Performance Analysis of Transmit Weights Optimization for Cooperative Communications in Wireless Networks

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ABSTRACT

Cooperative communications among users in multiple access wireless environments is an efficient way to obtain the powerful benefits of multi-antenna systems without the demand for physical arrays. This paper proposes a solution to optimize the weights of partnering users’ signals for the minimum error probability at the output of maximum likelihood (ML) detector under the transmit power constraints by taking advantage of channel state information (CSI) feedback from the receiver to the transmitter. Simulation programs are also established to evaluate the performance of the system under flat Rayleigh fading channel plus AWGN (Additive White Gaussian Noise).

Key Words: Cooperative Communications, Rayleigh Fading, AWGN, CSI, ML

1. Introduction

Signal fading due to multi-path propagation is a serious problem in wireless communications. Using a diversified signal in which information related to the same data appears in multiple time instances, frequencies, or antennas that are independently faded can reduce considerably this effect of the channel [11]. Among well-known diversity techniques, the spatial diversity has received a great deal of attention in recent years because of the feasibility of deploying multiple antennas at both transmitter and receiver [2]. However, when wireless mobiles may not be able to support multiple antennas due to size or other constraints [3], the spatial diversity is unobtainable. To overcome this restriction, a new technique, called cooperative communications, was born which allows single-antenna mobiles to gain some benefits of transmit diversity. The main idea is that in a multi-user network, two or more users share their information and transmit jointly as a virtual antenna array. This enables them to obtain higher diversity than they could have individually. The way the users share information is by tuning into each other’s transmitted signals and by processing information that they overhear. Since the inter-user channel is noisy and faded, this overheard information is not perfect. Hence, one has to carefully study the possible signaling strategies that can exploit the benefits of cooperative communications at most. There are three basic cooperative signaling methods [3] where amplify and

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forward strategy is the simplest and applicable in many wireless networks such as wireless sensor network, mobile communications network, ad-hoc network, relay network, etc. This is because in every wireless network, there must be a second independent propagation path through an idle user besides the direct link for the signal transmission to the destination. Thus, transmit diversity is obtained to combat shadowing and deep fading. Compared to single transmission, the amplify and forward-based cooperation showed a significant performance improvement and channel capacity increase [4]-[9]. However, a majority of the work on this protocol only concentrates on signal combining at the receiver to minimize the BER. It is well-known that transmit diversity systems can perform better if the knowledge about the channel can be exploited to adapt the weights for each transmit antenna in such a way that the SNR at the receiver is maximized [10]. Similarly, it is possible to apply this principle to the cooperative communications by considering each user’s antenna as an element of the physical antenna array and assigning amplification factors adaptively to the channel variation. These amplification factors are practically acquired through training sequences and feedback channel from the receiver to the transmitter. This is our motivation to develop an algorithm to optimize weights of user signals with the goal of the minimum error probability through ML detection. With optimum factors, the signal detection at the receiver is extremely simple. In fact, [7] investigated the similar optimization problem but the maximum ratio combining-based detection technique at the receiver is different from and more complicated than that in this paper. Moreover, the optimum factors found in [7] is just approximate, not exact as in our paper.

The rest of the paper is organized as follows. Section 2 presents a solution to optimize the transmit weights for a practical and generic wireless network of $M$ relays. Then simulation results that compare the performance of the proposed cooperation with non-cooperation (without a relay) are exposed in section 3. Moreover, this section also discusses thoroughly about the achieved results. Finally, the paper is closed in section 4 with a conclusion.

2. Weights Optimization

Consider a cooperative transmission in a generic wireless network consisting of single-antenna entities: a source (S), $M$ relays ($R_m$) and a destination (D) as depicted in (Fig. 1) where the function of the relay is simply to receive the signal from the source and then amplify it and finally, forward to the destination. To prevent the multi-access interference among active users, an orthogonal channel (e.g., a different time slot or a different frequency band or a different spreading code) is also allocated to each mobile unit in the network. Without the loss of generality, FDMA (Frequency Division Multiple Access) is used for channel allocation. Therefore, the destination receives ($M+1$) versions of the original signal, one from the source and the others with processing delays from the relays. Based on these data sequences, the ML detection is performed and as we will see later, this detector is very simple without the knowledge of channel state.

