FEKETE-SZEGÖ PROBLEM FOR SUBCLASSES
OF STARLIKE FUNCTIONS WITH
RESPECT TO SYMMETRIC POINTS

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ABSTRACT. In the present investigation, sharp upper bounds of
$|a_3 - \mu a_2^2|$ for functions $f(z) = z + a_2z^2 + a_3z^3 + \cdots$ belonging
to certain subclasses of starlike and convex functions with respect
to symmetric points are obtained. Also certain applications of the
main results for subclasses of functions defined by convolution with
a normalized analytic function are given. In particular, Fekete-
Szegö inequalities for certain classes of functions defined through
fractional derivatives are obtained.

1. Introduction

Let $\mathcal{A}$ denote the class of all analytic functions $f(z)$ of the form

\begin{equation}
    f(z) = z + \sum_{k=2}^{\infty} a_k z^k \quad (z \in \Delta := \{ z \in \mathbb{C} : |z| < 1 \})
\end{equation}

and $\mathcal{S}$ be the subclass of $\mathcal{A}$ consisting of univalent functions. For two
functions $f, g \in \mathcal{A}$, we say that the function $f(z)$ is subordinate to $g(z)$
in $\Delta$ and write $f \prec g$ or $f(z) \prec g(z)$ ($z \in \Delta$), if there exists an analytic
function $w(z)$ with $w(0) = 0$ and $|w(z)| < 1$ ($z \in \Delta$), such that $f(z) = g(w(z))$ ($z \in \Delta$). In particular, if the function $g$ is univalent in $\Delta$, the
above subordination is equivalent to $f(0) = g(0)$ and $f(\Delta) \subset g(\Delta)$.

Let $\phi(z)$ be an analytic function with positive real part on $\Delta$ with
$\phi(0) = 1$, $\phi'(0) > 0$ which maps the unit disk $\Delta$ onto a region starlike
with respect to 1 which is symmetric with respect to the real axis. Let

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$S^*(\phi)$ be the class of function $f \in S$ for which \(\frac{zf'(z)}{f(z)} < \phi(z), (z \in \Delta)\) and
\(C(\phi)\) be the class of functions $f \in S$ for which $1 + \frac{zf''(z)}{f'(z)} < \phi(z), (z \in \Delta)$. These classes were introduced and studied by Ma and Minda [7]. When $\phi(z) = (1 + Az)/(1 + Bz), (-1 \leq B < A \leq 1)$, the class $S^*(\phi)$ reduces to the class $S^*[A,B]$ studied by Janowski [6]. See also Silverman and Silvia [15]. Ma and Minda [7] have obtained the Fekete-Szegö inequality for the functions in the class $C(\phi)$. Since $f \in C(\phi)$ if and only if $zf' \in S^*(\phi)$, we get the Fekete-Szegö inequality for functions in the class $S^*(\phi)$.

Sakaguchi [14] introduced and studied the class $S_s^*$ of starlike functions with respect to symmetric points. The class $S_s^*(\phi)$ defined below is the generalization of the class $S_s^*$.

**DEFINITION 1.1.** [10] Let $\phi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \cdots$ be a univalent starlike function with respect to 1 which maps the unit disk $\Delta$ onto a region in the right half plane which is symmetric with respect to the real axis, and let $B_1 > 0$. The function $f \in A$ is in the class $S_s^*(\phi)$ if
\[
\frac{2zf'(z)}{f(z) - f(-z)} < \phi(z).
\]

The function $f \in A$ is in the class $C_s(\phi)$ if
\[
\frac{2zf'(z)}{f'(z) + f'(-z)} < \phi(z).
\]

When $\phi(z) = (1 + Az)/(1 + Bz), (-1 \leq B < A \leq 1)$, we denote the subclasses $S_s^*[A,B]$ and $C_s[A,B]$ respectively. For $0 \leq \alpha < 1$, let $S_s^*[\alpha] := S_s^*[1 - 2\alpha, -1]$ and $C_s(\alpha) := C_s[1 - 2\alpha, -1]$.

