ON A SEMI-ININVARIANT SUBMANIFOLD OF CODIMENSION 3 WITH CONSTANT MEAN CURVATURE IN A COMPLEX PROJECTIVE SPACE

SEONG-BAEK LEE

ABSTRACT. Let $M$ be a semi-invariant submanifold of codimension 3 with lift-flat normal connection in a complex projective space. Further, if the mean curvature of $M$ is constant, then we prove that $M$ is a real hypersurface of a complex projective space of codimension 2 in the ambient space.

0. Introduction

A submanifold $M$ is called a $\textit{CR submanifold}$ of a Kaehlerian manifold with complex structure $J$ if it is endowed with a pair of mutually orthogonal and complementary differentiable distribution $(\Delta, \Delta^\perp)$ such that for any point $p \in M$ we have $J\Delta_p = M_p$, $J\Delta^\perp_p \subset M^\perp_p$, where $M^\perp_p$ denotes the normal space of $M$ at $p$ ([1]). In particular, $M$ is said to be a semi-invariant submanifold of a Kaehlerian manifold if $\dim \Delta^\perp = 1$ ([2], [12]). In this case, $M$ admits an almost contact metric structure. Furthermore new examples of nontrivial semi-invariant submanifolds in a complex projective space $\mathbb{CP}^n$ are constructed in [7] and [11]. Therefore we may expect to generalize some results which are valid in a real hypersurface to a semi-invariant submanifold. From this point of view, a semi-invariant submanifold of codimension 3 in a complex projective space are studied in [4], [5], [6], [7], [13] and so on by using properties of the third fundamental forms of the submanifold and those of the induced almost contact metric structure. One of them Ki, Li and Lee ([6]) assert that the following:

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THEOREM K-L ([6]). Let $M$ be a semi-invariant submanifold of codimension 3 with lift-flat normal connection in a complex projective space $\mathbb{C}P^{n+1}$. If the scalar curvature of $M$ is constant, then $M$ is a real hypersurface in $\mathbb{C}P^{n}$.

The main purpose of the present paper is to prove that a semi-invariant submanifold $M$ of codimension 3 in a complex projective space is a real hypersurface provided that the mean curvature of $M$ is constant.

All manifolds in this paper are assumed to be connected and of class $C^\infty$ and the dimension of the submanifold is greater than 2.

1. Preliminaries

At first we review fundamental properties on a semi-invariant submanifold of codimension 3 in a Kaehlerian manifold.

Let $\tilde{M}$ be a real $2(n + 1)$-dimensional Kaehlerian manifold equipped with parallel almost complex structure $J$ and a Riemannian metric tensor $G$, and covered by a system of coordinate neighborhoods $\{\tilde{V}; y^A\}$.

Let $M$ be a real $(2n-1)$-dimensional Riemannian manifold covered by a system of coordinate neighborhoods $\{V; x^h\}$ and immersed isometrically in $\tilde{M}$ by the immersion $i : M \rightarrow \tilde{M}$. We represent the immersion $i$ locally by $y^A = y^A(x^h)$ and $B_j = (B_j^A)$ are $(2n - 1)$-linearly independent local tangent vectors of $M$, where $B_j^A = \partial_j y^A$ and $\partial_j = \partial/\partial x^j$.

Let $A, B, C, D$ and $E$ run over $1, 2, \cdots, 2n + 2$ and let $h, i, j, k, r$ and $s$ run from $1, 2, \cdots, 2n - 1$. The summation convention will be used with respect to those system of indices. Three mutually orthogonal unit normals $C, D$ and $E$ may be chosen. Since the immersion $i$ is isometric, the induced Riemannian metric tensor $g$ with components on $M$ is given by $g_{ji} = G(B_j, B_i)$.

Denoting by $\nabla_j$ the operator of van der Wareden-Bortolotti covariant differentiation with respect to $g$, equations of the Gauss for $M$ of $\tilde{M}$ is obtained:

\begin{equation}
\nabla_j B_i = A_{ji}C + K_{ji}D + L_{ji}E,
\end{equation}

where $A_{ji}, K_{ji}$ and $L_{ji}$ are components of the second fundamental forms in the direction $C, D$ and $E$ respectively. Equations of Weingarten are
also given by

\[ \nabla_j C = -A_j^h B_h + l_j D + m_j E, \]
\[ \nabla_j D = -K_j^h B_h - l_j C + n_j E, \]
\[ \nabla_j E = -L_j^h B_h - m_j C - n_j D, \]

where \( A = (A_j^h) \), \( A_{(2)} = (K_j^h) \) and \( A_{(3)} = (L_j^h) \), which are related by \( A_{ji} = A_j^r g_{ir} \), \( K_{ji} = K_j^r g_{ir} \) and \( L_{ji} = L_j^r g_{ir} \) respectively, and \( l_j, m_j \) and \( n_j \) being components of the third fundamental forms.

