A Framework of Rate Control and Power Allocation in Multipath Lossy Wireless Networks

Amr Radwan†, Hoon Kim‡

ABSTRACT

Cross-layer design is a concept, which captures the dependencies and interactions and enables information sharing among layers in order to improve the network performance and security. There are two key challenges in wireless networks, lossy features of links and power assumption of network nodes. Cross-layer design of congestion control and power allocation in wireless lossy networks has been studied in the existing literature; however, there has been no contribution proposed in the literature that exploits the path diversity. In this paper, we are motivated to develop a cross-layer design of congestion control and power allocation, which takes into account lossy features of wireless links and transmission powers of network nodes and can be implemented in a distributed manner. Numerical simulation is conducted to illustrate the performance of our proposed algorithm and the comparison with current alternative approaches.

Key words: Rate Control, Power Allocation, Lossy Links, Multipath Networks, Rayleigh-faded Channels.

1. INTRODUCTION

The concept of cross-layer has been introduced for over ten years to design optimal resource schemes in wired/wireless networks. Wireless ad hoc networks are usually constrained by limited resource, such as battery capacity, bandwidth, and spectrum. There are two fundamental tasks in wireless ad hoc networks: power allocation to prolong the battery life and rate control to maximize the total utility of the users.

Cross-layer designs of rate control and power allocation in wireless networks has received many attentions from network communities [1, 2]. In [3], the authors proposed a distributed algorithm of power control and congestion control in existing TCP protocols. The problem is relaxed and solved by the high-SIR approximation and the solution is therefore sub-optimal. In order to obtain the globally optimal solution to the problem in [3], a convexification method is proposed in [4], where the authors also introduced a novel successive convex approximation method so as to develop a distributed algorithm that can preserve existing TCP protocols. A similar optimization problem in fast-fading network was studied in [5], where constraints on rate outage probability are taken into consideration. However, the common point of the aforementioned literature is that the network utility is computed at the source rates and the transmission rate of a flow does not change over the route to the destination. This assumption is just suited for wired networks and not practical for wireless networks.

† Corresponding Author: Hoon Kim, Address: (10380) Juhwa-ro 170, Eunan-gu, Goyang-si, Gyeonggi-do, Korea. TEL: +82-31-910-7119, FAX: +82-31-910-7188, E-mail: mega@hanmail.net
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‡ Dept. of Information & Communication Eng., Inje University
†† Dept. of Emergency Medicine, Inje University Ilsan Paik Hospital

E-mail: amr_or@yahoo.com
Unlike wired networks where rate control is shielded from channel variations [6] and the assumption in, for example, [7], the transmission rate at the source node of a flow is always greater than or equal to that at the destination node. Based on this fact, Q. Gao et al. proposed the leaky-pipe model and the concept of effective utility, which associate with the rate at the destination node instead of the rate at the source node. Then, two optimization problems, one with link outage constraints and the other with path outage constraints, were studied. Followed from the notion of leaky-pipe model in [7], a number of algorithms for congestion control have contributed to different cross-layer issues. Considering wireless lossy networks, the authors in [8] took link delays into constraints on link capacity and proposed a distributed algorithm for rate control and power control. The ENUM algorithm in [7] was extended to the joint case of power allocation and rate control in fast-fading wireless lossy networks. To guarantee the globally optimal solution, instead of sub-optimal, to the problem in [8], a novel distributed algorithm was proposed in [9]. Considering a same problem as [8], the authors in [10] however take the timescale difference among power control, link delay control, and congestion control, into account. To exploit benefits of the path diversity, a distributed rate control algorithm in wireless multipath lossy networks was proposed in [11]. A common point derived from simulation results in the above mentioned works is that the total effective rate and the network effective utility are higher than that of the correspondingly compared ones.