(Fig. 1) Cooperative transmission model

For simplicity of exposition, we use complex baseband-equivalent models to express all the signals. So, at the destination the received signals at the time $n$ after being filtered with a square-root Nyquist filter and sampled at the symbol rate can be written as

$$y_{so}[n] = \alpha_{so} x_{s}[n] + n_{so}[n]$$  \hspace{1cm} (1)

$$y_{mo}[n] = \alpha_{mo}^2 x_{m}[n] n_{so}[n]$$  \hspace{1cm} (2)

where

- $y_{j}[n] = \alpha_{jo} x_{j}[n-d_j] + n_{so}[n-d_j]$
- $m=1,\ldots,M$
- $y_{j} (i=S,m; j=m, D)$: received signal at node $j$ when the transmitted signal is from node $i$.
- $x[n]$: modulated symbol generated from S.
- $n_{so}[n], n_{so}[n], n_{so}[n]$: noise samples corrupting the S-D channel, S-R_m channel and R_m-D channel. They are modeled as independent zero-mean complex Gaussian r.v.'s (ZMCRVs) with variances $\sigma_{so}^2, \sigma_{mo}^2, \sigma_{mo}^2$ correspondingly.
\[ y[n] = y_{SD}[n-d_{max}] + \sum_{m=1}^{N} y_{n}[n-(d_{max}-d_{m})] \]

\[ = \alpha_{SD} x_{m}[n-d_{max}] + n_{SD}[n-d_{max}] \]
\[ + \sum_{m=1}^{N} \left( \alpha_{n0} x_{n0}[n-d_{max}] + n_{n0}[n-d_{max}] \right) + \sum_{m=1}^{N} \left( \alpha_{n1} x_{n1}[n-d_{max}] + n_{n1}[n-d_{max}] \right) \]
\[ = \left( \alpha_{SD} + \sum_{m=1}^{N} \alpha_{n0} x_{n0} \right) x_{n} + \sum_{m=1}^{N} \left( \alpha_{n0} x_{n0} + \alpha_{n1} x_{n1} \right) n_{n} \]
\[ + n_{SD}[n-d_{max}] + n_{n0}[n-d_{max}] + n_{n1}[n-d_{max}] \]

where \( d_{max} \) is the maximum delay time among propagation paths from \( S \) and all relays to the destination.

For further simplification, we drop the time indices in the sequel. Therefore, \( y[n] \) can be written as
\[ y = Ax + N \]

where
\[ A = \left( \alpha_{SD} + \sum_{m=1}^{N} \alpha_{n0} x_{n0} \right) \]
\[ N = \sum_{m=1}^{N} \left( \alpha_{n0} x_{n0} + \alpha_{n1} x_{n1} \right) + n_{SD} \]

Since ZMCGRVs \( n_{SD}, n_{n0}, n_{n1} \) are mutually independent of each other, conditional on the fading realizations, \( N \) is also a ZMCGRV with the variance
\[ \sigma^{2} = \sum_{m=1}^{N} \left( \left( \alpha_{n0} x_{n0} \right)^2 + \left( \alpha_{n1} x_{n1} \right)^2 \right) \sigma_{SD}^2 \]

The ML detection of \( x \) given \( y \) amounts to minimizing the following metric:
\[ |y - Ax|^2 = |A|^{-2} |x - \bar{x}|^2 + B \]

where the constant term \( B \) does not depend on \( x \) and where the quantity
\[ \bar{x} = \frac{A^T y}{|A|^2} \]

which is also a complex Gaussian r.v.'s with mean \( x \) and variance \( \sigma^{2}/|A|^2 \) can be interpreted as the output of an AWGN channel with SNR:
\[ SNR = \frac{|A|^2}{\sigma^{2}} E[|x|^2] \]

The BER performance of the ML detectors depends only on the SNR. Thus, given the channel state at \( S \) and \( R \), we can optimize \( \omega \) and \( z_{b0} \) so as to maximize the SNR in \( \bar{x} \) (or equivalently, minimize the BER). For this optimization to be meaningful, we must constrain the transmit power, otherwise the problem would yield an unbounded solution (an arbitrarily high SNR can be achieved by choosing \( \omega \) and \( z_{b0} \) large enough). For power constraints, we use an automatic gain control (AGC) at the relays in order to comply with current standards [11], which require control on the output power at the mobile units. Summarily, the problem is addressed as
\[ \max_{\omega, z_{b0}} SNR \]
subject to \( E[|u|^2] = P_s \) and \( E[|u_{b0}|^2] = P_{b0} \)

where \( P_s \) and \( P_{b0} \) are average powers per symbol of the
source and relay $m$.