As is well-known, a submanifold of a Kaehlerian manifold \( \bar{M} \) is said to be a CR submanifold ([1], [14]) if it is endowed with a pair of mutually orthogonal complementary differentiable distribution \((\Delta, \Delta^{\perp})\) such that for any \( p \in \bar{M} \) we have \( J \Delta_p = M_p \), \( J \Delta_p^{\perp} \subset M_p^{\perp} \), where \( M_p^{\perp} \) denotes the normal space of \( M \) at \( p \). In particular, \( M \) is said to be a semi-invariant submanifold if \( \dim \Delta^{\perp} = 1 \), and the unit normal vector in \( J \Delta^{\perp} \) is called a distinguished normal to the submanifold and denoted this by \( C \) ([2], [12]). Then we can write

\[ J B_i = \phi_i^h B_h + \xi_i C, \quad J C = -\xi^h B_h, \quad J D = -E, \quad J E = D, \]

where we have put \( \phi_{ji} = G(J B_j, B_i) \), \( \xi_j = G(J B_j, C) \), \( \xi^h \) being associate components of \( \xi_h \) ([7]). A tensor field of type (1,1) with components \( \phi_j^h \) will be denoted by \( \phi \). By properties of the almost complex structure \( J \), it is, using (1.3), seen that

\[ \phi_i^r \phi_r^h = -\delta_i^h + \xi_i \xi^h, \quad \xi_i \phi_i^r = 0, \quad \xi^r \phi_r^h = 0, \]
\[ \xi_i \xi^r = 1, \quad g_{ir} \phi_j^r \phi_i^s = g_{ji} - \xi_j \xi_i. \]

In the sequel, we denote the normal components of \( \nabla^{\perp} C \) by \( \nabla_j C \). The distinguished normal is said to be parallel in the normal bundle if we have \( \nabla^{\perp} C = 0 \), that is, \( l_j \) and \( m_j \) vanish identically.

Since \( J \) is parallel, differentiating (1.3) covariantly along \( M \) and making use of (1.1), (1.2) and (1.3) itself, we find ([13])

\[ \nabla_j \phi_i^h = -A_{ji} \xi^h + A_j^h \xi_i, \]
\[ \nabla_j \xi_i = -A_{jr} \phi_i^r, \]
\[ K_{ji} = -L_{jr} \phi_i^r - m_j \xi_i, \]
\[ L_{ji} = K_{jr} \phi_i^r + l_j \xi_i. \]
REMARK 1. To write our formulas in a convention form, in what follows we denote by \( \alpha = A_{rs} \xi^r \xi^s \), \( \beta = A_{rs} \xi^r \xi^s h \), \( h = T_r A \), \( k = T_r A_{(2)} \), \( h_{(2)} = T_r A^2 \), \( K_{(2)} = T_r A_{(2)}^2 \), \( L_{(2)} = T_r A_{(3)}^2 \), and for a function \( f \) we denote by \( \nabla f \) the gradient vector field of \( f \).

We notice here that we may assume \( T_r A_{(3)} = 0 \) (see [7]). Thus, it is, using (1.6) and (1.7), verified that

\[
(1.8) \quad K_{jr} \xi^r = -m_j, \quad L_{jr} \xi^r = l_j,
\]

\[
(1.9) \quad m_r \xi^r = -k, \quad l_r \xi^r = 0.
\]

Further, we obtain

\[
(1.10) \quad \phi_{jr} m^r = -l_j, \quad \phi_{jr} l^r = m_j + k \xi_j,
\]

\[
(1.11) \quad K_{jr} L_i^r + K_{ir} L_j^r + l_j m_i + l_i m_j = 0.
\]

2. Auxiliary results

In order to prove our results we present in this section some notation, terminology and auxiliary results.

