Nonlinear Tidal Characteristics along the Uldolmok Waterway off the Southwestern Tip of the Korean Peninsula

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Abstract: Analyses of tidal observations and a numerical model of the \( M_2 \) and \( M_4 \) tides in the Uldolmok waterway located at the southwestern tip of the Korean Peninsula are described. This waterway is well known for its strong tidal flows of up to more than 10 knots at the narrowest part of the channel. Harmonic analysis of the observed water level at five tidal stations reveals dramatic changes in the amplitude and phase of the shallow water constituents at the station nearest the narrowest part, while survey results show a decreasing trend in local mean sea levels toward the narrow section. It was also observed that the amplitudes of semi-diurnal constituents, \( M_2 \) and \( S_2 \), are diminishing toward the narrowest part of the waterway. Two-dimensional numerical modeling shows that the \( M_2 \) energy flux is dominated by the component coming from the eastern boundary. The \( M_2 \) energy is inward from both open boundaries and is transported toward the narrow region of the channel, where it is frictionally dissipated or transferred to other constituents due to a strong non-linear advection effect. It is also shown that the \( M_4 \) generation is strong around the narrow region, and the abrupt decrease in the \( M_4 \) amplitude in the region is due to a cancellation of the locally generated \( M_4 \) with the component propagated from open boundaries. The superposition of both propagated and generated \( M_4 \) contributions also explains the discontinuity of the \( M_4 \) phase lag in the region. The tide-induced residual sea level change and the regeneration effect of the \( M_2 \) tide through interaction with \( M_4 \) are also examined.

Key words: nonlinear tidal phenomena, \( M_4 \) generation, Uldolmok waterway, numerical modeling.

1. Introduction

The Uldolmok waterway is a channel connecting the entrance of the South Sea and the Yellow Sea (Fig. 1). Its width is in the order of 1-2 km, its length is about 15 km, and the water depth is about 15 m. As shown in Fig. 1 and 4, the waterway has a narrow section of about 250 m width. This site is well known for its strong tidal flow and a pre-feasibility study for power generation utilizing these tidal currents was made in 1985-1986 by the Korea Ocean Research and Development Institute (KORDI 1986) and again through recent observation in 2002 (MOMAF 2002) where direct current measurement using ADCP was carried out.

Lagrangian current measurements were carried out in

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1985 under the bridge connecting the narrowest section of the channel. The current speed, measured with lagrangian method, in the narrowest section during spring and mean tides was estimated. The maximum spring current speed is about 6 m/s at flood, and 5.7 m/s at ebb, respectively, with 4.2 m/s speed during mean ebb tide. This surface magnitude was approximately verified in recent Eulerian observation using ADCP (MOMAF 2002), from which the surface \( M_2 \) and \( S_2 \) along-channel component are analyzed to be 3.70 m/s and 1.24 m/s, respectively.

About a month-long tide level recording was carried out at 4 stations in 1986 along the channel using two types of self-recorders. For the 3 stations 86T1, 86T3 and 86T4, Aanderaa WLR-5 instruments were used. Stations except for 86T4 (located beyond the study domain) are shown in the subset of Fig. 1. At 86T2, observations were recorded using a EPT-5 gauge on an analogue chart as well as in
digital form. During the period of the tide level measurements, tidal staff readings were made for 5 to 7 hours at all three sites during daylight in order to establish the relative tide levels from the four tidal bench marks (TBMs) buried near the tide stations. In order to find the absolute elevations of those TBMs, the BM 14-6-12-1 of the National Geography Institute (NGI) was connected to TBMs No. 1, 2 and 3. The water level recordings were compared with the staff readings and then connected to these land levelling nets. The theoretical sea levels, derived from the harmonic constants as well as mean sea levels, are shown in Fig. 2. The lowest mean sea level is 207 cm at 86T2, while it is 217 cm at 86T1 and 214 cm at 86T3, and gets higher outward from 86T2 station. The sea level set-down is remarkable at 86T2 near the narrowest part of the channel where the strong current occurs. The results of harmonic analysis for the tidal records in the channel are also described in Table 1. The low value of the \( M_2 \) tidal amplitude, as well as the extraordinarily small value of the higher harmonics (\( M_4 \) and \( MS_4 \)) at 86T2, are remarkable. The phase lag of the \( M_4 \) constituents at 86T2 shows an abrupt change that cannot be explained by simple wave propagation. As a result of a series of observations related to tides, tidal currents and tidal levelling, it was indicated that there exists some nonlinear phenomena related to higher and lower harmonics generation in association with strong current and large shear of the currents along the channel. However, no thorough analysis was made due to the scarcity of the available data.