Literature on resource allocation in multipath communication networks has showed that multipath communication can support traffic load balancing and bandwidth usage efficiency, enhance connection persistence, reliability, and increase the network performance [11]. However, to the best of found knowledge, there is no existing work dedicated to the joint problem of congestion control and power allocation in wireless multipath lossy networks. In light of this observation, a cross-layer approach of congestion control and power allocation in wireless multipath lossy networks is designed and a distributed algorithm called multipath Effective NUM with Power control (mENUMP) is proposed. In a nutshell, the main contributions of this paper can be summarized, as follows:

- In Section 2, we introduce the system model and then formulate a joint optimization problem, which is a non-convex problem.
- In Section 3, first we show that using appropriate approximation methods, we can transform the formulated non-convex problem into a sequence of convex problems. Then each convex problem can be solved optimally via the duality approach. What is more, we proposed a distributed algorithm and prove its convergence guarantee.
- In order to evaluate the performance of the proposed algorithm, we provide simulation results through numerical examples in Section 4. We also compare the performance of the proposed algorithm with existing procedures.

2. SYSTEM MODEL

2.1 Network Model

A multipath wireless networks of \( L \) logical links and \( S \) sources is considered. Denote by \( L(s) \) the set of links used by flow \( s \), \( S(l) = \{ s \in S | l \in L(s) \} \) the set of sources using link \( l \), \( J(l) \) the set of paths using link \( l \), and \( J(s) \) the set of paths of source \( s \). For source \( s \), the data rate of subflow \( j \) on link \( i \) is \( x_{ij} \). Each source \( s \) associates with a utility function \( U_s(x_s) \). In this paper, we consider a family of concave utility functions [12],

\[
U_s(x_s) = \frac{(1 - \alpha)^{-1} x_s^{1 - \alpha}}{\ln(x_s)} \quad \text{if } \alpha \geq 0, \alpha \neq 1,
\]

\[
= \frac{1}{\alpha} \quad \text{if } \alpha = 1,
\]

where \( \alpha \) is the degree of fairness, for example, optimal fairness with \( \alpha \rightarrow 0 \), proportional fairness with
$\alpha$-1, harmonic mean fairness with $\alpha$-$2$, and max–min fairness with $\alpha$-$\infty$. Let $Pr(\cdot)$ be the probability function, $e^{\cdot}$ the exponential function, and $X$: $Y$ be the set of $\{X, X+1,..., Y-1, Y\}$.

The link capacity is modeled by the Shannon capacity $C(P) = W \log(1 + K \gamma(P))$, where $P$ is the transmission power vector, $W$ is the bandwidth, $K$ is a constant value depending on particular modulation, coding scheme and bit-error-rate and $\gamma(P)$ is the instantaneous signal-to-interference-plus-noise ratio (SINR) on link $l$. SINR of link $l$ is

$$\gamma(P) = \frac{\sum_{k} p_l g_{lk}}{n_0 + \sum_i p_i g_{il}}$$

where $n_0$ is the thermal noise power at the receiver on link $l$, $p_l$ is the transmission power of link $l$, $g_{lk}$ is the instantaneous channel gain from the transmitter on link $k$ to the receiver on link $l$ with $g_l$ and $f_k$ representing slow-fading channel and fast-fading channel, respectively, and $\sum_k p_k g_{lk}$ denotes the total interference from the other links.

We consider the non-line-of-sight propagation environment, where we can employ the Rayleigh fading model. In this case, the average link SINR $\gamma_l(P)$ is

$$\gamma_l(P) = \frac{\mathbb{E}[sg_{lk}(f_k)]}{\mathbb{E}[n_0 + \sum_i p_i g_{il}]} = \frac{\gamma_{lk}}{n_0 + \sum_i p_i g_{il}}$$

where exponentially random variables $f_k$ are assumed to be independent and identically distributed (i.i.d.) and $\mathbb{E}[f_k] = 1$, $\forall k$.

### 2.2 Rate Outage Probability

For any algorithm in wireless networks, when the channel state changes, it is compelled to re-run the algorithm to obtain a new optimal solution. The channel state changes frequently, especially in fast-fading networks: algorithms in such environments are not practical since re-calculating the optimal solution increases the number of message passing in the network. To tackle this problem, we consider the concept of rate outage probability, where the network is enabled to suffer a tolerable rage of outage. Rate outage probability of a link occurs when the SINR of that link is lower than a target minimum SINR $\gamma_l^h$, i.e. $Pr(\gamma_l < \gamma_l^h)$, in that case the network performance is unacceptable.

