CHARACTERIZATIONS OF REAL HYPERSURFACES OF COMPLEX SPACE FORMS IN TERMS OF RICCI OPERATORS

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ABSTRACT. We prove that a real hypersurface $M$ in a complex space form $M_\mathbb{C}(c)$, $c \neq 0$, whose Ricci operator and structure tensor commute each other on the holomorphic distribution and the Ricci operator is $\eta$-parallel, is a Hopf hypersurface. We also give a characterization of this hypersurface.

0. Introduction

A complex $n$-dimensional Kaehlerian manifold of constant holomorphic sectional curvature $c$ is called a complex space form, which is denoted by $M_\mathbb{C}(c)$. A complete and simply connected complex space form consists of a complex projective space $P_n(\mathbb{C})$, a complex Euclidean space $\mathbb{C}^n$ or a complex hyperbolic space $H_n(\mathbb{C})$, according to $c > 0$, $c = 0$ or $c < 0$.

R. Takagi ([9]) classified all homogeneous real hypersurfaces in $P_n(\mathbb{C})$ into six model spaces $A_1$, $A_2$, $B$, $C$, $D$ and $E$ (see also [10]). J. Berndt ([2]) has completed the classification of homogeneous real hypersurfaces with principal structure vector fields in $H_n(\mathbb{C})$, which are divided into the model spaces $A_0$, $A_1$, $A_2$ and $B$. A real hypersurface of type $A_1$ or $A_2$ in $P_n(\mathbb{C})$ or that of $A_0$, $A_1$ or $A_2$ in $H_n(\mathbb{C})$ is said to be of type $A$ for simplicity.

We shall denote the induced almost contact metric structure of the real hypersurface $M$ in $M_\mathbb{C}(c)$ by $(\phi, \langle,\rangle, \xi, \eta)$. The Ricci operator of $M$ will be denoted by $S$, and the shape operator or the second fundamental tensor field of $M$ by $A$. If the structure vector field $\xi$ is principal, then $M$ is called a Hopf hypersurface. The holomorphic distribution $T_0$ of a real hypersurface $M$ in $M_\mathbb{C}(c)$ is defined by

$$T_0(p) = \{ X \in T_p(M) \mid \langle X, \xi \rangle_p = 0 \}.$$
where $T_p(M)$ is the tangent space of $M$ at $p \in M$. The Ricci operator $S$ is said to be $\eta$-parallel if
\begin{equation}
<(\nabla_X S)Y, Z> = 0
\end{equation}
for any vector fields $X$, $Y$ and $Z$ in $T_0$.

Many authors have occupied themselves with the study of geometrical properties of real hypersurfaces with $\eta$-parallel Ricci operators (see [1], [3], [4], [5], [6], [7], [8] and [9]). Recently, I.-B. Kim, K. H. Kim and the present author studied real hypersurfaces in $M_n(c)$ with certain conditions related to the Ricci operator and the structure tensor field $\phi$ in [3]. In [4], I.-B. Kim, H. J. Park and the present author gave a characterization of the real hypersurface with a special $\eta$-parallel Ricci operators. For the conditions on the $\eta$-parallel Ricci operator, Kimura and Maeda ([5]) and Suh ([8]) proved the following.

**Theorem A.** Theorem A ([5], [8]) Let $M$ be a real hypersurface in a complex space form $M_n(c)$, $c \neq 0$. Then the Ricci operator of $M$ is $\eta$-parallel and the structure vector field $\xi$ is principal if and only if $M$ is locally congruent to one of the model spaces of type $A$ or type $B$.

The purpose of this paper is to improve the results in the previous paper [4] and characterize the real hypersurfaces with $\eta$-parallel Ricci operator. Namely, we shall prove the followings.

**Theorem 1.** Let $M$ be a real hypersurface with $\eta$-parallel Ricci operator in a complex space form $M_n(c)$, $c \neq 0$, $n \geq 3$. If $M$ satisfies
\begin{equation}
<(S\phi - \phi S)X, Y> = 0,
\end{equation}
for any $X$ and $Y$ in $T_0$, then $M$ is a Hopf hypersurface.

**Theorem 2.** Let $M$ be a real hypersurface with $\eta$-parallel Ricci operator in a complex space form $M_n(c)$, $c \neq 0$, $n \geq 3$. If $M$ satisfies (0.2), then $M$ is locally congruent to one of the model spaces of type $A$ or type $B$.

