I. Introduction

The compensation for labor force has been a central focus of researchers in many fields, and a particular attention has been paid to understand heterogeneity across salesforce and firms using different research approaches. For example, Basu, Lal, Srinivasan and Staelin[1] focus on the differences in likely behaviors of the salesperson across firms to examine the presence of heterogeneous types of compensation plans using an agency theory framework. In contrast, Lal and Staelin[2] and Rao[3] address salesforce
heterogeneity and information asymmetry to find the optimal compensation scheme. The studies on the salesforce compensation have not been limited to the salary-commission structure[4-7], and empirical and experimental evidence have been provided to support the theoretical findings[8-11].

Yet little effort has been made to explain about a phenomenon where the structure of the compensation plan for a single salesperson may vary as he/she gains experience. The absence of such studies is particularly interesting in that it is frequently witnessed that the salesforce compensation plan depends solely on the base salary for a certain period of time at the beginning of a salesperson’s career. The structure of the salesforce compensation, then, turns to the usual compensation scheme, such as combined salary and commissions or quota-based compensation plan.

One explanation for such dynamics in the compensation scheme structure can be the training for new salespeople, in that they may need training on their new tasks and therefore they cannot expect to receive commissions during the training. This argument is intuitive for a salesperson at a retail store for frequently purchased goods, where this period is a relatively short term. However, it does not give a sufficient explanation for the case of durable goods, in which such period is relatively long and salesperson often start engaging in sales before the period ends. Under this restriction, conventional wisdom suggests that the salespersons do not have motivation for the sales without commissions during this period, and the expected outcome level for a new salesperson is consequently low. Building on this notion, we attempt to address the dynamics in salesforce compensation structure by considering the accumulated efforts level of a salesperson.

Human capital accumulation through learning and experience has been widely studied. However, surprisingly few studies in the agency literature have addressed this question in studying the salesforce compensation. This is because the optimal contract plan in a repeated moral hazard model becomes quickly intractable as the number of periods increases even when an agent’s performance depends upon his/her current effort[12]. However, one need to note that a distinctive feature of accumulated human capital that current effort not only affects the agents’ current outcome but also determines their future productivity, in designing the optimal salesforce compensation. Given this recognition of accumulated human capital, we take a look at the problem of experience accumulated by a salesperson’s efforts and convergence of a salesperson’s productivity for a given present effort level. The emphasis is made on developing a theory, based on standard agency model, which explains the dynamics in the compensation plan structure within and across industries.

The intuition behind this result is following: we assume that there are diminishing marginal returns to the accumulated efforts. Until the marginal returns of accumulated efforts are sufficient, in order to induce high an effort of a salesperson, a sales manager only need to promise the adequate amounts of commissions at the end of the last period. However, the amounts of the commissions at the last period are far less than the sum of the direct commissions. Because high effort increases salesperson’s productivity and increases the probability of winning commissions at the last period, a salesforce will make a high effort, without any explicit commissions in any previous periods.

In particular, this result predicts that a salesperson will make an effort at every period early in his/her career without any commissions in order to gain experience. However, if one cannot expect any
marginal returns of accumulated efforts, the sales manager must provide commissions on the high volume of sales at each period, to facilitate a high effort of a salesforce at earlier periods. In order to consider the zero marginal returns of the accumulated efforts, we make an emphasis upon that the probability of a high outcome is dependent both on the experience of a salesperson (e.g. accumulated effort) and on the effort at the present period.

The rest of the proposal is organized as follows: In section 2, we present a model and deal with the analysis and the results for strictly positive marginal returns of accumulated efforts. Section 3 then examines extensions of the basic model for zero marginal returns of accumulated efforts and heterogeneity in a salesforce. Section 4 contains a discussion of the results and section 5 finally concludes with a summary and directions for the future research.

II. Model

1. Assumption and Notation

The sales environment consists of a sales manager and a salesforce. The sales manager is the principal and the salespersons are agents in conventional principal agent problems. The risk-neutral sales manager designs the compensation scheme, which the risk-averse agents take as given and the agents’ actions affect the outcome as well as the actual compensation received by them. Additionally, the sales manager understands the salesperson does what is best for him/herself, given the compensation plan. The sales manager’s problem is therefore to design a scheme which maximizes the firm’s profits and the salesperson’s problem is to maximize his/her utility.