In the Rayleigh fading model, the exactly closed-form expression [13] of rate outage probability is given as

$$Pr(\gamma_l < \gamma_l^h) = 1 - \phi(P) = 1 - \exp \left( - \frac{n_0 \gamma_l^h}{P} \right) \prod_{k=1}^{\gamma_l} \left(1 + \frac{\gamma_l^h p_k g_{lk}}{P} \right)^{-1} \cdots (1)$$

Let $\epsilon_l$ is the outage probability threshold of link $l$, i.e., $Pr(\gamma_l < \gamma_l^h) \leq \epsilon_l$, Eq. (1) can be transformed equivalently into

$$\prod_{k=1}^{\gamma_l} \left(1 + \frac{\gamma_l^h p_k g_{lk}}{P} \right) \leq \Omega_l(y_k),$$

where $\Omega_l(y_k) = \frac{1}{1 - \epsilon_l} \exp \left( - \frac{n_0 \gamma_l^h}{P} \right)$.

### 2.3 Effective Rate

The leaky-pipe flow model [7] was originally designed to model lossy wireless links, where the transmission rate at the source node is called injection rate and that at the destination node is called effective rate. In that model, the transmission rate of a flow changes hop by hop and decreases along its route, i.e., the effective rate is always not larger than the injection rate. According to [7], the effective rate $y_s$ is calculated as the multiplication of the outage probability on links that flow $s$ traverses and the injection rate $x_s$, as $y_s = x_s \prod_{i \in L(s)} \left[1 - Pr(\bar{r}_i)\right]$, where $Pr(\bar{r}_i)$ is the rate outage probability on link $l$. Denote by $H_s$ the number of hops that subflow $j$ of source $s$ travels through before reaching the destination, we have $x_s^{i+1} = x_s^i (1 - Pr(\bar{r}_i)), i=1,2,...H_s$ where $Pr(\bar{r}_i)$ denotes the rate outage probability on link $i$.

### 2.4 Optimization Problem

In order to take the lossy features of wireless
3. DISTRIBUTED ALGORITHM

3.1 Convexification of the optimization problem

Firstly, to avoid the non-concavity of the utility function of path rates, we use the Jensen’s inequality to approximate it into a concave function of path rates. Let \( \theta = [\theta_s : \theta]\) and \( \theta_s = [\theta_{ss} : \theta_{ss'}] \) be respectively the approximation vector of all of the sources and of source \( s \), where \( \theta_{ss} > 0, \forall s, j \) and \( \sum_{j \in X_s} \theta_{ss} = 1 \).

We have

\[
\sum_s \sum_{j \in X_s} \theta_{ss} U_s \left( \frac{x_{ss}^{H_s+1}}{\theta_{ss}} \right) \geq \sum_s \sum_{j \in X_s} \theta_{ss} U_s \left( \frac{x_{ss}^{H_s+1}}{\theta_{ss}} \right),
\]

where the equality happens when \( \theta_{ss} = \frac{x_{ss}^{H_s+1}}{\sum_j x_{ss}^{H_s+1}} \).

The optimization problem (2) then becomes

\[
\max \sum_s \sum_{j \in X_s} \theta_{ss} U_s \left( \frac{x_{ss}^{H_s+1}}{\theta_{ss}} \right) - w \sum_j p_j \tag{4}
\]

s.t. \( m_j = x_{ss} \leq M \), \( \forall s, j, i \),

\[
\prod_{j \in X_s} \left( 1 + \frac{p_j^o \theta_j}{p_j^o} \right) \leq \Omega_j(p_j) \forall j,
\]

\[
x_{ss}^j \leq C(\gamma_j(P)) \forall j,
\]

\[
x_{ss}^j \leq x_{ss}^j(1 - Pr(i)), i = 1, 2, ..., H_s.
\]

