that MMSE detector minimizes the IDSI and Gaussian noise while best exploiting the frequency diversity benefits.

Finally, the detected symbols $a'_k$ are passed through a processing block and parallel-to-serial transformed to recover the original data block. The function of the processing block is to simply remove the last bits of the recovered data blocks with indices greater than 2. To explain its mechanism more clearly, we take a simple example of the system with $N=8$, two BPSK-modulated data groups and $b_m$ is chosen from the set $\{+1, -1\}$ as follows.

<table>
<thead>
<tr>
<th>Group index</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original data</td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group index</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmitted data $a$</td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

The vector $a$ as the input to the WHT block (see Fig. 1) is created by keeping the first group unchanged but inserting a bit $b_1$ into the last position of the second group as the side information. The resulting stream is given by

<table>
<thead>
<tr>
<th>Group index</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output of processing block</td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

Now we have two possible values of $b_1$ as $-1$ or $+1$. For each value, the optimizer will calculate the PAPR and finally, it selects $b_1$ which creates the lowest PAPR to fill in the last location in the second group. In this example, we assume $b_1$ to be $-1$.

Assuming that the received vector $a' = [a'_1\ a'_2\ \cdots\ a'_N]$ is $[+1\ -1\ +1\ +1\ +1\ -1\ -1]$. Then the processing block performs the reversed steps of the above. First, it remains the first group and afterward removes $b_1$ out of the received block.

As a consequence, even though the SF bit is wrongly decoded, the system performance is not affected: that is, it is independent of SF (vector $b$). This is a prominent feature of DGT over other PAPR-reducing techniques such as PTS and SLM in [1] and sequence selection in [4] which strongly depend on SF.
3. Simulation results and discussions

In this part, we will present simulation results of three systems: P-OFDM, C-OFDM and WHT-based OFDM (WH/OFDM) in a frequency selective fading channel. The complex baseband equivalent model is used for simulation. Assuming that the guard interval is longer than the maximum delay spread of the channel to completely suppress ISI and each sub-carrier only experiences a flat Rayleigh fading given in Eq. (11) with equal power expectation values.

To be convenient for the goal of comparison among the systems, computer simulations only examine the MMSE detectors. Moreover, BPSK transmission and no channel coding are considered and each element of the phase control vector \( b \) is selected from the set \( \{-1, +1\} \). In addition, the number of subcarriers is limited to \( N=16 \) and 32 for low computation complexity. Furthermore, the channel state information is assumed to be perfectly estimated at the receiver.

(Table 2) PAPR's parameters comparison for C-OFDM, WH/OFDM and P-OFDM (\( N=32-2 \) means 32 subcarriers and 2 groups)

<table>
<thead>
<tr>
<th>System</th>
<th>Mean</th>
<th>Variance</th>
<th>Max</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-OFDM</td>
<td>2.3669</td>
<td>0.1357</td>
<td>4.5011</td>
<td>1</td>
</tr>
<tr>
<td>N=16-2</td>
<td>2.3669</td>
<td>0.1357</td>
<td>4.5011</td>
<td>1</td>
</tr>
<tr>
<td>N=16-4</td>
<td>2.3669</td>
<td>0.1357</td>
<td>4.5011</td>
<td>1</td>
</tr>
<tr>
<td>N=16-8</td>
<td>2.3669</td>
<td>0.1357</td>
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<td>1</td>
</tr>
<tr>
<td>N=32-8</td>
<td>2.3669</td>
<td>0.1357</td>
<td>4.5011</td>
<td>1</td>
</tr>
<tr>
<td>C-OFDM</td>
<td>3.7192</td>
<td>1.3265</td>
<td>16.0000</td>
<td>1.7671</td>
</tr>
<tr>
<td>N=16</td>
<td>3.7192</td>
<td>1.3265</td>
<td>16.0000</td>
<td>1.7671</td>
</tr>
<tr>
<td>N=32</td>
<td>3.7192</td>
<td>1.3265</td>
<td>16.0000</td>
<td>1.7671</td>
</tr>
<tr>
<td>WH/OFDM</td>
<td>4.3643</td>
<td>1.4969</td>
<td>18.0000</td>
<td>1.9321</td>
</tr>
<tr>
<td>N=16</td>
<td>4.3643</td>
<td>1.4969</td>
<td>18.0000</td>
<td>1.9321</td>
</tr>
<tr>
<td>N=32</td>
<td>4.3643</td>
<td>1.4969</td>
<td>18.0000</td>
<td>1.9321</td>
</tr>
</tbody>
</table>