Without the loss of generality, let $E[|x|^2] = 1$. Then, the constraint conditions can be represented as

$$
|w|^2 = P_s
$$

$$
|z_m|^2 = \frac{P_{rn}}{|\alpha_m|^2 P_s + \sigma_m^2}
$$

(11)

where the expectation operation is only taken on Gaussian random variable.

Based on Eq. (11), we rewrite Eq. (10) in more compact form

$$
SNR = \frac{|\alpha_{SD}|^2 \left( \sum_{m=1}^{M} \frac{\alpha_{sm} \alpha_{SD} z_m}{|\alpha_m|^2 P_s + \sigma_m^2} \right)^2}{\sum_{m=1}^{M} \left( \frac{P_{rn}}{|\alpha_m|^2 P_s + \sigma_m^2} + \frac{\sigma_m^2 + \sigma_n^2}{\sigma_{SD}^2} \right) P_s}
$$

(12)

Now $SNR$ is a function of variables $z_{rn}$. By using the knowledge of geometry, we can find the optimal values of $z_{rn}$ that maximize $SNR$ as

$$
z_{rn, opt} = C_m \left( \frac{\alpha_{sm} \alpha_{SD}}{\alpha_{SD}} \right)
$$

(13)

With the constraint on magnitude in Eq. (11), the constant $C_m$ is given by

$$
C_m = \frac{P_{rn}}{|\alpha_m|^2 P_s + \sigma_m^2}
$$

(14)

Substituting $z_{rn}$ into $A$ in Eq. (5), we have

$$
A = \left( 1 + \sum_{m=1}^{M} \frac{\alpha_{sm} \alpha_{SD} z_m}{\alpha_{SD}} \right)^2
$$

$$
= \left( 1 + \sum_{m=1}^{M} \frac{\alpha_{sm} \alpha_{SD}}{\alpha_{SD}} \frac{P_{rn}}{|\alpha_m|^2 P_s + \sigma_m^2} \left( \frac{\alpha_{sm} \alpha_{SD}}{\alpha_{SD}} \right)^2 \right)^2
$$

$$
= \left( 1 + \sum_{m=1}^{M} \frac{P_{rn}}{|\alpha_m|^2 P_s + \sigma_m^2} \left( \frac{\alpha_{sm} \alpha_{SD}}{\alpha_{SD}} \right)^2 \right)^2
$$

(15)

To prevent the phase distortion caused by fading, the term $\alpha_{SD} w$ must have zero-phase. Therefore, combining with the condition in Eq. (11), the optimal value of $w$ is given by.

$$
w_{opt} = \frac{\alpha_{SD}^*}{|\alpha_{SD}|} \sqrt{P_s}
$$

(16)

From Eqs. (12)-(14), we obtain the optimal values of $z$ and $SNR$ as

$$
z_{rn, opt} = \sqrt{\frac{P_{rn}}{|\alpha_m|^2 P_s + \sigma_m^2} \left( \frac{\alpha_{sm} \alpha_{SD}}{\alpha_{SD}} \right)^2}
$$

(17)

$$
SNR_{opt} = \sum_{m=1}^{M} \frac{P_{rn}}{|\alpha_m|^2 P_s + \sigma_m^2} + \sigma_n^2 + \sigma_{SD}^2
$$

(18)

3. Simulation Results

Monte Carlo simulations are done to evaluate the BER performance of the proposed cooperation. In the presented results, we choose the BPSK transmission and set the noise variances equally as $\sigma_n^2 = \sigma_m^2 = \sigma_{SD}^2 = 1$. The x-axis of all figures represent the signal-to-noise ratio of the source which is defined as $P_s / \sigma_{SD}^2$. Also, we assume all entities in the network transmit the same power.

For the case of BPSK modulation, the data bit can be recovered easily by

$$
\tilde{x} = \text{sign}(\text{Re}(y))
$$

(19)

where $\text{sign}(\, \cdot \,)$: a signum function; $\text{Re}(\, \cdot \,)$: real part of a complex number.