In the rest of this paper we shall suppose that \( \hat{M} \) is a Kaehlerian manifold of constant holomorphic sectional curvature \( c \), which is called a complex space form and denoted by \( M_{n+1}(c) \). Then equations of Gauss and Codazzi are given by

\[
R_{kjih} = \frac{c}{4} (g_{kh} g_{ji} - g_{jh} g_{ki} + \phi_{kh} \phi_{ji} - \phi_{jh} \phi_{ki} - 2 \phi_{kj} \phi_{ih})
\]

\[
+ A_{kh} A_{ji} - A_{jh} A_{ki} + K_{kh} K_{ji} - K_{jh} K_{ki}
\]

\[
+ L_{kh} L_{ji} - L_{jh} L_{ki},
\]

\[
\nabla_k A_{ji} - \nabla_j A_{ki} - l_k K_{ji} + l_j K_{ki} - m_k L_{ji} + m_j L_{ki}
\]

\[
= \frac{c}{4} (\xi_k \phi_{ji} - \xi_j \phi_{ki} - 2 \xi_i \phi_{kj}),
\]
(2.3) \[ \nabla_k K_{ji} - \nabla_j K_{ki} + l_k A_{ji} - l_j A_{ki} - n_k L_{ji} + n_j L_{ki} = 0, \]
(2.4) \[ \nabla_k L_{ji} - \nabla_j L_{ki} + m_k A_{ji} - m_j A_{ki} + n_k K_{ji} - n_j K_{ki} = 0, \]
where \( R_{kjih} \) are covariant components of the Riemann-Christoffel curvature tensor of \( M \), and those of the Ricci by

(2.5) \[ \nabla_k l_j - \nabla_j l_k + A_{kr} K_{jr}^r - A_{jr} K_{kr}^r + m_k n_j - m_j n_k = 0, \]
(2.6) \[ \nabla_k m_j - \nabla_j m_k + A_{kr} L_{jr}^r - A_{jr} L_{kr}^r + n_k l_j - n_j l_k = 0, \]
(2.7) \[ \nabla_k n_j - \nabla_j n_k + K_{kr} L_{jr}^r - K_{jr} L_{kr}^r + l_k m_j - l_j m_k = \frac{c}{2} \phi_{kj}. \]

Now, we put \( U_j = \xi^r \nabla_r \xi_j \). Then \( U \) is orthogonal to the structure vector \( \xi \). Because of (1.5) and properties of the almost contact metric structure, it follows that

(2.8) \[ \phi_{jr} U^r = A_{jr} \xi^r - \alpha \xi_j, \]
(2.9) \[ U^r \nabla_j \xi_r = A_{jr} \xi^r - \alpha A_{jr} \xi^r. \]

From (2.8) we get \( g(U, U) = \beta - \alpha^2 \). Therefore we easily see that \( A \xi = \alpha \xi \) if and only if \( \beta - \alpha^2 = 0 \). Differentiating (2.8) covariantly and taking account of (1.4) and (1.5), we find

(2.10) \[ \xi_j (A_{kr} U^r + \alpha_k) + \phi_{jr} \nabla_k U^r = \xi^r \nabla_k A_{jr} - A_{jr} A_{ks} \phi^s + \alpha A_{kr} \phi_j^r, \]
where we put \( \alpha_k = \nabla_k \alpha \), which shows that

(\( \nabla_k A_{rs} \)) \( \xi^r \xi^s = 2 A_{kr} U^r + \alpha_k, \)

which together with (1.8), (1.9) and (2.2) implies that

(2.11) \[ (\nabla_r A_{js}) \xi^r \xi^s = 2 A_{jr} U^r + \alpha_j + 2 k l_j. \]

By means of (1.4), (1.5) and (2.11) it is verified that

(2.12) \[ \xi^r \nabla_r U_i = -3 U^r A_{rs} \phi_i^s + \alpha A_{rs} \xi^r - \beta \xi_i - \phi_{ir} \alpha^r - 2 k \phi_i l^r. \]

The normal connection of a semi-invariant submanifold of codimension 3 in a complex space form is said to be lift-flat if it satisfies \( dn = \frac{c}{2} \omega \), that is,

(2.13) \[ \nabla_j n_i - \nabla_i n_j = \frac{c}{2} \phi_{ji}, \]
where \( \omega(X, Y) = g(\phi X, Y) \) for any vectors \( X \) and \( Y \) on \( M \) (see [9]).
LEMMA 2.1. Let $M$ be a semi-invariant submanifold of codimension 3 in a complex space form $M_{n+1}(c)$. Then the normal connection of $M$ is lift-flat if and only if $A_{(2)}A_{(3)} = A_{(3)}A_{(2)}$.