For a given approximation vector \( \theta \), the optimization problem (4) is still not a convex problem due to the last three constraints. Therefore, a further step is needed in order to convexify (4). Let \( x_{ss}^{H_s+1} = e^{\epsilon_{ss}} \) and \( p_j = e^{\epsilon_j} \) and take the logarithm of both sides of all of the constraints, the optimization problem (4) can be converted to the problem, as follows:

\[
\max \sum_s \sum_{j \in X_s} \theta_{ss} U_s \left( e^{\epsilon_{ss}} \right) - w \sum_j \bar{p}_j \tag{5}
\]

s.t. \( \log(m_j) \leq \epsilon_{ss} \leq \log(M_j) \), \( \forall s, j, i \),

\[
\log(p_j^{max}) = \bar{p}_j \leq \log(p_j^{max}) \forall j,
\]

\[
\sum_j \log \left( 1 + \frac{p_j^o \theta_j}{p_j^o} \right) \leq \log \Omega_j(e^{\epsilon_j}) \forall j,
\]

\[
x_{ss}^j \leq \bar{x}_{ss}^j - \psi_j(e^{\epsilon_j}), i = 1, 2, ..., H_s
\]

where \( \psi_j(e^{\epsilon_j}) = -\log(\delta_j(e^{\epsilon_j})) \).
Lemma 1

For a given \( \theta \) and \( \alpha \geq 1 \), the problem (5) is a convex optimization problem.

Proof

According to Lemma 1 in [11], for a given \( \theta \) and \( \alpha \geq 1 \), the first part of the objective is concave with respect to the effective rate. In addition, the second part of the objective, sum-of-exp, is convex with respect to the transmission power. Therefore, the objective is a joint concave function with respect to the effective rate and transmission power. The first constraint and second constraint are line segments, they are hence convex. The Hessian of the third constraint is non-negative [5], it is therefore convex. In the fourth constraint, the left-hand side is convex with respect to transmission rates and the right-hand side is concave due to the subtraction of a linear term and a log-sum-exp term [14]. The Hessian of the last constraint in \( P \) is exactly that of the third constraint, thus it is convex with respect to \( P \). It is also linear in transformed transmission rates. Consequently, the fourth one is jointly convex in transmission rates and transmission powers. We therefore finally conclude that (5) is a convex optimization problem.

3.2 Rate Control and Power Allocation for a Given Approximation Vector

Due to convexity, the approximation problem (5) can be solved optimally by the Lagrangian dual method; the local optimal solution is also the globally optimal solution. Especially, when the fairness degree \( \alpha > 1 \), the problem (5) is strictly convex, the optimal solution is then unique. Applying the dual technique, the Lagrangian for (6) is given as the following

\[
L(x, P, \lambda, \mu, \nu) = \sum_{s=1}^{K} \sum_{l=1}^{S(s)} \theta_s U_s \left( \frac{e^{x_{s,l}}}{\theta_s} \right) - \sum_{l=1}^{L} e^{x_{l}} + \sum_{l=1}^{L} \lambda_l \left( \log_2(e^{x_{l}}) - \log \left( 1 + \frac{\theta_s e^{x_{s,l}}}{e^{P_l}} \right) \right)
+ \sum_{l=1}^{L} \mu_l \left( C_l(\gamma_l(e^{x_l})) - \sum_{s \in S(l)} e^{x_{s,l}} \right)
+ \sum_{s=1}^{S} \sum_{l=1}^{S(s)} \nu_{s,l} \left( e^{x_{s,l}} - x_{s,l} - \psi_l(\theta_s) \right),
\]

where \( \lambda = [\lambda_1 : \lambda_2], \mu = [\mu_1 : \mu_2] \), and \( \nu = [\nu_{s,l} : \nu_{s,l}^{(2)} : \ldots : \nu_{s,l}^{(S(s))}] \) are three dual variable vectors, which associate with the rate outage constraint, link capacity constraint, and rate reservation constraint, respectively, and can be considered as the outage price, congestion price, and lossy price. The Lagrange dual function is defined as the maximum value of the Lagrangian (6) over \((x, P)\), as follows:

\[
g(\lambda, \mu, \nu) = \max_{x, P} L(x, P, \lambda, \mu, \nu).
\]

Accordingly, the dual problem is given as

\[
\min_{\lambda, \mu, \nu} g(\lambda, \mu, \nu).
\]