In the present paper, we obtain the Fekete-Szegö inequality for functions in the subclasses $S_s^*(\phi)$ and $C_s(\phi)$. Also we give applications of our results to certain functions defined by convolution (or Hadamard product) and in particular we consider classes $S_s^*[\alpha, \lambda]$, $S_s^*[A, B, \lambda]$ and $C_s[\alpha, \lambda]$ defined by fractional derivatives.

To prove our main result, we need the following:

**LEMMA 1.2.** [7] If $p(z) = 1 + c_1 z + c_2 z^2 + \cdots$ is an analytic function with positive real part in $\Delta$, then
\[
|c_2 - vc_1^2| \leq \begin{cases} 
-4v + 2 & \text{if } v \leq 0, \\
2 & \text{if } 0 \leq v \leq 1, \\
4v - 2 & \text{if } v \geq 1.
\end{cases}
\]

When $v < 0$ or $v > 1$, the equality holds if and only if $p(z)$ is $(1 + z)/(1 - z)$ or one of its rotations. If $0 < v < 1$, then the equality holds
if and only if \( p(z) \) is \((1 + z^2)/(1 - z^2)\) or one of its rotations. If \( v = 0 \),
the equality holds if and only if
\[
p(z) = \left(\frac{1}{2} + \frac{1}{2} \lambda\right) \frac{1 + z}{1 - z} + \left(\frac{1}{2} - \frac{1}{2} \lambda\right) \frac{1 - z}{1 + z} \quad (0 \leq \lambda \leq 1)
\]
or one of its rotations. If \( v = 1 \), the equality holds if and only if \( p(z) \) is
the reciprocal of one of the functions such that the equality holds in the
case of \( v = 0 \).

Also the above upper bound is sharp, and it can be improved as
follows when \( 0 < v < 1 \):
\[
|c_2 - vc_1^2| + v|c_1|^2 \leq 2 \quad (0 < v \leq 1/2)
\]
and
\[
|c_2 - vc_1^2| + (1 - v)|c_1|^2 \leq 2 \quad (1/2 < v \leq 1).
\]

2. Fekete-Szegö problem

Our main result is the following:

**Theorem 2.1.** If \( f(z) \) given by (1.1) belongs to \( S_\phi^*(\phi) \), then
\[
|a_3 - \mu a_2^2| \leq \begin{cases}
\frac{1}{2} \left[B_2 - \frac{\mu}{2} B_1^2\right] & \text{if } \mu \leq \sigma_1, \\
\frac{B_1}{2} & \text{if } \sigma_1 \leq \mu \leq \sigma_2, \\
-\frac{1}{2} \left[B_2 - \frac{\mu}{2} B_1^2\right] & \text{if } \mu \geq \sigma_2,
\end{cases}
\]
where
\[
\sigma_1 := \frac{2(B_2 - B_1)}{B_1^2}, \quad \sigma_2 := \frac{2(B_2 + B_1)}{B_1^2}.
\]
The result is sharp.

**Proof.** For \( f \in S_\phi^*(\phi) \), let
\[
(2.1) \quad p(z) := \frac{2zf'(z)}{f(z) - f(-z)} = 1 + b_1 z + b_2 z^2 + \cdots.
\]
From (2.1), we obtain
\[
2a_2 = b_1 \quad \text{and} \quad 2a_3 = b_2.
\]
Since \( \phi(z) \) is univalent and \( p \prec \phi \), the function
\[
p_1(z) = \frac{1 + \phi^{-1}(p(z))}{1 - \phi^{-1}(p(z))} = 1 + c_1 z + c_2 z^2 + \cdots
\]
is analytic and has positive real part in $\Delta$. Thus we have

\[(2.2) \quad p(z) = \phi \left( \frac{p_1(z) - 1}{p_1(z) + 1} \right) \]

and from this equation (2.2), we obtain

\[b_1 = \frac{1}{2} B_1 c_1 \]

and

\[b_2 = \frac{1}{2} B_1 (c_2 - \frac{1}{2} c_1^2) + \frac{1}{4} B_2 c_1^2.\]