1. Preliminaries

Let $M$ be a real hypersurface immersed in a complex space form $(M_n(c), <, >, J)$ of constant holomorphic sectional curvature $c$, and let $N$ be a unit normal vector field on an open neighborhood in $M$. For a local tangent vector field $X$ on the neighborhood, the images of $X$ and $N$ under the almost complex structure $J$ of $M_n(c)$ can be expressed by
\[ JX = \phi X + \eta(X)N, \quad JN = -\xi, \]
where $\phi$ defines a linear transformation on the tangent space $T_p(M)$ of $M$ at any point $p \in M$, and $\eta$ and $\xi$ denote a 1-form and a unit tangent vector field on the neighborhood respectively. Then, denoting the Riemannian metric on $M$ induced from the metric on $M_n(c)$ by the same symbol $<, >$, it is easy to see that
\[ <\phi X, Y> + <\phi Y, X> = 0, \quad <\xi, X> = \eta(X) \]
for any tangent vector fields $X$ and $Y$ on $M$. The collection $(\phi, <, >, \xi, \eta)$ is
called an almost contact metric structure on $M$, and satisfies

$$
\begin{align*}
\phi^2 X &= -X + \eta(X)\xi, & \phi\xi &= 0, & \eta(\phi X) &= 0, & \eta(\xi) &= 1, \\
<\phi X, \phi Y >= &<X, Y> - \eta(X)\eta(Y).
\end{align*}
$$

(1.1)

Let $\nabla$ be the Riemannian connection with respect to the metric $<, >$ on $M$, and $A$ be the shape operator in the direction of $N$ on $M$. Then we have

$$
\nabla_X \xi = \phi AX, & (\nabla_X \phi)Y = \eta(Y)AX - <AX, Y> \xi.
$$

(1.2)

Since the ambient space is of constant holomorphic sectional curvature $c$, the equations of Gauss and Codazzi are given by

$$
R(X, Y)Z = \frac{c}{4} \{<Y, Z > X - <X, Z > Y + <\phi Y, Z > \phi X - <\phi X, Z > \phi Y \\
- 2 <\phi X, Y > \phi Z\} + <AY, Z > AX - <AX, Z > AY,
$$

(1.3)

$$
(\nabla_X A)Y - (\nabla_Y A)X = \frac{c}{4} \{\eta(X)\phi Y - \eta(Y)\phi X - 2 <\phi X, Y > \xi\}
$$

(1.4)

for any tangent vector fields $X$, $Y$ and $Z$ on $M$, where $R$ is the Riemannian curvature tensor field of $M$. Then it is easily seen from (1.3) that the Ricci operator $S$ of $M$ is expressed by

$$
SX = \frac{c}{4} \{(2n + 1)X - 3\eta(X)\xi\} + mAX - A^2 X,
$$

(1.5)

where $m = \text{trace} A$ is the mean curvature of $M$, and the covariant derivative of

(1.5) is given by

$$
(\nabla_X S)Y = - \frac{3c}{4} \{<\phi AX, Y > \xi + \eta(Y)\phi AX\} + (Xm)AY \\
+ m(\nabla_X A)Y - (\nabla_Y A)AY - A(\nabla_X A)Y,
$$

(1.6)

If the vector field $\phi\nabla_\xi$ does not vanish, that is, the length $\beta$ of $\phi\nabla_\xi$ is not
equal to zero, then it is easily seen from (1.1) and (1.2) that

$$
A\xi = \alpha\xi + \beta U,
$$

(1.7)

where $\alpha = <A\xi, \xi>$ and $U = -\frac{1}{\beta} \phi\nabla_\xi$. Therefore $U$ is a unit tangent vector
field on $M$ and $U \in T_0$. If the vector field $U$ can not be defined, then we may
consider $\beta = 0$ identically. Therefore $A\xi$ is always expressed as in (1.7).

