JORDAN ∗-HOMOMORPHISMS BETWEEN UNITAL C*-ALGEBRAS

Madjid Eshaghi Gordji, Norooz Ghobadipour, and Choonkil Park

Abstract. In this paper, we prove the superstability and the generalized Hyers-Ulam stability of Jordan ∗-homomorphisms between unital C*-algebras associated with the following functional equation

\[ f \left( \frac{-x+y}{3} \right) + f \left( \frac{x-3z}{3} \right) + f \left( \frac{3x-y+3z}{3} \right) = f(x). \]

Moreover, we investigate Jordan ∗-homomorphisms between unital C*-algebras associated with the following functional inequality

\[ \| f \left( \frac{-x+y}{3} \right) + f \left( \frac{x-3z}{3} \right) + f \left( \frac{3x-y+3z}{3} \right) \| \leq \| f(x) \|. \]

1. Introduction

The stability of functional equations was first introduced by Ulam [33] in 1940. More precisely, he proposed the following problem:

Given a group \( G_1 \), a metric group \( (G_2,d) \) and a positive number \( \epsilon \), does there exist a \( \delta > 0 \) such that if a function \( f : G_1 \to G_2 \) satisfies the inequality \( d(f(xy), f(x)f(y)) < \delta \) for all \( x, y \in G_1 \), then there exists a homomorphism \( T : G_1 \to G_2 \) such that \( d(f(x), T(x)) < \epsilon \) for all \( x \in G_1 \)?

As mentioned above, when this problem has a solution, we say that the homomorphisms from \( G_1 \) to \( G_2 \) are stable. In 1941, Hyers [7] gave a partial solution of Ulam’s problem for the case of approximate additive mappings under the assumption that \( G_1 \) and \( G_2 \) are Banach spaces. In 1978, Th. M. Rassias [27] generalized the theorem of Hyers by considering the stability problem with unbounded Cauchy differences. This phenomenon of stability that was introduced by Th. M. Rassias [27] is called generalized Hyers-Ulam stability or Hyers-Ulam-Rassias stability.
Theorem 1.1. Let \( f : E \rightarrow E' \) be a mapping from a norm vector space \( E \) into a Banach space \( E' \) subject to the inequality
\[
\|f(x + y) - f(x) - f(y)\| \leq \epsilon (\|x\|^p + \|y\|^p)
\]
for all \( x, y \in E \), where \( \epsilon \) and \( p \) are constants with \( \epsilon > 0 \) and \( p < 1 \). Then there exists a unique additive mapping \( T : E \rightarrow E' \) such that
\[
\|f(x) - T(x)\| \leq \frac{2\epsilon}{2 - 2p}\|x\|^p
\]
for all \( x \in E \). If \( p < 0 \), then the inequality (1.1) holds for all \( x, y \neq 0 \), and (1.2) for \( x \neq 0 \). Also, if the function \( t \mapsto f(tx) \) from \( \mathbb{R} \) into \( E' \) is continuous for each fixed \( x \in E \), then \( T \) is \( \mathbb{R} \)-linear.

Recently, C. Park and W. Park [26] applied the Jun and Lee’s result to the Jensen’s equation in Banach modules over a \( \mathbb{C}^* \)-algebra. B. E. Johnson [15, Theorem 7.2] also investigated almost algebra \( \ast \)-homomorphisms between Banach \( \ast \)-algebras: Suppose that \( \mathcal{U} \) and \( B \) are Banach \( \ast \)-algebras which satisfy the conditions of [15, Theorem 3.1]. Then for each positive \( \epsilon \) and \( K \) there is a positive \( \delta \) such that if \( T \in L(\mathcal{U}, B) \) with \( \|T\| < K \), \( \|T^\prime\| < \delta \) and \( \|T(x^\ast - T(x))\| \leq \delta \|x\| \), then there is a \( \ast \)-homomorphism \( T' : \mathcal{U} \rightarrow B \) with \( \|T' - T\| < \epsilon \). Here \( L(\mathcal{U}, B) \) is the space of bounded linear maps from \( \mathcal{U} \) into \( B \), and \( T^\prime(x, y) = T(xy) - T(x)T(y) \). See [15] for details.