In each period \( t(t = 1, ..., N) \), the salesperson can make one unit of effort \( i : i_t = 0 \) if the salesperson gives an effort, and \( i_t = 0 \) if the salesperson does not. There are two possible outcomes, \( y_1 \) and \( y_2 \) where \( y_1 < y_2 \) and the efforts are not observable to the sales manager whereas the outcomes are publicly observable. The probability distribution of the outcome depends on the effort given at the current stage as well as the accumulated efforts the salesperson has made.

We denote the probability of a high outcome \( y_2 \) by \( P_{k,i} \) when the accumulated effort level is \( k \) and the level of effort given at current stage is \( i \). The accumulated efforts level increases the probability of a high outcome at a given current effort level. The level of effort made at the current stage increases the probability of a high outcome at a given accumulated effort level, i.e. \( P_{k+1,i} \geq P_{k,i} \) and \( P_{k+1,1} \geq P_{k,0} \). We assume that there are diminishing marginal returns to accumulated effort level as commonly and well documented (e.g. [13]); i.e., \( P_{k+1,i} - P_{k,i} \leq P_{k,i} - P_{k-1,i} \), though we do not make any assumption on the shape of the probability for a given level of current effort. In addition, no specific distributional assumptions about the conditional distribution of sales for a given level of efforts are made.

The salesperson’s utility function is characterized by two separable components: the utility from the wealth which he/she receives from the sales of firm’s products and the disutility from the effort required to realize the sales. Thus, \( \Sigma k \cdot i_t = 1 \cdot \left( U(s(y_t)) - V(i_t) \right) \) where \( s(y_t) \) is the salesperson’s income for given sales, \( y_t \) in period \( t \). The salesperson’s utility for wealth \( U(\bullet) \) is a Von Neumann–Morgenstern utility function of \( S(\bullet) \) and increases at a decreasing rate with \( U'(\bullet) > 0 \) and \( U''(\bullet) < 0 \). The salesperson’s disutility \( V(\bullet) \) for effort increases with effort \( i_t \), \( h \) is defined as an inverse function of \( U(\bullet) \) and implies the sufficient amount of compensation for a given level of utility, \( h' \)
>0 and \( h^* > 0 \) due to the property of salesperson’s risk aversion.

We first consider only the case where \( P_{k-1,j} > P_{k,i} \) holds, and furthermore, for simplicity, we assume the property of homogeneity in ability of a salesforce. However, later we will discuss the case where \( P_{k-1,j} = P_{k,i} \) as a salesperson gains the sufficient experience. We will also incorporate the heterogeneity in ability of a salesforce under information symmetry and asymmetry between a salesforce and a sales manager.

2. Basic Model

2.1 One Period Case

In order to solve the optimal salesforce compensation plan, we initially consider a one-period model where the problem is identical to that in the standard agency model. Suppose that the salesperson’s experience level is \( k - 1 \) at the beginning of the period. In the one-period case, a salesforce compensation plan will take the following form; \( C = (u_1, u_2) \), where \( u_j \) is the utility payment for the outcome, \( y_j (j = 1, 2) \). In order to maximize the profit, a sales manager wants to induce an effort; a sales manager solves the following cost minimization problem subject to the two constraints.

\[
\begin{align*}
\text{subject to} & \quad \min_{u_1, u_2} (1 - P_{k-1,1}) h(u_1) + P_{k-1,1} h(u_2) \\
& \quad (1 - P_{k-1,1}) u_1 + P_{k-1,1} u_2 - V(1) \geq m \\
& \quad (1 - P_{k-1,1}) u_1 + P_{k-1,1} u_2 - V(1) \geq m \\
& \quad (1 - P_{k-1,0}) u_1 + P_{k-1,0} u_2 \\
& \quad h(u_1) + h(u_2) \geq 0
\end{align*}
\]