2. $\eta$-parallel Ricci operators

In this section we assume that the open subset

$$
U = \{p \in M \mid \beta(p) \neq 0\}
$$

is not empty. Then, in the previous paper [4], we have proved the followings.
Lemma 2.1. ([4]) Let $M$ be a real hypersurface with the $\eta$-parallel Ricci operator $S$ in a complex space form $M_n(c)$, $c \neq 0$, $n \geq 3$. If it satisfies (0.2), then we have
\begin{equation}
(2.1) \quad m = \text{trace} A = \alpha + \gamma,
\end{equation}
\begin{equation}
(2.2) \quad AU = \beta \xi + \gamma U,
\end{equation}
on $\mathcal{U}$, where we have put $\gamma = \langle AU, U \rangle$.

It follows from (1.1), (1.5), (1.7) and (2.2) that
\begin{equation}
(2.3) \quad S \xi = \left(\frac{n-1}{2}c + \alpha \gamma - \beta^2\right) \xi,
\end{equation}
\begin{equation}
SU = \left(\frac{2n+1}{4}c + \alpha \gamma - \beta^2\right) U,
\end{equation}
\begin{equation}
S\phi U = \left(\frac{2n+1}{4}c + \alpha \gamma - \beta^2\right) \phi U.
\end{equation}
Differentiating the second equation of (2.3) covariantly along any vector field $X$ in $T_0$, we obtain
\begin{equation}
(2.4) \quad (\nabla_X S)U = \left(\frac{2n+1}{4}c + \alpha \gamma - \beta^2\right) I - S \nabla_X U + X(\alpha \gamma - \beta^2) U.
\end{equation}
If we take inner product of (2.4) with $U$ and make use of (0.1) and (2.3), we get
\begin{equation}
(2.5) \quad X(\alpha \gamma - \beta^2) = 0 \quad \text{for} \quad X \in T_0.
\end{equation}

We put
\begin{equation}
Q = (\alpha + \gamma) A - A^2 = S - \frac{c}{4} (2n+1) I - 3\eta \otimes \xi.
\end{equation}
Then $Q$ is a symmetric endomorphism on the tangent space of $M$. Since we see from (0.2) and (2.3,1) that $S\phi = \phi S$ on $M$, we have $Q\phi = \phi Q$ on $M$. Moreover (2.3) is equivalent to
\begin{equation}
(2.6) \quad Q \xi = (\alpha \gamma - \beta^2) \xi, \quad QU = (\alpha \gamma - \beta^2) U, \quad Q \phi U = (\alpha \gamma - \beta^2) \phi U.
\end{equation}
Let $k_r$ be an eigenvalue of $Q$, and $Q(k_r)$ be the eigenspace of $Q$ associated with $k_r$, where $1 \leq r \leq 2n - 1$. If $\lambda$ is a principal curvature of $M$, then there is an eigenvalue $k_r$ of $Q$ such that $k_r = (\alpha + \gamma) \lambda - \lambda^2$. From this quadratic, we see that there are at most two distinct principal curvatures $\lambda_1$ and $\lambda_2$ of $M$ for a given eigenvalue $k_r$. Therefore we have
\begin{equation}
(2.7) \quad Q(k_r) = \left\{ A(\lambda_1) \right\} \otimes \left\{ A(\lambda_1) \oplus A(\lambda_2) \right\} \quad (\lambda_1 \neq \lambda_2),
\end{equation}
where $A(\lambda_j)$ is the eigenspace of $A$ associated with the principal curvature $\lambda_j (j = 1, 2)$ of $M$, and $\oplus$ indicates the direct sum of vector spaces. For a tangent vector field $X \in T_0$ such that $QX = k_r X$, we have $Q\phi X = k_r \phi X$ because of $Q\phi = \phi Q$. 

Let \( k_1, \ldots, k_s \) be the distinct eigenvalues of \( Q \), and let \( k_1 = \alpha \gamma - \beta^2 \). Then, by (2.6) and the above results, it is easily seen that the dimension of \( Q(k_1) \), denoted it by \( \dim Q(k_1) \), is odd and that of \( Q(k_r) \) is even for \( 2 \leq r \leq s \). Moreover we see from (1.7) that there are two distinct principal curvatures, say \( \lambda \) and \( \mu \), of \( M \) such that \( \xi \in A(\lambda) \oplus A(\mu) \), and hence \( Q(k_1) \) is given by \( Q(k_1) = A(\alpha) \oplus A(\mu) \).