Therefore we have

\[(2.3) \quad a_3 - \mu a_2^2 = \frac{B_1}{4} \left( c_2 - v c_1^2 \right),\]

where

\[v := \frac{1}{2} \left[ 1 - \frac{B_2}{B_1} + \frac{\mu}{2} B_1 \right].\]

Our result now follows by an application of Lemma 1.2. To show that these bounds are sharp, we define the functions $K_{\phi_n}$ ($n = 2, 3, \ldots$) by

\[\frac{2zK'_{\phi_n}(z)}{K_{\phi_n}(z) - K_{\phi_n}(-z)} = \phi(z^{n-1}), \quad K_{\phi_n}(0) = 0 = [K_{\phi_n}]'(0) - 1\]

and the function $F^\lambda$ and $G^\lambda$ ($0 \leq \lambda \leq 1$) by

\[\frac{2zF'_\lambda(z)}{F_\lambda(z) - F_\lambda(-z)} = \phi \left( \frac{z(z + \lambda)}{1 + \lambda z} \right), \quad F_\lambda(0) = 0 = [F_\lambda]'(0) - 1\]

and

\[\frac{2zG'_\lambda(z)}{G_\lambda(z) - G_\lambda(-z)} = \phi \left( -\frac{z(z + \lambda)}{1 + \lambda z} \right), \quad G_\lambda(0) = 0 = [G_\lambda]'(0) - 1.\]

Clearly the functions $K_{\phi_n}, F_\lambda, G_\lambda \in S_3^*(\phi)$. Also we write $K_\phi := K_{\phi_2}$. If $\mu < \sigma_1$ or $\mu > \sigma_2$, then the equality holds if and only if $f$ is $K_\phi$ or one of its rotations. When $\sigma_1 < \mu < \sigma_2$, the equality holds if and only if $f$ is $K_{\phi_3}$ or one of its rotations. If $\mu = \sigma_1$ then the equality holds if and only if $f$ is $F_\lambda$ or one of its rotations. If $\mu = \sigma_2$ then the equality holds if and only if $f$ is $G_\lambda$ or one of its rotations. \hfill \Box

If $\sigma_1 \leq \mu \leq \sigma_2$, then, in view of Lemma 1.2, Theorem 2.1 can be improved.
Theorem 2.2. Let \( f(z) \) given by (1.1) belongs to \( S^*_s(\phi) \). Let \( \sigma_3 \) be given by \( \sigma_3 := 2B_2/B_1^2 \). If \( \sigma_1 \leq \mu \leq \sigma_3 \), then

\[
|a_3 - \mu a_2^2| + \frac{1}{B_1^2} \left[ 2(B_1 - B_2) + \mu B_1^2 \right] |a_2|^2 \leq \frac{B_1}{2}.
\]

If \( \sigma_3 \leq \mu \leq \sigma_2 \), then

\[
|a_3 - \mu a_2^2| + \frac{1}{B_1^2} \left[ 2(B_1 + B_2) - \mu B_1^2 \right] |a_2|^2 \leq \frac{B_1}{2}.
\]

Example 2.3. Let \(-1 \leq B < A \leq 1\). If \( f(z) \) given by (1.1) belongs to \( S^*_s[A, B] \), then

\[
|a_3 - \mu a_2^2| \leq \begin{cases} \frac{B - A}{2} \left[ B + \frac{\mu}{2}(A - B) \right] & \text{if } \mu \leq -2 \left[ \frac{1 + B}{A - B} \right], \\ \frac{A - B}{2} & \text{if } -2 \left[ \frac{1 + B}{A - B} \right] \leq \mu \leq 2 \left[ \frac{1 - B}{A - B} \right], \\ \frac{A - B}{2} \left[ B + \frac{\mu}{2}(A - B) \right] & \text{if } \mu \geq 2 \left[ \frac{1 - B}{A - B} \right]. \end{cases}
\]

In particular, if \( f \in S^*_s(\alpha) \), then

\[
|a_3 - \mu a_2^2| \leq \begin{cases} (1 - \alpha)(1 - \mu(1 - \alpha)) & \text{if } \mu \leq 0, \\ (1 - \alpha) & \text{if } 0 \leq \mu \leq \frac{2}{1 - \alpha}, \\ -(1 - \alpha)(1 - \mu(1 - \alpha)) & \text{if } \mu \geq \frac{2}{1 - \alpha}. \end{cases}
\]

The results are sharp.