The first constraint is the participation constraint and implies that conditional on the current effort for given experience level \( k - 1 \), the expected utility of a salesperson is greater than reservation utility \( m \). The second constraint is the incentive constraint that implies for given experience level \( k - 1 \), the expected utility with an effort at current period is larger than the expected utility without an effort at the current period. The solution of the cost maximization problem subject to the participation constraint and incentive constraint is

\[
( u_1^*, u_2^* ) = (m + V(1) - (P_{k-1,1} - P_{k-1,0})V(1), m + V(1) + (1 - P_{k-1,1} - P_{k-1,0})V(1))
\]

where the difference between two utility payments is \( V(1) / [P_{k-1,1} - P_{k-1,0}] \). This optimal compensation plan will be valid only if the profit of a firm is higher with higher level of salesperson’s effort, and otherwise, the optimal compensation scheme will be \( (u_1^*, u_2^*) = (m, m) \) that gives same amount of utility payment as his/her reservation utility regardless of his/her outcome.

Because we are interested only in the case where a higher level of efforts of salesforce helps to realize higher profits of the firm, we assume that the following condition always holds.

\[
(1 - P_{k-1,1}) (y_1 - h(m + V(1)) + (P_{k-1,1} - P_{k-1,0})V(1)) + P_{k-1,1}(y_2 - h(m + V(1)) + (1 - P_{k-1,1} - P_{k-1,0})V(1)) > 0
\]

This assumption ensures that the optimal salesforce compensation level in one-period case will be given by (4).

2.2 Two Periods Case

Now suppose there are two periods in one accounting period and the marginal returns of accumulated efforts are positive. Therefore, the probability distribution of the sales at the second period is dependant on the salesperson’s effort at the first period. As an effort made by a salesforce increases the probability of a high outcome, the sales
manager’s optimal choice is to induce an effort in every period. For simplicity, we let \( m = 0 \) and the salesperson’s accumulated efforts, \( \kappa = 0 \) without loss of generality.

Also we let \( y_j \) and \( y_1 \) represent the outcome at the first and the second period, respectively, and we denote \( u_0 \) as the second period utility payment when the first period outcome is \( y_j \) and the second period outcome is \( y_1 \). As in one-period model, the optimal salesforce compensation will be the solution to the cost minimization problem subject to the participation and incentive constraints.

If a salesperson makes efforts in both periods as the sales manager intends, the salesperson’s experience level will be one at the end of the first period and salesperson will realize a high outcome with probability \( P_k \). His/her experience level at the end of the second period will be two and he/she then will gain a high outcome with probability \( P_{k,1} \). The sales manager now has a cost minimization problem subject to the participation and incentive constraints.

To explain how the first period constraint is characterized by above, we now adopt the following \textit{Lemma} proven by Kwon[14].

\textbf{Lemma 1.} In each period \( t \), if the incentive constraint holds for a salesperson with accumulated effort level \( k \), the incentive constraint also holds for a salesperson with accumulated effort level \( k' \), for all \( k' \geq k \).

\begin{proof}
Suppose the incentive constraint holds for a salesperson with initial accumulated effort level \( k \). That is \((1 - P_{k,1}) u_1 + P_{k,1} u_2 - V(1) \geq (1 - P_{k,0}) u_1 + P_{k,0} u_2 \) and this simplifies as \( u_2 - u_1 \geq V(1) / [P_{k,1} - P_{k,0}] \). We know, from assuming the diminishing marginal returns of the accumulated effort level, that \( P_{k,1} - P_{k,0} \geq P_{k,1} - P_{k,2} \) and therefore, \((1 - P_{k,1}) u_1 + P_{k,1} u_2 - V(1) \geq (1 - P_{k,0}) u_1 + P_{k,0} u_2 \) always holds. \( Q.E.D. \)
\end{proof}

\textbf{Lemma 1} shows that the salesperson’s optimal strategy is always making an effort regardless of the history of the effort choice if the incentive constraint is binding. In other words, in the second period case, provided that the second-period incentive constraint is satisfied, the salesperson will make an effort in the second period regardless of the level of effort at the first period.