Table 1. Tidal harmonic constants for semi, diurnal, quarter-diurnal and fortnight constituents (\( M_2, K_1, M_4, MS_4 \)) from the analysis of observed data in 1986, along the Uldolmok waterway, Korea. Phase lags in degrees are referred to the local transit time of each tide-generating body. Am and Ph denote amplitude and phase lag, respectively. For location of stations, see Fig. 1.

<table>
<thead>
<tr>
<th>Station</th>
<th>( M_2 ) am cm</th>
<th>( M_2 ) ph deg</th>
<th>( K_1 ) am cm</th>
<th>( K_1 ) ph deg</th>
<th>( M_4 ) am cm</th>
<th>( M_4 ) ph deg</th>
<th>( MS_4 ) am cm</th>
<th>( MS_4 ) ph deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>86T1</td>
<td>115.3</td>
<td>24.6</td>
<td>28.9</td>
<td>244.8</td>
<td>11.5</td>
<td>195.2</td>
<td>7.3</td>
<td>237.8</td>
</tr>
<tr>
<td>86T2</td>
<td>111.9</td>
<td>17.9</td>
<td>28.9</td>
<td>243.7</td>
<td>2.2</td>
<td>228.7</td>
<td>2.8</td>
<td>284.5</td>
</tr>
<tr>
<td>86T3</td>
<td>114.4</td>
<td>331.2</td>
<td>28.6</td>
<td>219.4</td>
<td>8.1</td>
<td>146.2</td>
<td>5.1</td>
<td>187.6</td>
</tr>
</tbody>
</table>
Therefore, the need for additional measurements and research work were raised to confirm if the observed phenomena is related to actual tidal phenomena.

In this manuscript, the tidal phenomena in the Uldolmok waterway under the influence of a strong current are investigated in terms of additional tidal measurements and two-dimensional tidal modelling. The non-linearity of tidal phenomena in the channel is investigated based upon the analysis of simultaneously measured data at five tidal stations along the channel. Furthermore, the characteristics of the $M_2$ tide, its energy flux and the non-linear generation of $M_2$ at the narrowest section are examined using the numerical tidal model. The contributions of both the channel boundary forcing and the local generation effect on the observed features of $M_2$ are investigated in relation to the abrupt change in the $M_2$ phase lag in the narrow section. The residual surface or mean sea level in the channel is also investigated using the numerical model and compared with observations. The regeneration of the $M_2$ by the interaction of $M_2$ and $M_4$ is also examined.

2. Measurements and data analyses

To further investigate the characteristics of the shallow water constituents in the Uldolmok waterway, tide level recordings were carried out at 5 stations along the channel during the period of January 15 to February 9-14, 1992. Simultaneous measurements were carried out in order to eliminate the introduction of monthly variability in the harmonic constants. This was found to be necessary since monthly variability in harmonic constants actually exists, as shown by Foreman et al. (1995) and Kang et al. (1995). Automatic water level recorders with the same kind of paroscientific sensors were deployed at 5 stations (T1-T5). The quality of observed data was carefully checked by plotting and visual inspection, and in general the data quality was quite satisfactory. Water level data were sampled every 10-minutes and the non-tidal high frequency components were filtered out to yield hourly data. The time series of about one month were then analysed using a standard least square technique (Murray 1963; Foreman 1977). For a month-long data analysis 8 related components as well as 27 major components were included, with Mokpo being a tidal station of permanent reference.

One point to note here is that even though we allow for the possibility of over (or under) corrections by nodal correction, as noted by Godin and Gutierrez (1986), we analyzed the monthly data with nodal correction, in order to obtain harmonic constants reflecting these nodal effects. This was done because nodal corrections do not influence the spatial trend of variability among monthly harmonic constants from several locations provided that the effect is treated uniformly for monthly analyses. A detailed description of harmonic analyses of monthly tidal data was made in Kang et al. (1995).

Characteristics of principal tidal components

The results of the analyses at 5 stations for the major components ($M_2$, $S_2$, $N_2$, $K_1$, $O_1$) are listed in Table 2.