Throughout this paper, let \( A \) be a unital \( \mathbb{C}^* \)-algebra with norm \( \|\cdot\| \) and unit \( e \), and \( B \) a unital \( \mathbb{C}^* \)-algebra with norm \( \|\cdot\| \). Let \( \mathcal{U}(A) \) be the set of unitary elements in \( A \), \( A_{sa} = \{x \in A | x = x^\ast\} \), and \( I_1(A_{sa}) = \{v \in A_{sa} | \|v\| = 1, v \text{ is invertible}\} \). During the last decades several stability problems of functional equations have been investigated by many mathematicians. A large list of references concerning the stability of functional equations can be found in [1]–[14], [18, 21, 30, 31, 32, 34].

Definition 1.2. Let \( A, B \) be two \( \mathbb{C}^* \)-algebras. A \( \mathbb{C} \)-linear mapping \( f : A \rightarrow B \) is called a Jordan \( \ast \)-homomorphism if
\[
\begin{align*}
(f(a^2) &= f(a)^2 \\
(f(a^\ast) &= f(a)^\ast
\end{align*}
\]
for all \( a \in A \).

C. Park [24] introduced and investigated Jordan \( \ast \)-derivations between unital \( \mathbb{C}^* \)-algebras associated with the following functional inequality
\[
\|f(a) + f(b) + k f(c)\| \leq \| kf \left( \frac{a + b}{k} + c \right) \|
\]
for some integer \( k \) greater than 1 and proved the generalized Hyers-Ulam stability of Jordan \( \ast \)-derivations between unital \( \mathbb{C}^* \)-algebras associated with the following functional equation
\[
f \left( \frac{a + b}{k} + c \right) = f(a) + f(b) + f(c)
\]
for some integer $k$ greater than 1 (see also [23, 19, 17, 20, 25]).

In this paper, we investigate Jordan $\ast$-homomorphisms between unital $C^\ast$-algebras associated with the following functional inequality
\[
\left\| f\left(\frac{b-a}{3}\right) + f\left(\frac{a-3c}{3}\right) + f\left(\frac{3a+3c-b}{3}\right) \right\| \leq \|f(a)\|.
\]

We moreover prove the generalized Hyers-Ulam stability of Jordan $\ast$-homomorphisms between unital $C^\ast$-algebras associated with the following functional equation
\[
f\left(\frac{b-a}{3}\right) + f\left(\frac{a-3c}{3}\right) + f\left(\frac{3a+3c-b}{3}\right) = f(a).
\]

2. Jordan $\ast$-homomorphisms

In this section, we investigate Jordan $\ast$-homomorphisms between unital $C^\ast$-algebras.

Lemma 2.1. Let $f : A \to B$ be a mapping such that
\[
(2.1) \quad \left\| f\left(\frac{b-a}{3}\right) + f\left(\frac{a-3c}{3}\right) + f\left(\frac{3a+3c-b}{3}\right) \right\|_B \leq \|f(a)\|_B
\]
for all $a, b, c \in A$. Then $f$ is additive.

Proof. Letting $a = b = c = 0$ in (2.1), we get
\[
\|3f(0)\|_B \leq \|f(0)\|_B.
\]
So $f(0) = 0$. Letting $a = b = 0$ in (2.1), we get
\[
\|f(-c) + f(c)\|_B \leq \|f(0)\|_B = 0
\]
for all $c \in A$. Hence $f(-c) = -f(c)$ for all $c \in A$. Letting $a = 0$ and $b = 6c$ in (2.1), we get
\[
\|f(2c) - 2f(c)\|_B \leq \|f(0)\|_B = 0
\]
for all $c \in A$. Hence
\[
f(2c) = 2f(c)
\]
for all $c \in A$. Letting $a = 0$ and $b = 9c$ in (2.1), we get
\[
\|f(3c) - f(c) - 2f(c)\|_B \leq \|f(0)\|_B = 0
\]
for all $c \in A$. Hence
\[
f(3c) = 3f(c)
\]
for all $c \in A$. Letting $a = 0$ in (2.1), we get
\[
\|f(b) + f(-c) + f(c - \frac{b}{3})\|_B \leq \|f(0)\|_B = 0
\]
for all $a, b, c \in A$. So
\[
f\left(\frac{b}{3}\right) + f(-c) + f(c - \frac{b}{3}) = 0
\]
for all \( a, b, c \in A \). Let \( t_1 = c - \frac{b}{3} \) and \( t_2 = \frac{b}{3} \) in the last equation, we get
\[
f(t_2) - f(t_1 + t_2) + f(t_1) = 0
\]
for all \( t_1, t_2 \in A \). This means that \( f \) is additive. \( \square \)

Now we prove the superstability problem for Jordan *-homomorphisms as follows.