In order to solve the cost minimization problem, we will follow the steps shown by Kwon[14]. We first ignore the first period incentive constraint (8) and show that the solution of the relaxed problem still satisfies the first-period incentive constraint (8). In other words, with constraints (7) and (9) only, the constraint (8) will be binding as in the one-period case. From the binding second-period incentive constraints.

\[ \begin{align*}
\text{Min} & \quad (1 - P_{k,0}) [h(u_0) + (1 - P_{k,1}) h(u_1) + P_{k,1} h(u_2)] + P_{k,1} [h(u_3) + (1 - P_{k,1}) h(u_4)] + \cdots \\
\text{subject to} & \quad (1 - P_{k,0}) [u_0 + (1 - P_{k,1}) u_1 + P_{k,1} u_2] + P_{k,1} [u_3 + (1 - P_{k,1}) u_4 + P_{k,1} u_5] - 2V(1) \\
& \quad \geq 0 \\
& \quad (1 - P_{k,0}) [u_0 + (1 - P_{k,1}) u_1 + P_{k,1} u_2] + P_{k,1} [u_3 + (1 - P_{k,1}) u_4 + P_{k,1} u_5] - 2V(1) > 0 \\
& \quad (1 - P_{k,0}) [u_0 + (1 - P_{k,1}) u_1 + P_{k,1} u_2] + P_{k,1} [u_3 + (1 - P_{k,1}) u_4 + P_{k,1} u_5] - V(1) \\
& \quad \geq 0 \\
& \quad (1 - P_{k,0}) u_0 + P_{k,0} u_2 \\
\end{align*} \]
constraint (9), we have \( u_{j2} - u_{j1} = V(1)/[P_{1,1} - P_{1,0}] \).

We can then write

\[
(u_{j1}, u_{j2}) = (m_{1}^{2} + \omega_{2}^{2}, m_{1}^{2} + \omega_{2}^{2}),
\]

where \( \omega_{1}^{2} - \omega_{2}^{2} = V(1)/[P_{1,1} - P_{1,0}] \) as proven at (4). By substituting these optimal payments into our objective function, we can solve the cost minimization problem subject to the participation constraint (7).

Because a salesperson is assumed to be risk-averse as \( h'' > 0 \), first order condition of the cost minimization problem provides that

\[
u_1 = u_2 \text{ and } m_1^2 = m_2^2.
\]

For simplicity, we now denote \( u \equiv u_1 = u_2 \) and \( m_2 \equiv m_1^2 = m_2^2 \). The participation constraint provides the condition, \( u = V(1) - m_2 \) and finally, \( m_2 \) is determined by the first order condition,

\[
- h(V(1) - m_2^2) + (1 - P_2) h'(m_2^2 + \omega_1^2) + P_2 h'(m_2^2 + \omega_2^2) = 0.
\]

Moreover, we have, from the second period and participation constraints,

\[
(1 - P_{1,1}) u_{j1} + P_{1,1} u_{j2} = m_2^2 + V(1) \quad (13)
\]

\[
(1 - P_{1,0}) u_{j1} + P_{1,0} u_{j2} = m_2^2, \quad (14)
\]

and these conditions verify that the first incentive constraint is binding with equality.

If a salesperson has made an effort at his/her first period, his/her expected utility in the second period will be \( (1 - P_{1,1}) u^{*1} + P_{1,1} u^{*2} - V(1) \) and otherwise, it will be \( (1 - P_{0,1}) u^{*1} + P_{0,1} u^{*2} - V(1) \). Furthermore, because the second period incentive constraint is binding with equality, the difference of these expected utilities is \( V(1) \), which is exactly the same as the disutility from making an effort. Therefore, the salesperson will make an effort at the first period without direct commissions at the first period and the commissions for the second period will be \( V(1)/[P_{1,1} - P_{1,0}] \). Furthermore, the commissions provided at the last period induce an effort at the first period as well as the second period and are much less than the sum of the direct commissions at the first and second period.