When comparing $M_2$ harmonic constants in 1992 with those in 1986, it is seen that the 1992 amplitude (phase lag) values for $T1$, $T2$ and $T3$, that is, 117.1 cm (22.1°), 109.1 cm (15.4°), and 114.1 cm (332.6°) are more or less similar to 1986 values at 86T1, 86T2 and 86T3, of 115.3 cm (24.6°), 111.9 cm (17.9°) and 114.4 cm (331.2°). Notice that the pattern of decreased amplitude at $T3$ (86T2) is equal in both observations. In addition, the $M_2$ amplitude (phase lag) in 1992, 11.1 cm (198.1°), 2.9 cm (205.3°) and 7.9 cm (154.1°) also shows a more or less similar pattern to those of 11.5 cm (195.2°), 2.2 cm (228.7°) and 8.1 cm (146.2°) in 1986. Even though there is some difference in the absolute values, an abrupt change in the $M_2$ amplitude and phase lag is still found in both observations. The difference in harmonic values between 1992 and 1986

<table>
<thead>
<tr>
<th>Station</th>
<th>$M_2$</th>
<th>$S_2$</th>
<th>$N_2$</th>
<th>$K_1$</th>
<th>$O_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>am cm</td>
<td>ph deg</td>
<td>am cm</td>
<td>ph deg</td>
<td>am cm</td>
</tr>
<tr>
<td>T1</td>
<td>117.1</td>
<td>22.1</td>
<td>39.9</td>
<td>66.0</td>
<td>23.1</td>
</tr>
<tr>
<td>T2</td>
<td>115.5</td>
<td>22.5</td>
<td>39.3</td>
<td>66.2</td>
<td>22.7</td>
</tr>
<tr>
<td>T3</td>
<td>109.1</td>
<td>15.4</td>
<td>37.6</td>
<td>58.0</td>
<td>21.8</td>
</tr>
<tr>
<td>T4</td>
<td>113.4</td>
<td>337.8</td>
<td>42.6</td>
<td>10.6</td>
<td>22.8</td>
</tr>
<tr>
<td>T5</td>
<td>114.1</td>
<td>332.6</td>
<td>43.7</td>
<td>5.8</td>
<td>23.1</td>
</tr>
</tbody>
</table>

Table 2. Tidal harmonic constants for major components of data measured in 1992 along the Uldolmok waterway. The locations are shown in Fig. 1.
may be related to the monthly variation of harmonic constants in the surrounding seas of Korean Peninsula as investigated by Kang et al. (1995) and Kang et al. (2002), in which the seasonal variability was shown in detail to be driven by seasonal stratification of the Yellow and East China Seas. This change in monthly harmonic values suggests that the contemporary data are necessary for investigating the spatial variation in the study area, as done in the present case. From both observations, such a consistency in the trend of spatial distribution of harmonic values, is also found in other constituents and this permits us to further investigate the tidal phenomena in the channel.

Based upon the results listed in Table 2, we proceed to examine the tidal characteristics of major constituents in some detail. Considering the phase lags of the \( M_s \), it is clear that the tidal waves propagate from T5 to T1. Notice that the amplitudes of the semi-diurnal components are considerably smaller at station T3, located at the narrowest section of the waterway. In the case of the diurnal constituents, the decreases in amplitude of T3 are quite small, and the amplitude of \( K_1 \) observed in 1986 does not show the same pattern of amplitude decrease. It is quite interesting to see that the diurnal constituents do not show the same clear pattern of amplitude decrease as the semi-diurnal constituents, even though \( N_s \) with smaller or nearly equal amplitude to the diurnal constituents, shows such a pattern.

Considering the phase lags of the constituents it is clear that the tidal waves propagate from T5 to T1. It is also shown that a large difference in phase lag occurs between T4 and T2. The differences in phase lags between T2 and T4 for \( M_2, S_2, N_2, K_1 \) and \( O_1 \) are 44.7°, 55.6°, 42.4°, 23.3° and 22.3°, respectively. It is noticeable that, in spite of the short distance between the stations (about 9 km), a big difference in phase lag occurs over the narrow part of the waterway. It seems that the tidal waves propagate faster than the usual speed of long wave propagation. It is also worth noting that the phase lag differences of semi-diurnal constituents are about two times as large as those of diurnal constituents. With relatively large amplitude of semi-diurnal constituents, this large phase lag will result in the stronger current speeds of semi-diurnal constituents over those of the diurnal constituents. Probably in the case of semi-diurnal constituents, it is likely that more potential energy is converted into kinetic energy through the narrow section, therefore inducing the smaller amplitude of semi-diurnal constituents. This will be further considered with numerical models.

**Characteristics of shallow water tides**

Shallow water tides are found in basins or rivers of shallow depth and they are generated as nonlinear interactions to external tidal oscillations (Godin 1972). The results of analyses at 5 stations for the shallow water tides are listed in Table 3.