The statistical data of PAPR for the different number of subcarriers \( N=16, 32 \) is illustrated in Table 2 in which PAPR is calculated for 500000 data symbol combinations whose components are randomly selected with a uniform possibility and the sample number per an OFDM symbol is 8N. Moreover, the group number under investigation is \( M=2, 4 \) and 8. It is noted that our system becomes the conventional WHT-based OFDM if \( M=1 \). As we can see, for any value of \( N \) and \( M \), the PAPR of the proposed system is considerably better than that of C-OFDM and WH/OFDM, particularly, its PAPR fluctuates slightly around the mean value (small variance), for example: P-OFDM's average PAPR is 2.9647 (\( N=32-2 \)) while 4.3643 for C-OFDM and P-OFDM's PAPR variance is 0.2884 in contrary to 1.4969 for C-OFDM. Furthermore, other parameters such as maximum PAPR and minimum PAPR are considerably less than those of C-OFDM and WH/OFDM.

(Fig. 5) cdf of PAPR for C-OFDM, WH/OFDM and P-OFDM (\( \text{cdf} = \Pr\{\text{PAPR} < P\} \))

(Table 2) also shows that PAPR's parameters increase with respect to an increase of \( N \). With small variance, the new system demonstrates that its PAPR is less sensitive with input data combinations than C-OFDM or WH/OFDM and changes almost slowly. In addition, the more groups the input data is divided into, the more effective the PAPR reduction. For P-OFDM, the probability that PAPR is greater than 6.1221 is zero for the case of 32 subcarriers and 8 groups: and similarly 2.3669 for \( N=16 \) and \( M=8 \). However, the cost to be paid for this improvement in PAPR is the bandwidth waste but BER performance is not degraded because it is not dependent of SI (see Figs. 7-8).

In order to have an insightful and intuitive view on PAPR statistics, Figs. 5-6 plot cdf (complementary cumulative density function) of PAPR for \( N=16, 32 \) for \( M=2, 4, 8 \) groups. Once again these figures reveal a dramatic improvement in alleviating a high PAPR in the new system over C-OFDM and WH/OFDM. For instance, a PAPR reducing enhancement of the P-OFDM is about 5dB over C-OFDM and approximately 1.5dB over WH/OFDM at cdf of \( 10^{-4} \) for \( N=16 \) and \( M=2 \).
BER performance in frequency selective Rayleigh fading channel with AWGN illustrated in Figs 7–8 shows that the performance of C-OFDM is significantly degraded by fading channel. However, for P-OFDM, due to frequency diversity benefit, its performance is considerably improved in comparison to C-OFDM with approximately 5dB SNR gain at BER $10^{-2}$ for any $N$ and $M$. Furthermore, BER enhancement accelerates with respect to an increase in the subcarrier number while C-OFDM’s BER is almost unchanged. This is because the frequency diversity gain also increases according to $N$. Obviously, this property cannot be found in C-OFDM. Figs. 7–8 also demonstrate that there is no performance degradation for P-OFDM as the number of groups increases because as reasoned in part 2.1 that P-OFDM’s performance is independent on the number of groups. Therefore, the performance of P-OFDM is also identical to that of WH/OFDM.

4. Conclusion

WHT-based OFDM system outperforms the conventional OFDM system in both aspects of BER performance and PAPR reduction without bandwidth expansion. By subdividing the input symbol stream into smaller groups, our proposed system corrects the disadvantage of WHT-based OFDM in high PAPR. The larger the number of groups, the more PAPR reduction increases and the less bandwidth utilization efficiency but not deteriorate error bit probability performance. Therefore, our system is suitable to apply for the transmitters of high frequency amplifiers with small dynamic range and operated in harsh environments as multi-path fading channels.

References


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