2.3 Two Periods Case

At the beginning, we have introduced a constraint that induces a salesperson to make an effort with \( k-1 \) level of accumulated efforts. From this, in order to motivate an effort at the period \( N \),

\[
(1 - P_{N-1,1}) u_{h1} + P_{N-1,1} u_{h2} - V(1) \geq (1 - P_{N-1,0}) u_{h1} + P_{N-1,0} u_{h2}
\]

must hold regardless of the history of his/her past performance, where \( h \) stands for the history of the past performance. From lemma 1, we know that the salesperson will make an effort every period regardless of his accumulated effort level, and that the effort at period \( t \) \(( t < N) \) makes one unit of accumulated effort level in the period \( N \). This fact consequently implies that the marginal benefits of an effort at period \( t \) must be larger than the marginal cost of an effort as shown in (15) and that the salesperson will make an effort at the period \( t \) without direct commissions when (15) is binding. Thus, analogous to the two period game, the commissions only for the last period are necessary to induce the efforts of the salesforce and the optimal amounts of commissions will be \( V(1)/[P_{N-1,1} - P_{N-1,0}] \).

Proposition 1. In the \( N \) period optimal contract, the payment to a salesforce is constant until the last period regardless of the volume of the sales. Commissions only at the end of the last period is provided to a salesforce and is optimal compensation plan.

The optimal compensation plan is exactly the same as the result achieved by Kwon[12]. The result implies that it is not necessary for the sales manager to offer direct commissions to encourage a salesperson to make an effort at any previous periods.
In addition, the manager does not backload the commissions on the last period but the only commissions for the last period are enough to induce an effort at the last period.

These properties provide an explanation to some empirical facts that the incentives are provided in the last period and are dependent only on the outcome of the last period. As exampled by Kwon[14], the graduate programs in many universities do not promote students to their thesis stage if they fail qualification test taken at the end of the course work.

3. Analysis

3.1 Convergence of a High Outcome Probability

From the cost minimization problem of the sales manager subject to the incentive and participation constraints, we have shown no need of explicit commissions in order to induce an effort of a salesperson at each stage. However, we note that under the property of the diminishing marginal returns of the accumulated effort level, the probability of a high outcome for a given effort level at the present period possibly converges to a certain stage. In other words, there possibly exists an experience level from which greater accumulated efforts do not raise the probability of a high outcome for a given level of present effort; the approach of accumulated effort level to such an experience level will disallow future productivity to depend on the decision of an effort at the present stage.

We denote the minimum number of efforts to achieve the sufficient experience level by \( \eta \) and write the probability of a high outcome as \( P_k,1 \) for all \( k \geq \eta \).

For example, when only a limited number of skills are required for a given job, the current effort of an agent with all required skills will not affect his/her performance in the next period. In addition when the required skills for the task remain unchanged, the necessary training period will not be infinite and moreover, especially when the given task for an agent is relatively simple, the period that an agent takes to reach the sufficient experience level will be relatively short. Therefore, from considering that the skills required for a salesperson are constant and not excessive, we infer that the present outcome of a salesforce then depends solely on the present effort of the salesperson if his/her accumulated experience level is sufficient; i.e. \( k \geq \eta \). This is from that the accumulated effort level from the previous periods and an effort level at present period determine the probability distribution for a high outcome.

We now solve the cost minimization problem subject to the participation constraint and the incentive constraint under the property of \( P_k,1 = P_k,1 \) to consider the case where \( k \geq \eta \). As we have shown earlier, suppose initially that one period is in one accounting period. The sales manager then has the same cost minimization problem subject to the same two constraints, which can be written as the following.

\[
\begin{align*}
\min & \quad u_1, u_2 \ (1 - P_{a1}) h(u_1) + P_{a1} h(u_2) \\
\text{subject to} & \\
(1 - P_{a1}) u_1 + P_{a1} u_2 - V(1) & \geq m \\
(1 - P_{a0}) u_1 + P_{a0} u_2 - V(1) & \geq 0
\end{align*}
\]

We have already shown that two constraints are binding as the objective function and two constraints are exactly identical to that of one period game. Consequently, the solution is identical to the earlier result as well:

\[
(u_1^*, u_2^*) = (m + V(1) - (P_{a1} / P_{a1} - P_{a0})V(1), m + V(1) + (1 - P_{a1} / P_{a1} - P_{a0})V(1)).
\]

Now we consider the two period case, where two
periods are in one accounting period. The objective function and the incentive and participation constraints are also the same and can be written as the following.