Since \( \lambda \) and \( \mu \) are distinct solutions of \( x^2 - (\alpha + \gamma)x - k_1 = 0 \), we have

\[
(2.8) \quad \lambda + \mu = \alpha + \gamma, \quad \lambda \mu = k_1 = \alpha \gamma - \beta^2.
\]

Now we shall prove

**Lemma 2.2.** Under the same assumptions of Lemma 2.1, there exist unit vector fields \( X \in A(\lambda) \) and \( Y \in A(\mu) \) such that

\[
(2.9) \quad \xi = fX + gY, \quad U = gX - fY,
\]

where \( f \) and \( g \) are smooth functions on \( U \), and satisfy \( f^2 + g^2 = 1 \) and \( fg \neq 0 \).

**Proof.** If \( A(\lambda) \) is spanned by \( \{X_1, \ldots, X_n\} \) and \( A(\mu) \) by \( \{Y_1, \ldots, Y_n\} \), then \( \xi \) is expressed by

\[
\xi = \sum_{i=1}^n a_iX_i + \sum_{j=1}^n b_jY_j.
\]

We can choose \( X \) and \( Y \) such as \( \sum a_iX_i = \| \sum a_iX_i \| X \) and \( \sum b_jY_j = \| \sum b_jY_j \| Y \). By putting \( f^2 = \| \sum a_iX_i \|^2 \) and \( g^2 = \| \sum b_jY_j \|^2 \), we have \( \xi = fX + gY \), \( f^2 + g^2 = 1 \) and \( fg \neq 0 \).

Since we have already seen that \( \xi = fX + gY \) and \( \beta U = -\phi \nabla \xi \xi \) on \( U \), it is easy to verify that

\[
\beta U = fg(\lambda - \mu)(gX - fY)
\]

by use of (1.2) and (1.7). Therefore we can choose \( f \) and \( g \) such that \( U = gX - fY \). \( \square \)

**Lemma 2.3.** Under the same assumptions of Lemma 2.1, the dimension of \( Q(k_1) \) is equal to 3 on \( U \).

**Proof.** We have already seen that \( \dim Q(k_1) \) is odd, and from (2.6) that \( \dim Q(k_1) \) is not less than 3.

Assume that \( \dim Q(k_1) \geq 5 \). Then, since \( Q(k_1) = A(\lambda) \oplus A(\mu) \), we may consider that \( \dim A(\lambda) > \dim A(\mu) \) and \( \dim A(\lambda) = 2\ell + 1(\ell \geq 1) \). For the vector fields \( X \in A(\lambda) \) and \( Y \in A(\mu) \) given in Lemma 2.2, we define the subspaces \( \Sigma, \Omega, \phi \Sigma \) and \( \phi \Omega \) of \( Q(k_1) \) by

\[
\Sigma = \{X_\lambda \in A(\lambda) \mid \langle X_\lambda, X_\lambda \rangle = 0\}, \quad \phi \Sigma = \{\phi X_\lambda \mid X_\lambda \in \Sigma\},
\]

\[
\Omega = \{Y_\mu \in A(\mu) \mid \langle Y_\mu, Y_\mu \rangle = 0\}, \quad \phi \Omega = \{\phi Y_\mu \mid Y_\mu \in \Omega\}.
\]

Then we see that \( Q(k_1) = \Sigma \oplus \Omega \oplus \text{span}\{X, Y\} \) and \( \dim \Sigma > \dim \Omega \).

Now we shall show that \( \phi \Sigma \subset \Omega. \) For any two orthogonal vector fields \( X_\lambda \) and \( Y_\lambda \) in \( \Sigma \), we see from Lemma 2.2 that both \( X_\lambda \) and \( Y_\lambda \) are orthogonal to
ξ. If we differentiate $AX_\lambda = \lambda X_\lambda$ covariantly along $Y_\lambda$ and make use of the equation of Coddazzi (1.4), then we obtain $X_\lambda \lambda = Y_\lambda \lambda = 0$ and

$$\left( A - \lambda I \right)[X_\lambda, Y_\lambda] = \frac{c}{2} < \phi X_\lambda, Y_\lambda > \xi.$$  

Taking inner product of (2.10) with $X$ and using (2.9), we get $< \phi X_\lambda, Y_\lambda > = 0$. This means that $\phi \Sigma \cap \Sigma = \{0\}$ and hence $\phi \Sigma \subset \Omega \oplus \text{span}\{X, Y\}$ because $\phi X_\lambda \in Q(k_1)$. Similarly, differentiating $AX_\lambda = \lambda X_\lambda$ covariantly along $X$ and taking account of (1.4), we also have $X_\lambda = 0$ and

$$\left( A - \lambda I \right)[X_\lambda, X] = \frac{c}{4} \{ \eta(X) \phi X_\lambda + 2 < \phi X_\lambda, X > \xi \}.$$  