Since the Lagrangian (6) is separable in transmission rates and transmission powers, (6) can be decomposed into two partial functions, as follows:

\[
L(x, P, \lambda, \mu, \nu) = L_e(x, \mu, \nu) + L_p(P, \lambda, \mu, \nu),
\]

where

\[
L_e(x, \mu, \nu) = \sum_{s=1}^{K} \sum_{l=1}^{S(s)} \theta_s U_s \left( \frac{e^{x_{s,l}}}{\theta_s} \right) - \sum_{l=1}^{L} \mu_l \sum_{s \in S(l)} e^{x_{s,l}}
+ \sum_{s=1}^{K} \sum_{l=1}^{S(s)} \nu_{s,l} \left( x_{s,l} - \psi_l(\theta_s) \right),
\]

and

\[
L_p(P, \lambda, \mu, \nu) = -\sum_{l=1}^{L} \mu_l \left( \gamma_l(e^{x_l}) - \sum_{s \in S(l)} \log \left( 1 + \frac{\theta_s e^{x_{s,l}}}{e^{P_l}} \right) \right)
+ \sum_{l=1}^{L} \mu_l \gamma_l(e^{x_l}) - \sum_{s=1}^{S} \sum_{l=1}^{S(s)} \mu_l \psi_l(\theta_s).
\]

Let \( \nu_{s,l} = 0, \forall s, l \), the partial Lagrangian (7) can be equally rewritten as

\[
L_e(x, \mu, \nu) = \sum_{s=1}^{K} \sum_{l=1}^{S(s)} \theta_s U_s \left( \frac{e^{x_{s,l}}}{\theta_s} \right) - \sum_{l=1}^{L} \mu_l \sum_{s \in S(l)} e^{x_{s,l}}
- \sum_{l=1}^{L} \mu_l \left( \gamma_l(e^{x_l}) - \sum_{s \in S(l)} \log \left( 1 + \frac{\theta_s e^{x_{s,l}}}{e^{P_l}} \right) \right)
+ \sum_{s=1}^{K} \sum_{l=1}^{S(s)} \nu_{s,l} \left( x_{s,l} - \psi_l(\theta_s) \right).
\]

Now, we can use the primal-dual method to update primal variables as well as dual variables and develop a distributed algorithm.

Power allocation: The transmission power of link \( l \) is updated, as follows:

\[
p_l(t+1) = \left\lfloor \frac{M_l(t) + (1 + \lambda_l(t) + \mu_l(t) + \psi_l(\theta_s)) e^{P_l(t)}}{1 + \sum_{m=0}^{M_l(t)} e^{x_l(t) + \lambda_l(t)} - \sum_{m=0}^{M_l(t)} e^{x_l(t) + \mu_l(t) + \psi_l(\theta_s)}} \right\rfloor.
\]
where \( x^b_s = \max \{ \min \{ x, b \}, a \} \), \( t \) is the iteration index of the inner loop, and \( \delta(t) \) is the step size, which is required to satisfy the diminishing rule [14]. In (8), we define

\[
M(t) = \mu(t) W \frac{K_{\gamma}(t)}{1 + K_{\gamma}(t)}, \quad m_i(t) = M(t) \frac{\gamma_i(t)}{\theta_{\gamma_i}(t)},
\]

\[
m_i^{th}(t) = \frac{\gamma_i(t)}{\theta_{\gamma_i}(t)}, \text{ which are message passing among links and } v = \sum_{s=1}^{S} \sum_{j=1}^{J} \nu_{ij}^o, \text{ which can regarded as the lossy price of link } l.
\]

**Transmission rates:** The transmission rate of subflow \( j \) of source \( s \) at link \( i \) is updated as

\[
\hat{z}_{ij}(t+1) = \hat{z}_{ij}(t) + \delta(t) \nabla L_{\gamma_i}(z_{ij}(t), \nu_{ij}^o)_{\lambda_{ij}(t)} \quad i = 1:H_s + 1,
\]