In the case of overtide \( (M_s) \), the magnitude of amplitude along the waterway is in the order of 10 cm except for T3 located near the narrowest section of the channel, and it is quite interesting that the variation in amplitude along the waterway is not smooth. Furthermore, the amplitude distributions of compound tides \( MS_f \), \( MN \) also show the same trend as that of \( M_s \). In the case of lower harmonics \( MS_f \) and \( MM \), the amplitudes at T3 are larger than those of other stations in contrast to higher harmonics. Such a significant amplitude variation of the shallow water constituents may be related to a non-linear phenomenon around the narrow part of the waterway.

The remarkable characteristic of quarter-diurnal tides or higher harmonics \( (M_n, MS_f, \text{ and } MN) \) takes place in the phase lag distribution of the above components. The phase lag patterns of quarter-diurnal components show that the quarter-diurnal tidal waves propagate from a T5 direction, but discontinuous behavior or abrupt jumps in phase lags is found at T3, again strongly suggesting that some nonlinear process takes place around the narrowest

<table>
<thead>
<tr>
<th>Station</th>
<th>( M_s )</th>
<th>( MS_f )</th>
<th>( MN )</th>
<th>( MS_f )</th>
<th>( MM )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>am (cm)</td>
<td>ph (deg)</td>
<td>am (cm)</td>
<td>ph (deg)</td>
<td>am (cm)</td>
</tr>
<tr>
<td>T1</td>
<td>11.1</td>
<td>198.1</td>
<td>9.8</td>
<td>262.4</td>
<td>4.6</td>
</tr>
<tr>
<td>T2</td>
<td>9.6</td>
<td>186.4</td>
<td>8.5</td>
<td>262.2</td>
<td>4.0</td>
</tr>
<tr>
<td>T3</td>
<td>2.9</td>
<td>205.3</td>
<td>3.7</td>
<td>294.9</td>
<td>0.6</td>
</tr>
<tr>
<td>T4</td>
<td>8.8</td>
<td>159.7</td>
<td>7.3</td>
<td>238.1</td>
<td>3.6</td>
</tr>
<tr>
<td>T5</td>
<td>7.9</td>
<td>154.0</td>
<td>6.9</td>
<td>224.8</td>
<td>3.3</td>
</tr>
</tbody>
</table>
part of the waterway. One point to keep in mind is that an external $M_4$ tidal wave propagating from T3 to T1 may exist, since its existence is expected to be possible by the propagation of a corresponding tidal wave around the southwestern end of the Korean Peninsula. The results of the two dimensional model for the $M_4$ tide in the Yellow and East China Seas (Kang et al. 1998; Lie 1996) support the interpretation that the external $M_4$ tide with 10 cm amplitude exists around the southwestern tip of the Korean Peninsula, as shown in Fig. 6a of Kang et al. (1998). In contrast to this external tidal wave, the locally-generated tidal wave is defined as one generated through a non-linear effect in the narrow part of the waterway. The phase lag and amplitude patterns of the quarter-diurnal components also suggest that the nonlinear process seems to be confined to the narrowest part, or the generated tidal wave appears to be strongly dissipated after generation. Therefore, the discontinuous behavior in amplitude and phase lag is probably not seen at T4 and T2.

In the case of lower harmonics, it is shown that the amplitude distributions of $MSf$ and $MM$ differ from those of $M_4$, $MS$, and $MN$. Relative high amplitudes appear at station T3 near the narrowest section of the waterway. This again implies that the external tidal waves from both boundary sides (T1, T5) are relatively weak and probably there is a source of energy for lower harmonics around the narrowest section of the waterway. Considering the amplitudes at T2 and T4 are larger than those at T1 and T5, the influence of generated lower harmonics seems to reach a little farther than those of higher harmonics.

For the sake of interpreting these findings, a hypothesis is proposed, in which the observed tidal phenomena in higher and lower harmonics result from the superposition of the co-oscillating (or external tide) from both the open boundaries of the channel and generated (internal) tides in the channel. Model experiments with such a hypothesis were carried out for the examination of the $M_4$ tide in the English Channel by Pingree and Maddock (1978). Inclusion of the generated wave to the hypothesis looks plausible, since the nonlinearity around the narrow channel will be large due to the existence of powerful and fast currents and a large current gradient. Numerical experiments have been carried out in order to qualitatively investigate the expected nonlinear phenomena in some detail, which will be treated in the following section. Also in the discussion section a simple analysis is made of the advection term in the equation for motion in order to examine the non-linear effect in order to understand the basic nonlinear mechanism in the channel.