**Theorem 2.2.** Let \( p < 1 \) and \( \theta \) be nonnegative real numbers, and let \( f : A \rightarrow B \) be a mapping satisfying \( f(0) = 0, f(3^n u x) = f(3^n u)f(x) \) for all \( u \in \mathcal{U}(A) \) and all \( x \in A \) and
\[
f( \frac{b - a}{3} ) + f( \frac{a - 3 \mu c}{3} ) + \mu f( \frac{3a + 3c - b}{3} ) \leq \| f(a) \| B,
\]
(2.2) \( \| f(3^n u^*) - f(3^n u)^* \| B \leq 2 \theta 3^{np} \),
(2.3) \( \| f(3^n u^*) \| B \leq \| f(0) \| B = 0 \)
for all \( \mu \in \mathbb{T} \) if \( : \{ \lambda \in \mathbb{C} : |\lambda| = 1 \} \), all \( u \in \mathcal{U}(A) \), \( n = 0, 1, 2, \ldots \) and all \( a, b, c \in A \). Then the mapping \( f : A \rightarrow B \) is a Jordan *-homomorphism.

**Proof.** Let \( \mu = 1 \) in (2.2). By Lemma 2.1, the mapping \( f : A \rightarrow B \) is additive. Letting \( a = b = 0 \) in (2.2), we get
\[
\| f(-\mu c) + \mu f(c) \| B \leq \| f(0) \| B = 0
\]
for all \( c \in A \) and all \( \mu \in \mathbb{T} \). So
\[
-f(\mu c) + \mu f(c) = f(-\mu c) + \mu f(c) = 0
\]
for all \( c \in A \) and all \( \mu \in \mathbb{T} \). Hence \( f(\mu c) = \mu f(c) \) for all \( c \in A \) and all \( \mu \in \mathbb{T} \).
By Theorem 2.1 of [22], the mapping \( f : A \rightarrow B \) is \( \mathbb{C} \)-linear. By (2.3), we get
\[
f(u^*) = \lim_{n \rightarrow \infty} \frac{1}{3^n} f(3^n u^*) = \lim_{n \rightarrow \infty} \frac{1}{3^n} f(3^n u)^* = \left( \lim_{n \rightarrow \infty} \frac{1}{3^n} f(3^n u) \right)^* = f(u)^*
\]
for all \( u \in \mathcal{U}(A) \). Since \( f \) is \( \mathbb{C} \)-linear and each \( x \in A \) is a finite linear combination of unitary elements (see [16, Theorem 4.1.7], i.e., \( x = \sum_{i=1}^{m} \lambda_i u_i \) \( \lambda_i \in \mathbb{C}, u_i \in \mathcal{U}(A) \)),
\[
f(x^*) = f \left( \sum_{i=1}^{m} \lambda_i u_i^* \right) = \sum_{i=1}^{m} \lambda_i f(u_i^*) = \sum_{i=1}^{m} \lambda_i f(u_i)^* = \left( \sum_{i=1}^{m} \lambda_i u_i \right)^* = f(x)^*
\]
for all \( x \in A \). Since \( f(3^n u x) = f(3^n u)f(x) \) for all \( u \in \mathcal{U}(A), x \in A \) and all \( n = 0, 1, 2, \ldots \),
\[
f(u x) = \lim_{n \rightarrow \infty} \frac{1}{3^n} f(3^n u x) = \lim_{n \rightarrow \infty} \frac{1}{3^n} f(3^n u)f(x) = f(u)f(x)
\]
for all \( u \in U(A) \), \( x \in A \). Since \( f \) is \( \mathbb{C} \)-linear and each \( x \in A \) is a finite linear combination of unitary elements, i.e., \( x = \sum_{i=1}^{m} \lambda_i u_i \) (\( \lambda_i \in \mathbb{C}, u_i \in U(A) \)),

\[
\begin{align*}
    f(xy) &= f \left( \sum_{i=1}^{m} \lambda_i u_i y \right) = \sum_{i=1}^{m} \lambda_i f(u_i y) = \sum_{i=1}^{m} \lambda_i f(u_i) f(y) \\
    &= f \left( \sum_{i=1}^{m} \lambda_i u_i \right) f(y) = f(x)f(y)
\end{align*}
\]

(2.4)

for all \( x, y \in A \). Replacing \( y \) by \( x \) in (2.4), we get \( f(x^2) = f(x)^2 \) for all \( x \in A \). Therefore, the mapping \( f : A \to B \) is a Jordan *-homomorphism, as desired.