(9)

where \( \nabla L_{\gamma_i}(z_{ij}(t), \nu_{ij}^o) \) is the gradient of the partial Lagrangian (6) with respect to \( \hat{z}_{ij} \). If \( i = H_s \), we have

\[
\nabla L_{\gamma_i}(z_{ij}(t), \nu_{ij}^o) = \nu_{ij}^o - \nu_{ij}^{o-1} - \mu_i \hat{z}_{ij}^o
\]

and if \( i = H_s + 1 \), we have

\[
\nabla L_{\gamma_i}(z_{ij}(t), \nu_{ij}^o) = e^{\gamma_i(t)} \left[ \frac{e^{\gamma_i(t)}}{\theta_{\gamma_i}(t)} \nu_{ij}^o - \mu_i e^{\gamma_i(t)} \right].
\]

**Outage prices:** Outage prices can be updated as

\[
\lambda_i(t+1) = \left[ \lambda_i(t) - \delta(t) \nabla L(\lambda_i(t)) \right]^+, \quad \text{(10)}
\]

where \( \langle z \rangle^+ = \max \{ z, 0 \} \) and \( \nabla \lambda_i \) is the gradient of the Lagrangian (6) with respect to \( \lambda_i \), which is

\[
\nabla L(\lambda_i(t)) = \log f_i(e^{\gamma_i(t)}) - \sum_k \log \left( 1 + \frac{e^{\gamma_i(t) \theta_{\gamma_i}(t)}}{e^{\gamma_i(t)}} \right).
\]

**Congestion prices:**

\[
\rho_i(t+1) = \left[ \rho_i(t) - \delta(t) \left[ C(\gamma_i(t)) - \sum_{s=1}^{S} \sum_{j=1}^{J} \hat{z}_{ij}(t) \right]^+ \right]^+
\]

(11)

**Lossy prices:**

\[
\nu_{ij}^o(t) = \left[ \nu_{ij}^o(t) - \delta(t) \left[ \hat{z}_{ij}(t) - \hat{z}_{ij}^{o-1}(t) - \psi(\nu_{ij}^o(t)) \right]^+ \right]^+
\]

(12)

**3.3 Proposed mENUMP Algorithm**

For a given approximation vector \( \theta \), transmission powers can be updated according to (8), transmission rates are updated via (9), dual variables as outage prices, congestion prices, and lossy prices can be respectively updated based on (10), (11), and (12). After obtaining the stationary values, we update the approximation vector \( \theta \) for the underlying problem (2) via the equality in (3). Denote by \( \tau \) the iteration index of the outer loop. The value of \( \theta \) at the iteration \( \tau+1 \) is obtained from the previous stationary values of effective rates, as follows:

\[
\theta_{\gamma_i}(\tau+1) = \frac{x_{\gamma_i}^{\tau+1} - x_{\gamma_i}^{\tau}}{x_{\gamma_i}^{\tau+1} - x_{\gamma_i}^{\tau}}, \quad \sum_{j=1}^{J} x_{\gamma_i}^{\tau+1} - x_{\gamma_i}^{\tau} = 1.
\]

The algorithm for solving the underlying problem (2) is summarized in Alg. 1, where \( obj \) denotes the value of the objective of the approximation optimization problem (4). In the following, we give some remarks on our proposed algorithm.

**Remark 1**

For a given approximation vector \( \theta \), transmission rates and lossy prices are updated with only local information while transmission powers, outage prices, and congestion prices can be updated distributedly with help of coordination among network nodes via message-passing. For example, in order to update the transmission power of link \( l \), other links need to send their message-passing components to the receiver of link \( l \), such as \( m_i(t), m_i^{th}(t), \nu^o_i, \), and \( \lambda_i(t) \); after finishing the update of the transmission rate and power, the transmitter of link \( l \) disseminates these information to receivers of other nodes. Therefore, the proposed algorithm can be implemented in a distributed fashion.

**Remark 2**

Message passing among nodes enables the proposed algorithm mENUMP to be implemented dis-
Algorithm 1 mENUMP Algorithm

Initialization
1) Initialize \( t = 0 \), \( x'_s = x'_s(0) \), \( p_i = p_i(0) \), \( \lambda_i = \lambda_i(0) \), \( \mu_i = \mu_i(0) \), \( \nu'_s = \nu'_s(0) \). Initial transmission rates and powers are chosen randomly, according to their feasible sets, and prices are required to be non-negative. Set \( \text{FLAG}_{\text{init}} = 0 \) and \( r = 1 \).