PROOF. Suppose that (2.13) is valid on $M$. Then we have by (2.7)

$$K_{jr}L_i^r - K_{ir}L_j^r + l_jm_i - l_i m_j = 0$$

or using (1.11)

$$K_{jr}L_i^r + l_j m_i = 0.$$ 

From this, (1.8) and (1.9), it is seen that $L_{ir}m^r = 0$ and hence $(m_{ir}m^r)l_j = 0$. Thus it follows that $l_j = 0$ because of (1.10). Therefore (2.14) is reduced to $A_{(2)}A_{(3)} = A_{(2)}A_{(3)}$.

Conversely, if $A_{(2)}$ and $A_{(3)}$ mutually commutes, then (1.11) turns out to be

$$2K_{jr}L_i^r + l_j m_i + l_i m_j = 0,$$

which together with (1.8), (1.9) and (1.10) gives

$$2K_{jr}r^r = kl_j, \quad 2L_{ir}m^r = -kl_i.$$

From the last three equations, we see that

$$l_j\{2m_{ir}m^r - k^2\} = 0,$$

which connected with the first equation of (1.9) implies that

$$\{\|m_i\|^2 + \|m_i + k\xi_i\|^2\}l_j = 0.$$ 

If we take account of (1.10) and the last equation, then we verify that $l_j = 0$. Thus (2.16) becomes $K_{jr}L_i^r = 0$, which together with (1.6) yields $K_{ji}^2 - k\xi_j\xi_i = 0$ and hence $K_{(2)} = k^2$ and $K_{jr}\xi^r = k\xi_j$. From these relationships, it is clear that

$$K_{ji} = k\xi_j\xi_i.$$ 

Thus (2.7) is reduced to $\nabla_j n_i - \nabla_i n_j = \frac{s}{2}\phi_{ji}$ since we have $l_j = 0$. This completes the proof. $\square$
3. Semi-invariant submanifolds satisfying $A_{(2)}A_{(3)} = A_{(3)}A_{(2)}$

In the rest of this paper we shall suppose that $M$ is a real $(2n - 1)$-dimensional semi-invariant submanifold of codimension 3 in a complex projective space $\mathbb{C}P^n$ and that $A_{(2)}A_{(3)} = A_{(3)}A_{(2)}$ is satisfied on $M$. Then we have $l_i = 0$ and

$$(3.1) \quad K_{ji} = n\xi_j \xi_i.$$ 

Further, we have

$$(3.2) \quad m_j = -n\xi_j,$$

$$(3.3) \quad L_{ji} = 0$$

because of (1.7) and (1.10). Thus (2.4) and (2.6) are reduced respectively to

$$(3.4) \quad k\{\xi_j A_{ki} - \xi_k A_{ji} + (n_k \xi_j - n_j \xi_k) \xi_i\} = 0,$$

$$(3.5) \quad \nabla_j m_i - \nabla_i m_j = 0.$$ 

Multiplying $\xi_j \xi_i$ to (3.4) and summing for $j$ and $i$, we find

$$(3.6) \quad k\{n_k - (n_i \xi^i) \xi_k + A_{kr} \xi^r - \alpha \xi_k\} = 0.$$ 

Now, let $\Omega$ be a set of points such that $k \neq 0$ on $M$ and suppose that $\Omega$ be nonavoid. Then (3.4) and (3.6) imply

$$(3.7) \quad A_{ji} = \xi_j A_{ir} \xi^r + \xi_i A_{jr} \xi^r - \alpha \xi_j \xi_i$$

on $\Omega$. From now on, we discuss our arguments on the open set $\Omega$ on $M$. Since the vector $U$ is orthogonal to $\xi$, it is seen that

$$(3.8) \quad A U = 0.$$ 

Transforming (3.7) by $\phi_k^i$ and making use of (1.5), we get

$$(3.9) \quad \nabla_k \xi_j = \xi_k U_j.$$ 

If we transform this by $\phi^k_j$ and use (1.5), then

$$(3.10) \quad h - \alpha = 0.$$ 

Multiplying (3.7) with $A^{ji}$ and summing for $j$ and $i$, we also find

$$(3.11) \quad h_{(2)} = 2\beta - \alpha^2.$$
Remark 2. We notice here that $\beta - \alpha^2$ does not vanish on $\Omega$. In fact, if not, then we have $A \xi = \alpha \xi$ and hence $A_{ji} = \alpha \xi_j \xi_i$ because of (3.7). From this fact and (3.9) we obtain $\nabla_k A_{ji} = \alpha_k \xi_j \xi_i$, which together with (2.2) and (3.3) gives

$$(\alpha_k \xi_j - \alpha_j \xi_k) \xi_i = \frac{c}{4} (\xi_k \phi_{ji} - \xi_j \phi_{ki} - 2 \xi_i \phi_{kj}),$$

a contradiction.