Since \( f \in C_s(\phi) \) if and only if \( zf' \in S^*_s(\phi) \), Theorem 2.1, with an obvious change of the parameter \( \mu \), leads to the following Corollary.

Corollary 2.4. If \( f(z) \) given by (1.1) belongs to \( C_s(\phi) \), then

\[
|a_3 - \mu a_2^2| \leq \begin{cases} \frac{1}{6} \left[ B_2 - \frac{3}{8} \mu B_1^2 \right] & \text{if } \mu \leq \sigma_1, \\ \frac{B_1}{6} & \text{if } \sigma_1 \leq \mu \leq \sigma_2, \\ -\frac{1}{6} \left[ B_2 - \frac{3}{8} \mu B_1^2 \right] & \text{if } \mu \geq \sigma_2, \end{cases}
\]

where

\[
\sigma_1 := \frac{8}{3} \frac{(B_2 - B_1)}{B_1^2}, \quad \sigma_2 := \frac{8}{3} \frac{(B_2 + B_1)}{B_1^2}.
\]

The result is sharp.
Example 2.5. If $f(z)$ given by (1.1) belongs to $C_s[A, B]$, then

$$|a_3 - \mu a_2^2| \leq \begin{cases} 
\frac{B - A}{6} \left[ B + \frac{3}{8} \mu (A - B) \right] & \text{if } \mu \leq -\frac{8}{3} \left[ \frac{1 + B}{A - B} \right], \\
\frac{A - B}{6} & \text{if } -\frac{8}{3} \left[ \frac{1 + B}{A - B} \right] \leq \mu \leq \frac{8}{3} \left[ \frac{1 - B}{A - B} \right], \\
\frac{A - B}{6} \left[ B + \frac{3}{8} \mu (A - B) \right] & \text{if } \mu \geq \frac{8}{3} \left[ \frac{1 - B}{A - B} \right].
\end{cases}$$

In particular, if $f \in C_s(\alpha)$, then

$$|a_3 - \mu a_2^2| \leq \begin{cases} 
\frac{1 - \alpha}{3} \left[ 1 - \frac{3}{4} \mu (1 - \alpha) \right] & \text{if } \mu \leq 0, \\
\frac{1 - \alpha}{3} & \text{if } 0 \leq \mu \leq \frac{8}{3(1-\alpha)}, \\
\frac{1 - \alpha}{3} \left[ 1 - \frac{3}{4} \mu (1 - \alpha) \right] & \text{if } \mu \geq \frac{8}{3(1-\alpha)}.
\end{cases}$$

The results are sharp.

If $\sigma_1 \leq \mu \leq \sigma_2$, then, in view of Lemma 1.2, Corollary 2.4 can be improved.

Corollary 2.6. Let $f(z)$ given by (1.1) belongs to $C_s(\phi)$. Let $\sigma_3$ be given by $\sigma_3 := \frac{8}{3} \frac{B_2}{B_1^2}$.

If $\sigma_1 \leq \mu \leq \sigma_3$, then

$$|a_3 - \mu a_2^2| + \frac{8}{3} \frac{1}{B_1^2} \left[ B_1 - B_2 + \frac{3}{8} \mu B_1^2 \right] |a_2|^2 \leq \frac{B_1}{6}.$$

If $\sigma_3 \leq \mu \leq \sigma_2$, then

$$|a_3 - \mu a_2^2| + \frac{8}{3} \frac{1}{B_1^2} \left[ B_1 + B_2 - \frac{3}{8} \mu B_1^2 \right] |a_2|^2 \leq \frac{B_1}{6}.$$