Taking the inner product of the above equation with $X$ and using (2.9) yields

$$< \phi X_\lambda, X > = 0.$$  

Since we get $< \phi X_\lambda, \xi > = f < \phi X_\lambda, X > + g < \phi X_\lambda, Y > = 0$ by (2.9), it follows from (2.11) that

$$< \phi X_\lambda, Y > = 0.$$  

Therefore it is easily seen from (2.11) and (2.12) that $\phi \Sigma \cap \text{span}\{X, Y\} = \{0\}$ and hence $\phi \Sigma \subset \Omega$. This shows that $\dim \phi \Sigma \leq \dim \Omega$, and give rise to a contradiction because $\dim \Sigma = \dim \phi \Sigma$. Thus we have $\dim Q(k_1) = 3$. \hfill $\square$

By Lemma 2.2, it is easy to see that $\phi U$ is orthogonal to both $X$ and $Y$. Since we have $\phi U \in Q(k_1) = A(\lambda) \oplus A(\mu)$ by (2.6) and $\dim Q(k_1) = 3$ by Lemma 2.3, we may consider that $\phi U \in A(\mu)$, that is,

$$A \phi U = \mu \phi U.$$  

**Lemma 2.4.** Under the same assumptions of Lemma 2.1, we have

$$< \nu + \kappa, \phi X_\nu, X_\kappa > = 0$$  

on $U$, where the non-zero vector fields $X_\nu$ and $X_\kappa$ are orthogonal to $\xi$, $U$ and $\phi U$, and satisfy $AX_\nu = \nu X_\nu$ and $AX_\kappa = \kappa X_\kappa$.

*Proof.* By Lemmas 2.2 and 2.3, we see that the principal curvatures $\nu$ and $\kappa$ of $M$ never equal to $\lambda$ and $\mu$. Let $X_\nu \in Q(k_\nu)$, that is, $k_\nu = (\alpha + \gamma) \nu - \nu^2$. Then we see from Lemma 2.3 that $k_\gamma \neq k_\nu = \alpha \gamma - \beta^2$. Therefore, if we multiply (2.4) by $X_\nu$ and take account of (0.1), (1.5) and (2.5), then we obtain

$$< \nabla X U, X_\nu > = 0 \quad \text{for} \quad X \in T_0.$$  

This means that the vector field $\nabla X U$ is expressed by a linear combination of $\xi$, $U$ and $\phi U$ only. Since we have $< \nabla X U, \xi > = \mu < X, \phi U >$ by taking account of (1.2) and (2.13), we see that

$$\nabla X U = \mu < X, \phi U > \xi + < \nabla X U, \phi U > \phi U$$  

on $U$. Now differentiating (2.2) covariantly along $X_\nu$ and using (2.15), we obtain

$$(\nabla X_\nu A) U = (X_\nu \beta) \xi + (X_\nu \gamma) U + (\gamma - \mu) < \nabla X_\nu U, \phi U > \phi U + \beta \nu \phi X_\nu,$$
from which
\[ \langle (\nabla_{X_\nu} A) X_\kappa, U \rangle = \beta \nu < \phi X_\nu, X_\kappa > . \]

As a similar argument as the above, we also have
\[ \langle (\nabla_{X_\kappa} A) X_\nu, U \rangle = \beta \kappa < \phi X_\kappa, X_\nu > . \]

Therefore, from the last two equations and the equation of Codazzi (1.4), we can verify (2.14). \hfill \box

3. Proof of Theorems

In this section, we shall prove Theorems 1 and 2.