Now, put $A \xi = \alpha \xi + \mu W$, where $\mu$ is a function on $M$ which is not vanish on $\Omega$ and $W$ is a unit vector field orthogonal to the structure vector field $\xi$. Then we have

$$(3.12) \quad \phi_{jr} U^r = \mu W_j$$

and $\mu^2 = \beta - \alpha^2$ because of (2.8). Thus $W$ is also orthogonal to $U$. Thus (3.7) turns out to be

$$(3.13) \quad A_{ji} = \mu (\xi_j W_i + \xi_i W_j) + \alpha \xi_j \xi_i.$$

We notice here that it is, using (3.9), verified that

$$(3.14) \quad \xi^r \nabla_k W_r = 0$$

because $\xi$ is orthogonal to $W$.

Differentiating (3.13) covariantly along $\Omega$ and making use of (3.9), we find

$$\nabla_k A_{ji} = \mu_k (\xi_j W_i + \xi_i W_j) + \mu \{(U_j W_i + U_i W_j) \xi_k + \xi_j \nabla_k W_i + \xi_i \nabla_k W_j\}$$

$$+ \alpha_k \xi_j \xi_i + \alpha (U_j \xi_i + U_i \xi_j) \xi_k,$$

from which, taking the skew-symmetric part with respect to indices $k$ and $j$, and using (2.2) with $l_j = 0$ and (3.3),

$$\frac{c}{4} (\xi_k \phi_{ji} - \xi_j \phi_{ki} - 2 \xi_i \phi_{kj})$$

$$= \mu_k (\xi_j W_i + \xi_i W_j) - \mu_j (\xi_k W_i + \xi_i W_k)$$

$$+ \mu \{(U_j W_i + U_i W_j - \nabla_j W_i) \xi_k - (U_k W_i + U_i W_k - \nabla_k W_i) \xi_j\}$$

$$+ \mu (\nabla_k W_j - \nabla_j W_k) \xi_i + (\alpha_k \xi_j - \alpha_j \xi_k) \xi_i + \alpha (U_j \xi_k - U_k \xi_j) \xi_i.$$
Applying $\xi^i$ to this and taking account of (3.14), we find
\begin{equation}
\mu_k W_j - \mu_j W_k + \mu(\nabla_k W_j - \nabla_j W_k) + \alpha_k \xi_j - \alpha_j \xi_k
\end{equation}
\begin{equation}
+ \alpha(U_j \xi_k - U_k \xi_j) + \frac{c}{2} \phi_{kj} = 0.
\end{equation}

From the last two equations it follows that
\begin{equation}
\frac{c}{4} (\xi_k \phi_{ji} - \xi_j \phi_{ki}) + (\mu_j \xi_k - \mu_k \xi_j) W_i
\end{equation}
\begin{equation}
= \mu\{(U_j W_i + U_i W_j - \nabla_j W_i) \xi_k - (U_k W_i + U_i W_k - \nabla_k W_i) \xi_j\},
\end{equation}
which together with (3.14) implies that
\begin{equation}
\frac{c}{4} \phi_{ji} + (\mu_j - (\mu \xi^i) \xi_j) W_i
\end{equation}
\begin{equation}
= \mu\{(U_j W_i + U_i W_j - \nabla_j W_i + (\xi^i \nabla_i W_i) \xi_j\}.
\end{equation}

If we apply this by $W^i$ and use (3.12), then we obtain
\begin{equation}
\mu \mu_j = \mu(\mu \xi^i) \xi_j + (\mu^2 + \frac{c}{4}) U_j.
\end{equation}

Multiplying (3.16) with $\xi^k W^j$ and summing for $k$ and $j$, and making use of (3.14) and (3.18), we have
\begin{equation}
\mu \xi^i = \alpha^i W^i.
\end{equation}

**Lemma 3.1.** Let $M$ be a semi-invariant submanifold of codimension 3 with lift-flat normal connection in a complex projective space $\mathbb{C}P^{m+1}$. If the mean curvature of $M$ is constant, then $\nabla_j U_i - \nabla_i U_j = 0$ on $\Omega$.