Then the probability of error is given by

$$
BER = Q(\sqrt{2SNR_{opt}})
$$

(20)

Here $Q(\, \cdot \,)$ is the Q-function.

Eqs. (3)-(19) demonstrates that the control of amplification factors at transmit sides makes the detection at the receiver extremely simple.

(Fig. 2) compares the BER performance between non-cooperation (without a relay) and the proposed collaboration with $\lambda_{sm}^2 = \lambda_{rn}^2 = 1$ and $\lambda_{rn}^2 = 0.5$, 1, 2. In this figure and those following, the numbers corresponding to the model in the legend box, for example, Cooperation-M=1-0.5 mean the performance of the cooperation strategy with
respect to the varying parameters: specifically, $\lambda_{\text{co}}=0.5$ in this example. It is found that weights optimization significantly contributes to BER improvement over the single transmission. For instance, at the target of BER $5\times10^{-5}$, the cooperation with optimum weights outperforms non-cooperation with the SNR gain of approximately 6dB, 9dB and 11dB correspondingly to the number of relays 1, 2, 3. Moreover, the performance of the proposed model changes negligibly with respect to the degradation in the quality of the channels $R_mD$.

Therefore, the properties of transmit diversity with the CSI priorly known at the transmitter for the physical antenna array also holds for the scenario of the virtual array gained from collection of single antennas of cooperative users. This once again asserts that the cooperation among single-antenna users in the wireless networks can obtain all benefits of spatial diversity.

The investigation of performance degradation in the quality of channels $S-R_m$ is depicted in (Fig. 3) where $\lambda_{\text{co}}=1$ and $\lambda_m=0.5$, 1, 2. It reveals that the performance of non-cooperation does not depend on channels $S-R_m$ while the cooperation considerably depends on the channels $S-R_m$ even though the weights optimization at the relays has been done. This is obvious since the direct transmission pays no attention to channels $S-R_m$ but the cooperation must take advantage of these channels to achieve the spatial diversity. In fact, this result has been foreseen from analytical expression in Eq. (18) in which $\sigma_m$ is one of the parameters that determine the quality of service of the cooperation. An alternative explanation for this performance degradation is that using the intermediate nodes to relay the source signal also causes the noise amplification at these nodes. However, under any condition of inter-user channel, the cooperation always outperforms non-cooperation and this gain increases when the channel quality is better and many more idle users to function as relays participate in forwarding the original signal of the source to the destination. This is certain because larger diversity gain can be obtained through many independent propagation paths.

(Fig. 3) BER performance of non-cooperation and the proposed model with different number of relays and the changing $S-R_m$ channel.

(Fig. 2) BER comparison between non-cooperation and the proposed model with different number of relays and the changing $R_mD$ channels.

(Fig. 4) BER comparison between non-cooperation and the proposed model with different number of relays and the changing $S-D$ channel.
It is evident that the direct transmission can only obtain low probability of error when the channel \( S-D \) is good. This is demonstrated in (Fig. 4) where the fading variance of the channel \( S-D \) is changed from 0.5 to 2 (\( \sigma_{\text{sp}} = 0.5, 1, 2 \)) while the others are unchanged \( \sigma_{\text{sp}} = \sigma_{\text{ex}} = 1 \). Compared to non-cooperation, the proposed cooperative transmission yields a better BER performance of about 6dB, 9dB and 10dB at BER of 5x10\(^{-3} \) for the value \( \sigma_{\text{sp}} = 1 \) and \( M=1, 2, 3 \) respectively. Moreover, this improvement keeps increasing in the increase of \( S-D \) channel quality and the number of relays.

4. Conclusion

The algorithm to optimize the amplification coefficients at the relay and source with the CSI known in advance at the transmitter to maximize the BER performance was proposed. This algorithm is applicable to an arbitrary wireless network with multiple relay nodes. The simulation results under the Rayleigh fading channel plus Gaussian noise demonstrate that the proposed cooperation considerably improves the performance of about more than 6dB over the non-cooperation regardless of the fading and noisy inter-user channels and the different number of relays. Moreover, the receiver structure with ML detector can be implemented with negligible hardware complexity. Therefore, the cooperation scheme with the proposed optimization algorithm is feasible and is a promising technique for the future wireless networks where there exist idle users to take advantage of system resources efficiently.

References


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