**Theorem 2.3.** Let \( p > 1 \) and \( \theta \) be a nonnegative real number, and let \( f : A \to B \) be a mapping satisfying (2.2) and (2.3). Then the mapping \( f : A \to B \) is a Jordan *-homomorphism.

**Proof.** The proof is similar to the proof of Theorem 2.2.

We prove the generalized Hyers-Ulam stability of Jordan *-homomorphisms between unital \( C^* \)-algebras.

**Theorem 2.4.** Suppose that \( f : A \to B \) is a mapping for which there exists a function \( \varphi : A \times A \times A \to \mathbb{R}^+ \) such that

\[
\begin{align*}
    &\sum_{i=0}^{\infty} 3^i \varphi \left( \frac{a}{3^i}, \frac{b}{3^i}, \frac{c}{3^i} \right) < \infty, \\
    &\lim_{n \to \infty} 3^2n \varphi \left( \frac{a}{3^n}, \frac{b}{3^n}, \frac{c}{3^n} \right) = 0, \\
    &\|f(3^n u^*) - f(3^n u)^*\|_B \leq \varphi(3^n u, 3^n u, 3^n u), \\
    &\left\| f \left( \frac{\mu b - a}{3} \right) + f \left( \frac{a - 3c}{3} \right) + \mu f \left( \frac{3\mu a - b}{3} + c \right) - f(a) + f(c^2) - f(c)^2 \right\|_B \\
    &\quad \leq \varphi(a, b, c)
\end{align*}
\]

for all \( a, b, c \in A \) and all \( \mu \in \mathbb{T}^1 \). Then there exists a unique Jordan *-homomorphism \( h : A \to B \) such that

\[
\|h(a) - f(a)\|_B \leq \sum_{i=0}^{\infty} 3^i \varphi \left( \frac{a}{3^i}, \frac{2a}{3^i}, 0 \right)
\]

for all \( a \in A \).

**Proof.** Letting \( \mu = 1 \), \( b = 2a \) and \( c = 0 \) in (2.8), we get

\[
\left\| 3\varphi \left( \frac{a}{3} \right) \right\|_B \leq \varphi(a, 2a, 0)
\]
for all $a \in A$. Using the induction method, we have

$$
\| 3^n f \left( \frac{a}{3^n} \right) - f(a) \| \leq \sum_{i=0}^{n-1} 3^i \varphi \left( \frac{a}{3^i}, \frac{2a}{3^i}, 0 \right)
$$

for all $a \in A$. In order to show the functions $h_n(a) = 3^n f \left( \frac{a}{3^n} \right)$ form a convergent sequence, we use the Cauchy convergence criterion. Indeed, replace $a$ by $\frac{a}{3^n}$ and multiply by $3^m$ in (2.10), where $m$ is an arbitrary positive integer. We find that

$$
\| 3^{m+n} f \left( \frac{a}{3^{m+n}} \right) - 3^m f \left( \frac{a}{3^m} \right) \| \leq \sum_{i=m}^{m+n-1} 3^i \varphi \left( \frac{a}{3^i}, \frac{2a}{3^i}, 0 \right)
$$

for all positive integers. Hence by the Cauchy criterion the limit $h(a) = \lim_{n \to \infty} h_n(a)$ exists for each $a \in A$. By taking the limit as $n \to \infty$ in (2.10) we see that

$$
\| h(a) - f(a) \| \leq \sum_{i=0}^{\infty} 3^i \varphi \left( \frac{a}{3^i}, \frac{2a}{3^i}, 0 \right)
$$

and (2.9) holds for all $a \in A$. Let $\mu = 1$ and $c = 0$ in (2.8), we get

$$
\| f \left( \frac{b-a}{3} \right) + f \left( \frac{a}{3} \right) + f \left( \frac{3a-b}{3} \right) - f(a) \|_B \leq \varphi(a,b,0)
$$

for all $a, b, c \in A$. Multiplying both sides (2.12) by $3^n$ and Replacing $a, b$ by $\frac{a}{3^n}, \frac{b}{3^n}$, respectively, we get