**Proof.** Differentiating (3.2) covariantly along $\Omega$, and using (3.9), we find
\[-\nabla_k m_j = \xi_j (\nabla_k k) + k \xi_k U_j,
\]
which together with (3.5) yields
\[\xi_j (\nabla_k k) - \xi_k (\nabla_j k) + k(\xi_k U_j - \xi_j U_k) = 0.
\]
Thus, it is seen that
\begin{equation}
\nabla_j k = (\xi^i \nabla_i k) \xi_j + k U_j.
\end{equation}
Since the mean curvature of $M$ is assumed to be constant, it is, taking account of $T_r A_{(3)} = 0$ and (3.10), seen that $k^2 + \alpha^2 = \text{const.}$, which unable us to obtain
\begin{equation}
k\nabla_j k + \alpha \alpha_j = 0.
\end{equation}
Because of the fact that $W$ is orthogonal to $U$ and $\xi$, we have from (3.20) and (3.21) $\alpha W^t = 0$. Thus (3.18) turns out to be
\begin{equation*}
\frac{1}{2} \nabla_j \mu^2 = (\mu^2 + \frac{c}{4}) U_j,
\end{equation*}
where we have used (3.19). From this, we obtain
\begin{equation*}
\frac{1}{2} \nabla_k \nabla_j \mu^2 = 2(\mu^2 + \frac{c}{4}) U_j U_k + (\mu^2 + \frac{c}{4}) \nabla_k U_j,
\end{equation*}
which implies $(\mu^2 + \frac{c}{4})(\nabla_j U_i - \nabla_i U_j) = 0$. This completes the proof because we have $\mu^2 + \frac{c}{4} > 0$.

Finally, we are proved. \qed

**Theorem 3.2.** Let $M$ be a real $(2n - 1)$-dimensional semi-invariant submanifold of codimension 3 with lift-flat normal connection in a complex projective space $\mathbb{C}P^{n+1}$. If the mean curvature of $M$ is constant, then $M$ is a real hypersurface in a complex projective space $\mathbb{C}P^n$.

**Proof.** Since the normal connection of $M$ is lift-flat, we have $l_j = 0$, (3.8) and (3.13) are valid. Thus (2.12) is reduced to
\begin{equation}
\xi^r \nabla_r U_i = \mu(\alpha W_i - \mu \xi_i) - \phi_{ir} \alpha^r.
\end{equation}
On the other hand, we have $U^r \nabla_j \xi_r + \xi^r \nabla_r U_j = 0$ by Lemma 3.1. Thus, it is, using (3.9) and (3.22), verified that $\phi_{ir} \alpha^r = \alpha \mu W_j$. So we have
\begin{equation*}
\alpha_j = (\alpha \xi^r) \xi_j + \alpha U_j,
\end{equation*}
where we have used (3.12). From (3.20), (3.21) and the last equation, we obtain
\begin{equation*}
\{ k(\xi^r \nabla_r k) + \alpha(\alpha \xi^r) \} \xi_j + (k^2 + \alpha^2) U_j = 0,
\end{equation*}
which shows that $\mu(k^2 + \alpha^2) = 0$ and hence $k = 0$, a contradiction. Hence $\Omega$ is empty. Thus, by (3.1) $\sim$ (3.3) it follows that $A_{(2)} = A_{(3)} = 0$ and $\nabla^\perp C = 0$ on $M$.

Let $N_0(p) = \{ \eta \in M_{\nu} | A_{\eta} = 0 \}$ and $H_0(p)$ the maximal $J$-invariant subspace of $N_0(p)$. Then, the orthogonal complement of $H_0(p)$ is invariant under parallel translation with respect to the normal connection because we have $\nabla^\perp C = 0$. Therefore, by the reduction theorem in [3] or [10], we see that $M$ is a real hypersurface of $\mathbb{C}P^n$ in $\mathbb{C}P^{n+1}$. Hence we arrive at conclusion. \qed
References


Department of Mathematics
Chosun University
Kwangju 502-759, Korea
E-mail: sblee@chosun.ac.kr