$$\min (1 - P_{a_1}) [ h(u_1) + (1 - P_{a_1}) h(u_2) + P_{a_1} h(u_3) ] + P_{a_1} [ h(u_4) + (1 - P_{a_1}) h(u_5) + (1 - P_{a_1}) h(u_6)] \quad (20)$$

subject to

$$(1 - P_{a_1}) [ u_1 + (1 - P_{a_1}) u_{11} + P_{a_1} u_{12} ] + P_{a_1} (u_2 + (1 - P_{a_1}) u_{21} + P_{a_1} u_{22}) - 2V(1) \quad (21)$$

$$\geq 0$$

$$(1 - P_{a_0}) [ u_1 + (1 - P_{a_0}) u_{11} + P_{a_0} u_{12} ] + P_{a_0} (u_2 + (1 - P_{a_0}) u_{21} + P_{a_0} u_{22}) - 2V(1)$$

$$> (1 - P_{a_1}) u_1 + (1 - P_{a_1}) u_{11} + P_{a_1} u_{12} \quad (22)$$

$$u_{12} + P_{a_0} u_2 + (1 - P_{a_0}) u_{21} + P_{a_0} u_{22} - V(1)$$

$$(1 - P_{a_0}) u_{11} + P_{a_0} u_{12} - V(1) \geq 0 \quad (23)$$

When $k < n$, in order to obtain the solution for the cost minimization problem of sales manager subject to three constraints, we have shown that the first period incentive is binding at the last step while initially ignoring it. However, we now simplify the first period incentive constraint at our first step and we achieve

$$u_2 - u_1 > V(1) / [P_{a_1} - P_{a_0}] \quad (24)$$

(24) implies that the compensation for a high outcome at the first period is larger than the one for a low outcome at the first period at least by $V(1) / [P_{a_1} - P_{a_0}]$. This result is conflicting with the earlier result as we expected. Furthermore, the solution for optimal commissions for the second period is

$$u_{12} - u_1 > V(1) / [P_{a_1} - P_{a_0}] \quad (25)$$

that is the same as the amounts of first period optimal commissions.

**Proposition 2.** When a salesperson achieves the sufficient experience and therefore, the decision of the current effort does not affect the future productivity of a salesperson, a commission is required to motivate a salesperson to give an effort at each period.

Because when $P_{a_1} > P_{a_0}$, the future productivity depends on the decision of the current effort, the difference between two probabilities induces an effort of the salesperson without any direct compensation at the first period. However, current effort does not affect future outcome under $P_{a_1} = P_{a_0}$, and commissions at the end of each period is consequently necessary. Therefore the convergence of the probability of a high outcome for a given present effort results in the dynamics in the salesforce compensation structure because commissions only at the end of the last period do not encourage a salesperson to make an effort at any previous periods as the salesperson achieves sufficient experience.

### 3.2 Heterogeneity in a Saleforce

As discussed by Lal and Staelin[2] and Rao[3], salespeople are possibly heterogeneous in their sales abilities and learning speeds. We initially suppose that salespeople are heterogeneous only in their learning speed but later in the next section, we will discuss heterogeneity in salespersons’ sales abilities. We now assume that a salesforce consists of two types of salespersons: fast-learning salespersons and slow-learning salespersons; we denote fast-learning salespersons as $f$ and slow-learning salespersons as $l$. In addition, the sales manager does not identify the salesperson’s type, but the salesperson observes his/her own type. Therefore, a type $f$ salesperson takes a relatively short period to achieve sufficient skills/experience and a type $l$ salesperson takes comparatively long term to gain the same skills/experience. Thus we write $\eta_f < \eta_l$ where $\eta_f$ and $\eta_l$
represents the minimum number of efforts required for a fast-learning salesperson to achieve sufficient skills/experience, and $\eta_l$ characterizes the minimum number of efforts necessary for a slow-learning salesperson to be proficient. Finally, we assume that the sales manager holds a fair expectation on $\eta_l$, and $\eta_l$ from his/her experience.

In order to minimize the cost, the sales manager changes the salesforce compensation scheme when the experience of salesforce exceeds the minimum sufficient level when a salesforce is assumed homogeneous. However, when the property of homogeneity in a salesforce is released, the timing is rather vague when to convert the salary-only-salesforce compensation scheme to a common one combined with salary and commissions because different types of salespersons achieve sufficient experience at different periods. When a sales manager can tell the types of salespeople, he/she can alter the compensation plan at the different stages according to the types of salespersons, but we assume a sales manager can not. Therefore, a sales manager must provide a single plan that maximizes firm’s profits under information asymmetry.