$$
\| 3^n f \left( \frac{b-a}{3^n+1} \right) + 3^n f \left( \frac{a}{3^n+1} \right) + 3^n f \left( \frac{3a-b}{3^n+1} \right) - 3^n f \left( \frac{a}{3^n} \right) \|_B
$$

$$
\leq 3^n \varphi \left( \frac{a}{3^n}, \frac{b}{3^n}, 0 \right)
$$

for all $a, b, c \in A$. Taking the limit as $n \to \infty$, we obtain

$$
h \left( \frac{b-a}{3} \right) + h \left( \frac{a}{3} \right) + h \left( \frac{3a-b}{3} \right) - h(a) = 0
$$

for all $a, b, c \in A$. Putting $b = 2a$ in (2.14), we get $3h \left( \frac{a}{3} \right) = h(a)$ for all $a \in A$. Replacing $a$ by $2a$ in (2.14), we get

$$
h(b - 2a) + h(6a - b) = 2h(2a)
$$

for all $a, b \in A$. Letting $b = 2a$ in (2.15), we get $h(4a) = 2h(2a)$ for all $a \in A$. So $h(2a) = 2h(a)$ for all $a \in A$. Letting $3a - b = s$ and $b - a = t$ in (2.14), we get

$$
h \left( \frac{t}{3} \right) + h \left( \frac{s+t}{6} \right) + h \left( \frac{t}{3} \right) = h \left( \frac{s+t}{2} \right)
$$

for all $s, t \in A$. Hence $h(s) + h(t) = h(s + t)$ for all $s, t \in A$. So, $h$ is additive. Letting $a = c = 0$ in (2.12) and using the above method, we have $h(\mu b) = \mu h(b)$.
for all \( b \in A \) and all \( \mu \in \mathbb{T} \). Hence by Theorem 2.1 of [22], the mapping \( f : A \rightarrow B \) is \( \mathbb{C} \)-linear.

Now, let \( h : A \rightarrow B \) be another \( \mathbb{C} \)-linear mapping satisfying (2.9). Then we have

\[
\|h(a) - h'(a)\|_B = 3^n\left\|h\left(\frac{a}{3^n}\right) - h'(\frac{a}{3^n})\right\|_B \\
\leq 3^n\left(\left\|h\left(\frac{a}{3^n}\right) - f\left(\frac{a}{3^n}\right)\right\|_B + \left\|h'(\frac{a}{3^n}) - f\left(\frac{a}{3^n}\right)\right\|_B\right) \\
\leq 2\sum_{i=0}^{\infty} 3^i \varphi\left(\frac{a}{3^i}, \frac{2a}{3^i}, 0\right) \\
= 0
\]

for all \( a \in A \). By (2.6), (2.7), (2.8) and similar to the proof of Theorem 2.2, the mapping \( h : A \rightarrow B \) is a Jordan \(*\)-homomorphism. \(\square\)

**Corollary 2.5.** Suppose that \( f : A \rightarrow B \) is a mapping with \( f(0) = 0 \) for which there exist constant \( \theta \geq 0 \) and \( p_1, p_2, p_3 > 1 \) such that

\[
\left\|f\left(\frac{\mu b - a}{3}\right) + f\left(\frac{a - 3c}{3}\right) + \mu f\left(\frac{3a - b}{3} + c\right) - f(a) + f(c^2) - f(c^2)\right\|_B \\
\leq \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3}),
\]

\[
\|f(3^n a^*) - f(3^n a^*)\|_B \leq \theta(3^{np_1} + 3^{np_2} + 3^{np_3})
\]

for all \( a, b, c \in A \) and all \( \mu \in \mathbb{T} \). Then there exists a unique Jordan \(*\)-homomorphism \( h : A \rightarrow B \) such that

\[
\|f(a) - h(a)\|_B \leq \frac{\theta\|a\|^{p_1}}{1 - 3^{1-p_1}} + \frac{\theta 2^{p_2}\|a\|^{p_2}}{1 - 3^{1-p_2}}
\]

for all \( a \in A \).