3. A Two-dimensional tidal modelling

In order to investigate the possible nonlinear tidal phenomenon, two dimensional tidal modeling has been carried out. The governing equations are the depth-averaged, two dimensional equations of shallow water waves, which can be alternatively represented in terms of mass flux as follows:

$$\frac{\partial \eta}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \quad (1)$$

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial x} \left( \frac{U^2}{h} \right) + \frac{\partial}{\partial y} \left( \frac{U V}{h} \right) = -g \frac{\partial \eta}{\partial x} + f V - \frac{K_s}{h} \sqrt{U^2 + V^2} + F_x \quad (2)$$

$$\frac{\partial V}{\partial t} + \frac{\partial}{\partial x} \left( \frac{U V}{h} \right) + \frac{\partial}{\partial y} \left( \frac{V^2}{h} \right) = -g \frac{\partial \eta}{\partial y} - f U - \frac{K_s}{h} \sqrt{U^2 + V^2} + F_y \quad (3)$$

where $(x,y)$ is (east-west, north-south) direction coordinates, $h$ total water depth, $\eta$ surface elevation, $U$ volume transport in $x$, $V$ volume transport in $y$, $f$ Coriolis parameter, $K_s = g/ C^2$, $g$ gravity acceleration, and $C$=Chezy coefficient. The full set of non-linear equations is solved with an ADI finite difference scheme (Kang 1991; Kang et al. 1998), which displays a second order accuracy in space and time. The characteristics of the numerical scheme are the same as the S21 scheme and described in detail by Abbott et al. (1981).

The momentum diffusion term was considered in terms of a subgrid-scale modelling technique, since momentum diffusion induced from the subgrid term is often larger than the dispersion effect (Elder, 1959) obtained from depth averaging of three dimensional motion. All of the motions induced by small-scale processes not directly resolved by a model grid (subgrid-scale) are parameterized in terms of horizontal diffusion processes. The terms $F_x$ and $F_y$ represent these unresolved processes and can be written as

$$F_x = \frac{\partial}{\partial x} \left( 2A_M \frac{\partial U}{\partial x} \right) + \frac{\partial}{\partial y} \left[ A_M \left( \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right) \right] \quad (4a)$$

$$F_y = \frac{\partial}{\partial y} \left( 2A_M \frac{\partial V}{\partial y} \right) + \frac{\partial}{\partial x} \left[ A_M \left( \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right) \right] \quad (4b)$$

where, with constant $C_i$ and velocity component $(u, v)$.
\[ A_M = C_s \Delta x \Delta y \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 / 2 + \left( \frac{\partial v}{\partial y} \right)^2 \right]^{1/2} \]  
(4c)

For the coefficient of subgrid scale stress we use the Smagorinsky (1963) diffusion idea. The effect of subgrid-scale stress (SGS) modelling is described in detail by Abbott and Basco (1989). In particular, the effect of SGS modelling in the present study was found to be effective in preventing the numerical instability from occurring during calculation, as the momentum diffusion term does.

The bottom friction term has been expressed as a quadratic friction form following the general applicability of the quadratic friction law for semi-diurnal tide modelling (Pingree and Griffiths 1987). The Chezy coefficient was chosen for several values such as

\[ C = 59.191 \text{ and } 44.294 \text{ m}^2/\text{s} \]  
(5a)

These values lead to the friction coefficients, respectively, \( K_s = 0.0028 \) and \( 0.0050 \)  
(5b)

A relatively large coefficient was chosen to reflect the strong dissipation in the narrow channel and the damping effect of other constituents to the \( M_2 \) tide. Due to the relatively small range of water depth, a constant bottom friction coefficient was used instead of a depth-dependent form as used in Kang et al. (1998). The results with different frictions are more or less similar to each other, giving the same trend in results and major results presented for \( K_s = 0.005 \). With an artificially high bottom friction nearly locked is the flow for the tidal flat area, where datum depth is less than 0 m.

**Computations**

The present model domain consists of a 262(13.1

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**Fig. 3.** Grid system for the modelled area. Tidal stations (T1-T5) for open boundary and verification along with central point M are illustrated in the figure. S1-SS are the section for calculating \( M_2 \) energy flux. C is the point where Eulerian measurement is carried out in 2002.

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**Fig. 4.** Depth contour of the model area (unit: m).
km) \times 103(5.2 \text{ km}) \text{ grid with T1 and T5 being two open boundaries. The grid interval is 50 meters. The narrowest section of the channel consists of 5 grid cells, as shown in Fig. 3. Model depths are generally less than 30 m or so, along with some tidal flat regions, as seen in Fig. 4. The time step used is 5.45825 seconds, and the Courant number \( C_r \), defined as

\[
C_r = \frac{\Delta t \sqrt{g(h + \eta)}}{\Delta x}, \tag{6}
\]

reaches to about 1.9 for the deep region of \( h + \eta = 32.1 \text{ m} \). This small Courant number was chosen to maintain accuracy in the rather complicated geography. A Coriolis parameter was calculated for the latitude of \( \phi = 34^\circ 34' \text{N} \), based upon the f-plane assumption.