If all new salespeople are assumed initially to contain zero experience/skills, we know, from section 2, that every salesperson will make an effort without any direct commissions at the period $t \in [0, \eta_l)$ and consequently, an optimal plan must be identical to the case with homogeneity in a salesforce. In addition, at the period $t^* \in [\eta_l, \infty)$, as every salesperson has achieved sufficient skills/experience, the direct commissions are required to stimulate each salesperson. However, at period $t^* \in [\eta_l, \eta_l)$, to induce an effort of type $h$ salesperson, direct commissions are necessary at each period while type $l$ salesperson makes an effort regardless of the existence of direct commissions. Therefore, as Lal and Staelin[2] and Rao[3] show, a sales manager provides a set of two contracts in such a way that each type of salesperson maximizes his/her utility under each contract while minimizing the cost to the firm. We now denote the compensation plan for a fast-learning salesperson by $C_l = (u_l^1, u_l^2)$ and the one for a slow-learning salesperson by $C_l = (u_l^0, u_l^1)$. The sales manager then has two cost minimization functions subject to two constraints as follows.

$$\min (1 - P_{k,1}^l) h(u_l^1) + P_{k,1}^l h(u_l^2)$$ (26)

subject to

$$\left(1 - P_{k,1}^l\right) u_l^1 + P_{k,1}^l u_l^2 - V(1) \geq m$$ (27)
$$\left(1 - P_{k,1}^l\right) u_l^1 + P_{k,1}^l u_l^2 - V(1) \geq$$ (28)

$$\min (1 - P_{a,1}^h) h(u_h^1) + P_{a,1}^h h(u_h^2)$$ (29)

subject to

$$\left(1 - P_{a,1}^h\right) u_h^1 + P_{a,1}^h u_h^2 - V(1) \geq m$$ (30)
$$\left(1 - P_{a,1}^h\right) u_h^1 + P_{a,1}^h u_h^2 - V(1) \geq (1 -$$ (31)

The objective functions (26) and (29) are cost minimization problems of a sales manager for a slow-learning salesperson and for a fast-learning salesperson, respectively and two constraints for each objective function are the participation and incentive constraints. We have seen exactly the same problems and constraints when we solve the cost minimization function for one period case, and we know the optimal compensation plan for each salesperson is

$$\left( u_l^1, u_l^2, u_h^1, u_h^2 \right) =$$

$$\left( m + V(1) - \left( P_{k,1}^l / P_{k,1}^l - P_{k,0}^l \right) V(1), \right.$$ (32)
$$\left. m + V(1) + (1 - P_{k,1}^l / P_{k,1}^l - P_{a,0}^l) V(1), \right.$$ (32)
$$\left. m + V(1) - \left( P_{a,1}^1 / P_{a,1}^1 - P_{a,0}^1 \right) V(1), \right.$$ (32)
$$\left. m + V(1) + (1 - P_{a,1}^1 / P_{a,1}^1 - P_{a,0}^1) V(1). \right)$$

Furthermore, the above optimal solution satisfies the two conditions below,
(1 - \( P_{k,1}^l \)) u_1^l + P_{k,1}^l u_2^l \leq \quad (33)
(1 - \( P_{k,1}^h \)) u_1^h + P_{k,1}^h u_2^h
(1 - \( P_{k,1}^l \)) u_1^h + P_{k,1}^l u_2^h \leq \quad (34)

in order that each type of salesperson will be better off with the salesforce compensation plan designed for him/herself given that a salesperson makes an effort.

This shows that a manager should employ a menu of compensation plans that is implemented by announcing a payment for a low outcome, and a commission rate for compensating a high outcome for each type of a salesperson. The announcement is made prior to the period \( \eta_f \), and the salesperson is asked to choose a type from the menu to maximize his/her utility at the same time. Thereafter, both types of salespersons will make efforts without any direct commissions until the period \( \eta_l \), and a sales manager only provides a commission at the period \( \eta_f \) to induce an effort for every salesperson. The salesforce compensation scheme then turns to a set of two contracts during the period \( t' \in [\eta_f, \eta_l] \), and afterwards, a single compensation plan will be provided for both types of salespersons.