**Proof.** Letting \( \varphi(a, b, c) := \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3}) \) in Theorem 2.4, we obtain the result. \(\square\)

**Theorem 2.6.** Suppose that \( f : A \rightarrow B \) is a mapping with \( f(0) = 0 \) for which there exists a function \( \varphi : A \times A \times A \rightarrow B \) satisfying (2.7), (2.8) and (2.8) such that

(2.16) \[
\sum_{i=1}^{\infty} 3^{-i} \varphi(3^i a, 3^i b, 3^i c) < \infty,
\]

(2.17) \[
\lim_{n \rightarrow \infty} 3^{-2n} \varphi(3^i a, 3^i b, 3^i c) = 0
\]

for all \( a, b, c \in A \). Then there exists a unique Jordan \(*\)-homomorphism \( h : A \rightarrow B \) such that

(2.18) \[
\|h(a) - f(a)\|_B \leq \sum_{i=1}^{\infty} 3^{-i} \varphi(3^i a, 3^i b, 3^i c) \leq \infty
\]

for all \( a \in A \).
Proof. Letting $\mu = 1$, $b = 2a$ and $c = 0$ in (2.8), we get
\begin{equation}
\left\| 3f \left( \frac{a}{3} \right) - f(a) \right\|_B \leq \varphi(a, 2a, 0)
\end{equation}
for all $a \in A$. Replacing $a$ by $3a$ in (2.19), we get
\begin{equation}
\left\| 3^{-1}f(3a) - f(a) \right\|_B \leq 3^{-1}\varphi(3a, 2(3a), 0)
\end{equation}
for all $a \in A$. On can apply the induction method to prove that
\begin{equation}
\|3^{-n}f(3^n a) - f(a)\|_B \leq \sum_{i=1}^{n} 3^{-i}\varphi(3^i a, 2(3^i a), 0)
\end{equation}
for all $a \in A$. In order to show the functions $h_n(a) = 3^{-n}f(3^n a)$ form a convergent sequence, we use the Cauchy convergence criterion. Indeed, replace $a$ by $3^m a$ and multiply by $3^{-m}$ in (2.20), where $m$ is an arbitrary positive integer. We find that
\begin{equation}
\|3^{-(m+n)}f(3^{m+n} a) - 3^{-m}f(3^m a)\| \leq \sum_{i=m+1}^{m+n} 3^{-i}\varphi(3^i a, 2(3^i a), 0)
\end{equation}
for all positive integers. Hence by the Cauchy criterion the limit $h(a) = \lim_{n \to \infty} h_n(a)$ exists for each $a \in A$. By taking the limit as $n \to \infty$ in (2.20) we see that
\begin{equation}
\|h(a) - f(a)\| \leq \sum_{i=1}^{\infty} 3^{-i}\varphi(3^i a, 2(3^i a), 0)
\end{equation}
and (2.18) holds for all $a \in A$.

The rest of the proof is similar to the proof of Theorem 2.4. \qed

Corollary 2.7. Suppose that $f : A \to B$ is a mapping with $f(0) = 0$ for which there exist constant $\theta \geq 0$ and $p_1, p_2, p_3 < 1$ such that
\begin{equation}
\left\| f \left( \frac{\mu b - a}{3} \right) + f \left( \frac{a - 3c}{3} \right) + \mu f \left( \frac{3a - b}{3} + c \right) - f(a) + f(c^2) - f(c)^2 \right\|_B \\
\leq \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3}),
\end{equation}
\begin{equation}
\|f(3^n a^*) - f(3^n a^*)\|_B \leq \theta(3^{np_1} + 3^{np_2} + 3^{np_3})
\end{equation}
for all $a, b, c \in A$ and all $\mu \in \mathbb{T}$. Then there exists a unique Jordan *-homomorphism $h : A \to B$ such that
\begin{equation}
\|f(a) - h(a)\|_B \leq \frac{\theta\|a\|^{p_1}}{3^{1-p_1}} + \frac{\theta2^{p_2}\|a\|^{p_2}}{3^{1-p_2}}
\end{equation}
for all $a \in A$.

Proof. Letting $\varphi(a, b, c) := \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3})$ in Theorem 2.7, we obtain the result. \qed
JORDAN *-HOMOMORPHISMS BETWEEN UNITAL C*-ALGEBRAS

References


Madjid Eshaghi Gordji
Department of Mathematics
Semnan University
P. O. Box 35195-363, Semnan, Iran
E-mail address: madjid.eshaghi@gmail.com

Norooz Ghobadipour
Department of Mathematics
Semnan University
P. O. Box 35195-363, Semnan, Iran
E-mail address: ghobadipour.n@gmail.com

Choonkil Park
Department of Mathematics
Hanyang University
Seoul 133-791, Korea
E-mail address: baak@hanyang.ac.kr