repeat (outer loop to update the approximation vector \( \theta \))
Set \( \text{FLAG}_n = 0 \) and \( t = 1 \)
Each source \( s \) calculates \( \hat{\theta}_s \) based on Eq. (13)
repeat (inner loop to update primal and dual variables for a given \( \hat{\theta} \))
Each source \( s \) updates its path transmission rates by Eq. (9)
Outage prices are updated according to Eq. (10)
Congestion prices are updated via Eq. (11)
The lossy price at link \( i \) on path \( j \) of source \( s \) is updated based on Eq. (12)
if \( \frac{|ob^{(r)}(s) - ob^{(r-1)}(s)|}{ob^{(r)}(s)} \leq \epsilon_n \) then
\( \hat{x}^*(s) = x^{(r)}(s) \) and \( P^*(s) = P^{(r)}(s) \) is the optimal solution obtained at the iteration \( r \)
Set \( \text{FLAG}_n = 1 \)
else
Set \( t = t + 1 \)
until \( \text{FLAG}_n = 1 \)
if \( \frac{|ob^{(r)}(s) - ob^{(r-1)}(s)|}{ob^{(r)}(s)} \leq \epsilon_{out} \) then
\( \hat{x}^* = \hat{x}^*(s) \) and \( P^* = P^{(r)}(s) \) is the optimal solution to the approximation problem (1)
Set \( \text{FLAG}_{\text{out}} = 1 \)
else
Set \( r = r + 1 \)
until \( \text{FLAG}_{\text{out}} = 1 \)
The result \( \hat{x}^* \) is transformed to the \( x \)-space. The optimal solution to the original problem (5) is \((x^*, P^*)\)

tributed, which can however decrease the network performance. Our focus in this paper is a joint scheme of congestion control and power allocation in wireless networks; therefore, the number of links, sources, and the number of multipath sources are often not large [7, 11] and then the number of information sharing and computational complexity of the algorithm is reasonable. To further limit the number of message passing, an optimal scheme based on a simplified outage can be considered [5, 10].

3.4 Convergence guarantee of mENUMP

For the inner loop of mENUMP (the approximation vector \( \theta \) is fixed), if the step size satisfies the diminishing rule, the sequences generated by the algorithm finally converge to the optimal points [14]. Regarding convergence of the outer loop of mENUMP, we have the following theorem.

Theorem 1

For a given \( \theta \) and \( a \geq 1 \), the outer loop of mENUMP monotonically improves the objective value. The optimal solution achieved by the mENUMP converges to a stationary point and satisfies the KKT optimality conditions.

Proof

When \( \theta \) is fixed, the optimal solution \((x^*, P^*)\) is
unique due to (strictly) concavity of the optimization problem (5). Let

$$V(x,P) = \sum_{k=1}^{S} U\left( \sum_{j=1}^{N} x_{k,j} \right) = \sum_{l=1}^{L} p_l$$

(14)

we have the following relationship

$$V(x^{(r-1)},p^{(r-1)}) = \sum_{k=1}^{S} U\left( \sum_{j=1}^{N} x_{k,j}^{(r-1)} \right) + \sum_{l=1}^{L} p_l^{(r-1)}$$

$$\leq \sum_{k=1}^{S} U\left( \sum_{j=1}^{N} x_{k,j}^{(r)} \right) + \sum_{l=1}^{L} p_l^{(r)}$$

(15)

In (15) the first equality is due to (14), the second equality can be achieved by substituting the approximation vector in (3) into the objective of the approximation problem. The third one (an inequality) is suitable since \((x^{(r)},p^{(r)})\) is the optimal solution to the approximation problem for a given \(\theta(r)\). The last one (an inequality) is followed from the Jensen’s inequality (3). In addition \(V(x,P)\) is bounded, therefore, the sequences generated by the proposed procedure eventually converge.

The second part of Theorem can be easily proved by constructing Lagrangian and KKT optimality conditions for the approximation problem (5) and the original problem (2). KKT conditions of these two problems are then compared one-by-one.

4. SIMULATION RESULTS

In this section, we show the numerical simulation of the proposed algorithm, in comparison with existing frameworks, for examples, [6], [9], and [11]. We consider two wireless multihop topology, a single-path network, used for simulation of compared frameworks [6, 9], and a multipath network, used for simulation of the algorithm in [11] and our proposed algorithm.