The initial and boundary conditions required for the solution of the equations (1) to (3) are described as follows. At \( t = 0 \), \( U(x, y, t) = V(x, y, t) = 0 \) are specified at all points. Boundary conditions for the tidal model consist of an absence of flow to the coast and specified elevations along the open boundaries. The open boundaries consist of two sections encompassing T1 and T5. Along the open boundaries \( M_2 \) and \( M_4 \) constituents are considered in the some combination, with constant values along the open boundary point listed using the observed values listed in Tables 2 and 3. The open boundary elevations are prescribed for the two constituents according to the expressions:

\[
\eta(t) = \sum f_j A_j \cos(\omega_j t - G_j + V_{0j} + U_j) \tag{7}
\]

where \( f_j \) and \( U_j \) are the astronomical amplitude and phase corrections and \( V_{0j} \) is the astronomical argument related to Greenwich. In this study \( f_j \) was set at 1.0 and \( V_{0j} + U_j \) to zero, as these are reasonable approximations for \( M_2 \) and \( M_4 \).

Even though shallow water constituents are not included in the area of boundary forcing, the nonlinear terms in the governing equations generate the higher harmonics such as the \( M_4 \) constituent as well as the residual surface or tide-induced sea level disturbance. This result without \( M_4 \) boundary forcing could be compared with the results with \( M_4 \) boundary forcing.

In this study, the Smagorinsky eddy viscosity model was extended to the two-dimensional free surface flow model, and the empirical constant \( C_s \) of equation (4c),

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**Fig. 5.** (a) Computed flood current during mean tide (2.5 hr after low water), (b) Computed ebb current during mean tide (2.5 hr after high water).
dependent only on Kolmogorov’s universal constant, will differ from the values of $C_s = 0.2$ in the three dimensional studies, established by Deardorff (1970) and others. The constant for the Smagorinsky formula is chosen as $C_s = 0.38$ in the present study, which is a little larger than the value used by Lilly (1967) and Deardorff (1971) for the three dimensional study.

After an initial spin-up, one $M_2$ cycle run was carried out. The computed time series data are subject to the data analysis to calculate amplitudes and phases of the $M_2$ and $M_4$ constituents along with residual surface.

**Current features**

The computed flood and ebb current fields are shown in Fig. 5a and 5b, respectively. During a flood tide, the tidal current flows from the right to the left boundary, and vice versa during an ebb tide. It is shown that the tidal current moves like a jet stream along the channel. During a flood, the two eddies are seen to be formed in the left domain of the narrowest section, and the large eddy tends to move downstream as time passes. The axis pattern of the flow jet in Fig. 5a seems to imply that the major axis is controlled by the strong large eddy. During ebb flows, the eddies are seen to form behind the small islands in the right section of the channel. Comparison of the eddies in Fig. 5b and the eddies computed one hour after (not shown here) shows that the time varying pattern of eddies behind the small islands is remarkable. The example of the time varying pattern of eddies behind the headland has been shown in research work by Geyer and Signell (1990; 1991) and Signell and Geyer (1991). The formation of eddies is thought to be due to the strong current shear behind the island or headland type of geography in the present case. The detailed verification for the eddy structures using instruments such as an ADCP has not been attempted yet, since the present study is focused on the nonlinear tidal dynamics in the strong current environments. Therefore, the verification for the performance of the present model is made in terms of the comparison between both the observed and computed harmonic values of $M_2$ and $M_4$.

**Results and energy flux calculation for the $M_2$**

The numerical experiment for the $M_2$ tide was carried out for the four combinations with/without the convection term, and with $M_2$ or $(M_2 + M_4)$ boundary forcing. The computation was first carried out with imposing one constituent $M_2$ at both open boundaries with/without the convection term. These are for investigating the pure generation effect of the $M_2$ pressure that occurs near the $M_2$ boundary according to the existence or non-existence of a convection term. Next, experiments were carried out by imposing the $(M_2 + M_4)$ at both the boundaries with or without advection term, which could be used to the regeneration effect of the $M_2$ tide through $M_2$ and $M_4$ interaction.