3. Applications to functions defined by fractional derivatives

For two analytic functions $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ and $g(z) = z + \sum_{n=2}^{\infty} g_n z^n$, their convolution (or Hadamard product) is defined to be the function $(f * g)(z)$ given by $(f * g)(z) = z + \sum_{n=2}^{\infty} g_n a_n z^n$. For a fixed $g \in \mathcal{A}$, let $S_s^g(\phi)$ be the class of functions $f \in \mathcal{A}$ for which $(f * g) \in S_s^g(\phi)$ and $C_s^g(\phi)$ be the class of functions $f \in \mathcal{A}$ for which $(f * g) \in C_s(\phi)$. 
DEFINITION 3.1 (see [9, 8]; see also [17, 18]). Let \( f(z) \) be analytic in a simply connected region of the \( z \)-plane containing the origin. The fractional derivative of \( f \) of order \( \lambda \) is defined by

\[
D_z^\lambda f(z) := \frac{1}{\Gamma(1-\lambda)} \frac{d}{dz} \int_0^z \frac{f(\zeta)}{(z - \zeta)^{\lambda}} d\zeta \quad (0 \leq \lambda < 1),
\]

where the multiplicity of \( (z - \zeta)^{-\lambda} \) is removed by requiring that \( \log(z - \zeta) \) is real for \( z - \zeta > 0 \).

Using the above Definition 3.1 and its known extensions involving fractional derivatives and fractional integrals, Owa and Srivastava [9] introduced the operator \( \Omega^\lambda : \mathcal{A} \rightarrow \mathcal{A} \) defined by

\[
(\Omega^\lambda f)(z) = \Gamma(2 - \lambda) z^\lambda D_z^\lambda f(z), \quad (\lambda \neq 2, 3, 4, \ldots).
\]

The classes \( S^\lambda_s(\phi) \) and \( C^\lambda_s(\phi) \) consist of functions \( f \in \mathcal{A} \) for which \( \Omega^\lambda f \in S^\lambda_s(\phi) \) and \( \Omega^\lambda f \in C^\lambda_s(\phi) \), respectively. The class \( S^\lambda_s(\phi) \) is the special case of the class \( S^\lambda_s(\phi) \) and the class \( C^\lambda_s(\phi) \) is the special case of the class \( C^\lambda_s(\phi) \) when

\[
g(z) = z + \sum_{n=2}^{\infty} \frac{\Gamma(n + 1)\Gamma(2 - \lambda)}{\Gamma(n + 1 - \lambda)} z^n.
\]

Since \( f \in S^\mu_s(\phi) \) (\( C^\mu_s(\phi) \)) if and only if \( f * g \in S^\mu_s(\phi) \) (\( C^\mu_s(\phi) \)), we obtain the coefficient estimates for functions in the classes \( S^\mu_s(\phi) \) and \( C^\mu_s(\phi) \), from the corresponding estimates for functions in the classes \( S^\mu_s(\phi) \) and \( C^\mu_s(\phi) \).

Applying Theorem 2.1 for the function \( (f * g)(z) = z + g_2 a_2 z^2 + g_3 a_3 z^3 + \cdots \), we get the following theorem after an obvious change of the parameter \( \mu \):

THEOREM 3.2. Let \( g(z) = z + \sum_{n=2}^{\infty} g_n z^n \) \((g_n > 0)\). If \( f(z) \) given by (1.1) belongs to \( S^\sigma_s(\phi) \), then

\[
|a_3 - \mu a_2^2| \leq \begin{cases} \frac{1}{2g_3} \left( B_2 - \frac{\mu g_3}{2g_2} B_1^2 \right) & \text{if } \mu \leq \sigma_1, \\ \frac{B_1}{2g_3} & \text{if } \sigma_1 \leq \mu \leq \sigma_2, \\ -\frac{1}{2g_3} \left( B_2 - \frac{\mu g_3}{2g_2} B_1^2 \right) & \text{if } \mu \geq \sigma_2, \end{cases}
\]

where

\[
\sigma_1 := \frac{2g_2^2 (B_2 - B_1)}{g_3 B_1^2}, \quad \sigma_2 := \frac{2g_2^2 (B_2 + B_1)}{g_3 B_1^2},
\]
The result is sharp.