Proof of Theorem 1. We can choose a local orthonormal frame field

\[ \{X_1, X_2, \ldots, X_{2n-1}\} \]

on \( \mathcal{U} \) such that \( X_1 = X \) and \( X_2 = Y \) are given in Lemma 2.2, \( X_3 = \phi U \) and \( AX_i = \lambda_i X_i \) for \( 4 \leq i \leq 2n - 1 \). For any \( X_i(i \geq 4) \) in (3.1), there exists an eigenvalue \( k_r(2 \leq r \leq s) \) of \( Q \) such that \( X_i \in Q(k_r) \). Since \( \phi = \phi Q \), we see that \( \phi X_i \in Q(k_r) \). As we have already seen in (2.7) and (2.8), we see that either \( Q(k_r) = A(\lambda_i) \) or \( Q(k_r) = A(\lambda_i) \oplus A(\alpha + \gamma - \lambda_i) \).

Let \( Q(k_r) = A(\lambda_i) \oplus A(\alpha + \gamma - \lambda_i) \). Since \( \phi X_i \in Q(k_r) \), there are two non-zero vector fields \( X_{\lambda_i} \in A(\lambda_i) \) and \( X_{\alpha + \gamma - \lambda_i} \in A(\alpha + \gamma - \lambda_i) \) such that

\[ \phi X_i = a X_{\lambda_i} + b X_{\alpha + \gamma - \lambda_i} , \]

where \( a \) and \( b \) are smooth functions on \( \mathcal{U} \).

If \( ab \neq 0 \), then we have \( \lambda_i = 0 \) by putting \( X_\nu = X_i \) and \( X_\kappa = X_{\lambda_i} \) into (2.14) of Lemma 2.4, and \( \alpha + \gamma = 0 \) by putting \( X_\nu = X_i \) and \( X_\kappa = X_{\alpha + \gamma - \lambda_i} \) into (2.14). This means that \( \lambda_i = \alpha + \gamma - \lambda_i = 0 \), that is, \( Q(k_r) = A(0) \) and a contradiction. Therefore we have either \( \phi X_i \in A(\lambda_i) \) or \( \phi X_i \in A(\alpha + \gamma - \lambda_i) \).

If \( \phi X_i \in A(\lambda_i) \), then we obtain \( \lambda_i = 0 \) by putting \( X_\nu = X_i \) and \( X_\kappa = \phi X_i \) into (2.14), and \( Q(k_r) = A(0) \oplus A(\alpha + \gamma) \). For a non-zero vector field \( X_{\alpha + \gamma} \in A(\alpha + \gamma) \), we have either \( \phi X_{\alpha + \gamma} \in A(0) \) or \( \phi X_{\alpha + \gamma} \in A(\alpha + \gamma) \). In each case, using (2.14), it is easily seen that \( \alpha + \gamma = 0 \), and a contradiction.

Thus we see that \( \phi X_i \in A(\alpha + \gamma - \lambda_i) \). Putting \( X_\nu = X_i \) and \( X_\kappa = \phi X_i \) into (2.14), we get \( \alpha + \gamma = 0 \). Hence we have \( Q(k_r) = A(\lambda_i) \oplus A(-\lambda_i) \). Moreover we see that the multiplicity of \( \lambda_i \) is equal to that of \( -\lambda_i \).

If \( Q(k_r) = A(\lambda_i) \), then we have \( \phi X_i \in A(\lambda_i) \), and hence \( \lambda_i = 0 \) from (2.14).

Summing up the above results, for the vector fields \( X_i(4 \leq i \leq 2n - 1) \) given in (3.1), there are two cases where all the principal curvatures \( \lambda_i \) associated with \( X_i \) are equal to zero on \( \mathcal{U} \), and where the multiplicity of a non-zero principal curvature \( \lambda_i \) associated with \( X_i \) is equal to that of \( -\lambda_i \) (associated with \( \phi X_i \)), and trace \( A = \alpha + \gamma = 0 \).

The former implies that trace \( A = \alpha + \gamma = \lambda + 2\mu \), and we see from (2.8) that \( \mu = 0 \) identically on \( \mathcal{U} \). Thus the type number at any point of \( \mathcal{U} \) is not greater than 1, and this does not occur (for instance, see [7]). The latter shows that trace \( A = \alpha + \gamma = \lambda + 2\mu = 0 \), and from (2.8) that \( \mu = 0 \) and \( k_1 = \alpha \gamma - \beta^2 = 0 \).
on \( \mathcal{U} \). Therefore we have \( \alpha^2 + \beta^2 = 0 \) and a contradiction. Thus the subset \( \mathcal{U} \) must be empty.

Proof of Theorem 2. Theorem 2 follows from Theorem A and Theorem 1. □

References


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