The observed and computed harmonic values of the $M_2$ tide for the 4 cases are given in Table 4 at the points of T2, T3, T4 and middle point (M) of the narrowest section, as marked in Fig. 3. The computed results are presented except for the tidal flat area. Fig. 8a shows the $M_2$ tidal chart for the $M_2$ input condition with advection and Fig. 8b for the $(M_2 + M_4)$ input condition with advection. The solid line denotes co-amplitude and the dotted the co-phase line. Results in both Figures reveal that the amplitudes become smaller toward the narrow section. The co-amplitude lines in Fig. 6a vary from 116 cm at the left boundary to 108 cm at the narrow part, and in Fig. 6b from 116 cm to 110 cm. With regard to the phase lag, the co-phase line is plotted every 10 degrees. Over the short distance appears a large phase lag difference from 340° to 20° in both results. This large phase lag difference over a short distance indicates that the tidal wave in the narrow section does not propagate at the usual gravity wave speed.

Table 4 also shows that the observed and computed amplitudes of the $M_2$ tide decrease toward the narrowest section in all experiments, with the results of all cases roughly coinciding with observations. Generally, the decrease in amplitude around the narrow region (T3 and M) is larger in the case of experiments without the advection terms. To interpret this, it would be interesting to see how large the current speeds are for the cases with and without convection terms.

Co-speed charts of the $x$ component of $M_2$ tidal currents are presented in Fig. 7a and 7b for without and with convection terms, respectively. It is clearly seen that the speed in the result without convection terms is much stronger than the results with convection terms. The $x$ component of currents at central channel M is about 4.1 m/s, while it reaches about 6.7 m/s for the results without the advection term. This is probably because the experiment without advection cannot generate lots of eddies and higher harmonics as well as a change in the residual surface to be shown later. This results in much less energy dissipation than the experiments with advection. The $x$ component of calculated current at C (Fig. 3) is 3.9 m/s, which is in quite good agreement with the observed depth-mean tidal speed of 3.3 m/s (MOMAF 2002).
Fig. 6. Computed $M_2$ tidal chart for (a) $M_2$ forcing and (b) ($M_2 + M_4$) forcing. Unit of amplitude (solid line) is cm and phase lag (dotted line) degree. Results for narrow channel region is also given in inset.

Table 4. Computed results of the $M_2$ tide with different forcing constituents at open boundary (FCOB) with convection terms included(Y) or excluded(N).

<table>
<thead>
<tr>
<th>FCOB</th>
<th>Convection terms</th>
<th>T2 am (cm)</th>
<th>T2 ph (deg)</th>
<th>T3 am (cm)</th>
<th>T3 ph (deg)</th>
<th>Mid point(M) am (cm)</th>
<th>Mid point(M) ph (deg)</th>
<th>T4 am (cm)</th>
<th>T4 ph (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_2$</td>
<td>N</td>
<td>113.5</td>
<td>20.5</td>
<td>107.5</td>
<td>13.5</td>
<td>106.2</td>
<td>5.8</td>
<td>113.4</td>
<td>338.6</td>
</tr>
<tr>
<td></td>
<td>Y</td>
<td>115.0</td>
<td>22.2</td>
<td>112.2</td>
<td>21.2</td>
<td>107.2</td>
<td>0.8</td>
<td>114.5</td>
<td>336.0</td>
</tr>
<tr>
<td>$M_2 + M_4$</td>
<td>N</td>
<td>113.5</td>
<td>20.5</td>
<td>107.5</td>
<td>13.5</td>
<td>106.1</td>
<td>5.9</td>
<td>113.4</td>
<td>338.6</td>
</tr>
<tr>
<td></td>
<td>Y</td>
<td>115.1</td>
<td>22.2</td>
<td>112.7</td>
<td>21.0</td>
<td>109.0</td>
<td>0.6</td>
<td>114.4</td>
<td>336.0</td>
</tr>
<tr>
<td>observed value</td>
<td></td>
<td>115.5</td>
<td>22.5</td>
<td>109.1</td>
<td>15.4</td>
<td>-</td>
<td>-</td>
<td>113.4</td>
<td>333.8</td>
</tr>
</tbody>
</table>
However, the relatively large inequality between observation and calculation at C, however, may be in part due to the neglect of frictional dissipation by other components or irrelevant representation of bottom friction by quadratic form. This seems to imply that some fundamental research for friction effect, along with increased consideration of tidal harmonics, is necessary in order to examine the tidal dynamics of the Ulldomok channel in the future, which is beyond the scope of the present study.

Meanwhile, the relatively small $M_2$ amplitude and high speed at the narrow region (T3-M) for the non-advection case imply that more potential energy (than in the case of advection terms) is converted to kinetic energy. However, in the case of experiments with advection terms, the $M_2$ amplitude at M still decreases to a great extent. This also implies that the potential energy is converted to kinetic energy at the narrowest section, inducing the strong current speed there.