**Corollary 3.3.** Let \( g(z) = z + \sum_{n=2}^{\infty} g_n z^n \) \((g_n > 0)\). If \( f(z) \) given by (1.1) belongs to \( C_s^g(\phi) \), then

\[
|a_3 - \mu a_2^2| \leq \begin{cases} 
\frac{1}{6g_3} \left( B_2 - \frac{3}{8} \frac{\mu g_3}{g_2^2} B_1^2 \right) & \text{if } \mu \leq \sigma_1, \\
\frac{B_1}{6g_3} & \text{if } \sigma_1 \leq \mu \leq \sigma_2, \\
-\frac{1}{6g_3} \left( B_2 - \frac{3}{8} \frac{\mu g_3}{g_2^2} B_1^2 \right) & \text{if } \mu \geq \sigma_2,
\end{cases}
\]

where

\[
\sigma_1 := \frac{8g_2^2 (B_2 - B_1)}{3g_3 B_1^2}, \quad \sigma_2 := \frac{8g_2^2 (B_2 + B_1)}{3g_3 B_1^2}.
\]

The result is sharp.

Since

\[
(\Omega^\lambda f)(z) = z + \sum_{n=2}^{\infty} \frac{\Gamma(n+1)\Gamma(2-\lambda)}{\Gamma(n+1-\lambda)} a_n z^n,
\]

we have

\[
(3.2) \quad g_2 := \frac{\Gamma(3)\Gamma(2-\lambda)}{\Gamma(3-\lambda)} = \frac{2}{2-\lambda}
\]

and

\[
(3.3) \quad g_3 := \frac{\Gamma(4)\Gamma(2-\lambda)}{\Gamma(4-\lambda)} = \frac{6}{(2-\lambda)(3-\lambda)}.
\]

For \( g_2 \) and \( g_3 \) given by (3.2) and (3.3), Theorem 3.2 reduces to the following:

**Theorem 3.4.** Let \( \lambda < 2 \). If \( f(z) \) given by (1.1) belongs to \( S_s^\lambda(\phi) \), then

\[
|a_3 - \mu a_2^2| \leq \begin{cases} 
\frac{(2-\lambda)(3-\lambda)}{12} \left[ B_2 - \frac{3(2-\lambda)}{4(3-\lambda)} \mu B_1^2 \right] & \text{if } \mu \leq \sigma_1, \\
\frac{(2-\lambda)(3-\lambda)}{12} B_1 & \text{if } \sigma_1 \leq \mu \leq \sigma_2, \\
-\frac{(2-\lambda)(3-\lambda)}{12} \left[ B_2 - \frac{3(2-\lambda)}{4(3-\lambda)} \mu B_1^2 \right] & \text{if } \mu \geq \sigma_2,
\end{cases}
\]

where

\[
\sigma_1 := \frac{4(3-\lambda)(B_2 - B_1)}{3(2-\lambda) B_1^2}, \quad \sigma_2 := \frac{4(3-\lambda)(B_2 + B_1)}{3(2-\lambda) B_1^2}.
\]
For \( g_2 \) and \( g_3 \) given by (3.2) and (3.3), Corollary 3.3 reduces to the following:

**Corollary 3.5.** Let \( \lambda < 2 \). If \( f(z) \) given by (1.1) belongs to \( C_{\lambda}^\lambda(\phi) \), then

\[
|a_3 - \mu a_2^2| \leq \begin{cases} \frac{(2 - \lambda)(3 - \lambda)}{36} \left[ B_2 - \frac{9(2 - \lambda)}{16(3 - \lambda)} \mu B_1^2 \right] & \text{if } \mu \leq \sigma_1, \\
\frac{(2 - \lambda)(3 - \lambda)}{36} B_1 & \text{if } \sigma_1 \leq \mu \leq \sigma_2, \\
-\frac{(2 - \lambda)(3 - \lambda)}{36} \left[ B_2 - \frac{9(2 - \lambda)}{16(3 - \lambda)} \mu B_1^2 \right] & \text{if } \mu \geq \sigma_2,
\end{cases}
\]

where

\[
\sigma_1 := \frac{4(3 - \lambda)}{(2 - \lambda)} \frac{(B_2 - B_1)}{B_1^2}, \quad \sigma_2 := \frac{4(3 - \lambda)}{(2 - \lambda)} \frac{(B_2 + B_1)}{B_1^2}.
\]

**References**


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