4.1 Simulation settings

Two considered network topology are illustrated in Fig. 1, where nodes are separated and positioned equally at a distance \(d = 50\) m. The maximal outage probability and SINR threshold are respectively set to 0.20 and 1.0 for all links. The fast-fading channel change is i.i.d. and the slow-fading channel is computed as \(g_b = g_r(d_p/50)^{-AF}\), where \(g_r\) is a reference channel gain at a distance 50 m and \(AF\) is the path loss attenuation factor [5], [10]. Unless otherwise stated, the fairness degree is set to 6.

4.2 Performance of mENUMP and compared framework

First, we examine the convergence guarantee of the proposed algorithm. Fig. 2 illustrates that the proposed algorithm eventually converges to the optimal solution and the injection rate of a flow is larger than its effective rate, which is due to the lossy features of wireless links. It can be also seen that the effective rate of a flow traversing a larger number of hops is higher than that of a flow traveling less hops, e.g., flow 1 travels 1 link, flow 2 travels 2 link and both sub-flows of source 4 travel 4 links. However, this property is not reversed for the injection rates.

Second, the performance of the proposed algorithm is compared with three other ones: an algorithm, namely ENUMPs, of congestion control and power allocation in fast-fading wireless lossy networks [6]; an algorithm of congestion control in multipath lossy wireless networks, namely
mENUM [11]; and an algorithm of congestion control, power allocation, and link delay in fast-fading wireless lossy networks, namely, nRENUM [9]. Fig. 3 shows the comparison among our proposed algorithm and two other algorithms in term of the total power consumption. ENUMP consumes much more powers than nRENUM and mENUM. This has been explained in [10] as the original problem of ENUMP does not consider the power consumption in the objective function, fix links’ SINR rigidly, and the problem is just to maximize overall utility. The power consumption in our proposed method is lower than that in nRENUM since mENUM can exploit the path diversity so as to improve the network performance. To compare mENUM with the algorithm of congestion control in multipath lossy networks, mENUM, we change the degree of fairness and then observe the total injection rate and the total effective rate. Fig. 4a, represents that the total injection rate in mENUM is always higher than that in mENUM. The total effective rate achieved by mENUM is however higher than that in mENUM only when the fairness index is greater than seven.

5. CONCLUSION

The problem of joint congestion control and
power allocation for wireless multipath lossy networks was studied in this paper. The formulated non-convex problem is transformed into a sequence of convex ones by the Jensen’s inequality. After that, a distributed, namely, mENUMP, was proposed. The simulation results showed that our proposed algorithm is superior to the algorithms in single-path networks in term of the total power consumption and in multipath networks in term of the transmission rate. As a future work of this paper, we try to conduct the comparison in term of transmission rates among our proposed algorithm and other frameworks of rate control and power allocation in single-path wireless lossy networks.

REFERENCE


Amr Radwan received his B.S. and M.S. degrees in Mathematics from Sohag University, Sohag, Egypt, in 1999 and 2005. He received the Ph.D. degree in Mathematics from Humboldt University zu Berlin, Germany in 2012. From Aug. 2012–Sep. 2014, he worked as a lecturer at Mathematics Department, Faculty of Science, Sohag University, Egypt. Dr. Radwan was awarded an NRF Postdoctoral Research Fellowship (2014, Oct.–2015, Feb.), and joined Prof. Won–Joo Hwang’s research group in the Department of Information and Communications Engineering at Inje University, where he worked problems related to wireless networks. Since Mar. 2015, he has been an assistant professor at Department of Information and Communications Engineering, Inje University, Gyeongnam, Korea. His research interests are in the area of nonlinear optimization, optimal control problem, automatic differentiation, evolutionary computing and wireless networks.

Hoon Kim Received the MSc Degree from Graduate School, College of Medicine, Inha University, Korea in 2006. He received his M. D. degree from Inha University, Republic of Korea in 2002. Since September 2010, he has been a professor at Inje University, Republic of Korea. His research interests are in Health Information System and International Cooperation in Healthcare.