On the other hand, we note that the payment to the salesforce is a function of probability where the probability continues to vary every period. This variation in probability then result in the changes in the commissions to a high outcome at every period during \( t' \in [\eta_f, \eta_l] \), but we hardly see this type of the compensation plan in practice. The discrepancy between our model and the practice can be explained as follows: in our model, we do not consider the management cost from changing the payment plan too frequently due to the management cost.

4. Discussion

In our paper, the optimal salesforce compensation plan is proven to vary with the accumulated efforts of the salesperson as we see in practice, and under the heterogeneity in the learning speed, a sales manager’s optimal compensation plan includes a menu of contracts that are designed for different types of salespersons.

Moreover, our model is flexible and robust to an extension and by adding minor modifications, we can address few other concerns that have not been incorporated. For example, our model can address the heterogeneity in the sales ability of a salesforce. To introduce the heterogeneity in the sales ability of a salesforce, first suppose the salespersons differ in their returns to the effort. If a salesperson can be categorized as a high ability salesperson or a low ability salesperson, a set of twice as many contracts is provided except during the period \( t \in [0, \eta_f] \). During the period \( t \in [0, \eta_f] \), without direct commissions, a salesperson will make an effort at every period in order to achieve the commissions at the period \( \eta_f \) as shown earlier and each salesperson selects a plan for the period \( t' \in [\eta_f, \eta_l] \) prior to the period \( \eta_f \). Meanwhile, a sales manager offers four distinctive compensation plans. The salesforce compensation plan is then converted to a set of two contracts after the period \( \eta_l \) as the both types of salespeople gain sufficient level of experience.

Furthermore, our model is robust with the length of the learning period. First, assume that the salesforce is homogeneous for simplicity and the learning period
is longer than one accounting period but shorter than two accounting periods. Then as we have shown in section 2, a salesforce will make an effort without any direct commissions during the first accounting period if a sales manager promises a commission for a high outcome at the end of one accounting period. Moreover, if a commission is promised at the end of the learning period, a salesperson will give an effort because the current effort determines the future productivity. The salesforce compensation plan will then be transformed to the salary-commissions combined compensation plan as we have shown earlier in this section.

In summary, given that the analysis shown by Kwon[12] is useful in incorporating the effect of accumulated efforts but cannot handle the dynamics in the salesforce compensation structure, the framework used in our model allows a more comprehensive treatment of the salesforce compensation plan. The result that the dynamics in salesforce compensation plan originates from the convergence of marginal returns of accumulated efforts is the key contribution.

5. Conclusion

In this paper, we analyze the problem of designing salesforce compensation plans to capture the dynamics in compensation structure. When a new salesforce starts the career without insufficient experience, a sales manager provides a commission only at the end of salesperson’s learning period. The amount of the commission is proven far less than the sum of direct commissions but enough to induce an effort at every stage during the learning period. After a salesperson gains the sufficient experience, a sales manager is then required to offer a commission at the end of every period because the current effort decision does not affect salesperson’s future productivity. We have also shown the optimal compensation plan of a sales manager under the heterogeneity in salesperson’s learning speed and sales ability. The results of our model give an explanation about the dynamics in the salesforce compensation plan that has not been considered at the past marketing literature. Our theoretical model fills the gap between the practice and the marketing theoretical model while characterizing the optimal salesforce compensation plan.

However, although our model first explains how the salesforce compensation scheme for a single salesperson varies, our model has few limitations. First, in contrast to the past marketing literature on the salesforce compensation, the outcome of a salesperson and the level of effort made by a salesperson take Bernoulli distribution. This simplification is not avoidable in order to achieve mathematical tractability of accumulated efforts, but on the other hand, the implementation of our model therefore becomes limited. In addition, our model does not consider the effect of the sales uncertainty, the marginal cost of production, the changes in the reservation utility, or the effectiveness of a sales effort. As shown by Basu, Lal, Srinivasan and Staelin[1] and Lal and Staelin[2], above variables often have the substantial effect on the design of salesforce compensation and therefore we encourage the further research on that subject.