It is also worth noting that there exists a difference in the $M_2$ amplitude between Fig. 6a and 6b with the different boundary forcing. A careful look at the co-amplitude lines of Fig. 6 reveals some difference around the narrow section: e.g., co-amplitude line of 108 cm is not shown in Fig. 6b. This implies that the non-linear interaction between $M_2$ and $M_t$ play a role in regenerating $M_2$. An analysis of advection terms for the possibility of the $M_2$ regeneration has been made in the discussion section.

The energy flux over the five sections (S1-S5) of the channel shown in Fig. 3 have been calculated in order to estimate the power transfer through the channel and to investigate the energy balance along the channel. The vertically integrated energy flux (Foreman et al. 1995) is given by

$$ F_x = \frac{1}{2} \int_0^T (H + \eta) \rho \left( \frac{1}{2} |u|^2 + g \eta \right) u_x dt $$

$$ = \frac{1}{2} \int_0^T \left[ \frac{1}{2} \rho H |u|^2 u_x + \rho g H \eta u_x + \frac{1}{2} \rho |u|^2 u_x + \rho g \eta |u|^2 u_x \right] dt $$

$$ = \frac{1}{2} \rho g H U \cos(\phi_x - \phi_x) + \frac{1}{16} \rho Z U^3 \cos(\phi_x - \phi_x) $$

$$ + \frac{1}{16} \rho Z U^2 \cos(\phi_x - \phi_x - 2 \phi_x) + \frac{1}{8} \rho Z U^2 \cos(\phi_x - \phi_x) $$

(9)

The term $\frac{1}{2} \rho g H U \cos(\phi_x - \phi_x)$ is due to the potential energy flux, as is also shown by Ye and Robinson (1983), and the other terms are due to kinetic energy flux. Using the above relations the mean energy flux over a cycle of the $M_2$ tide is given in Table 5 for five sections.

Table 5 shows that the major energy flux (about 2/3) is from the eastern boundary (T3) and the energy flux decreases sharply in section 3. The ratios of kinetic energy flux (KEF) to total energy flux (TEF) are 0.2, 0.7, 3.7, 0.6 and 0.0%, which are quite small compared with the potential energy contribution. However, the ratio 3.7% at the narrowest section 3 are relatively large, indicating that more energy at section 3 than at others is transported in terms of kinetic energy, possibly resulting in a decrease in the $M_2$ amplitude of elevation.

The directions of energy flux deserve special attention. The directions of energy flux in sections 1 and 2 are opposite to those of sections 4 and 5. The propagation of energy flux from both sections 1 and 5 toward the central channel could be plausible since both sections are linked to coastal oceans, as shown in Fig. 4. This means that energy fluxes are toward the narrow section and the energy amounting to the difference in energy flux between sections 2 and 4 is consumed in the area surrounded by sections 2 and 4, or the energy convergence occurs there. The consumed energy reaches to $4.701 \times 10^7$ joule which is

<table>
<thead>
<tr>
<th>Section number</th>
<th>PEF</th>
<th>KEF</th>
<th>TEF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-4.206</td>
<td>-0.009</td>
<td>-4.215</td>
</tr>
<tr>
<td>2</td>
<td>-2.935</td>
<td>-0.022</td>
<td>-2.956</td>
</tr>
<tr>
<td>3</td>
<td>-0.800</td>
<td>-0.031</td>
<td>-0.830</td>
</tr>
<tr>
<td>4</td>
<td>1.734</td>
<td>0.011</td>
<td>1.745</td>
</tr>
<tr>
<td>5</td>
<td>2.093</td>
<td>0.001</td>
<td>2.094</td>
</tr>
</tbody>
</table>
about 75% of the energy flux difference between sections 1 and 5.

In order to estimate the energy dissipation contribution from bottom friction, the tidal dissipation has been calculated using time series data reproduced by harmonic constants, since integration of the bottom friction term is not easy. Following Glorioso and Simpson (1994), the mean rate of tidal energy dissipation per unit area due to bottom friction can be estimated by the expression

$$E_d(i,j) = C_d \rho N^2 \sum \left[ u'(i,j)^2 + v'(i,j)^2 \right]$$

(10)

where velocity components \(u'(i,j), v'(i,j)\) are values at the center of each grid and \(N\) an iteration number. The energy dissipation was integrated over the area between sections S2 and S4 in Fig. 3. The average dissipation of tidal power over the domain is \(1.772 \times 10^7\) joules/s. This is about 38% of the total energy expected to be consumed there. An energy flux of \(2.929 \times 10^7\) joules/s, roughly equal to the energy flux from the eastern boundary still needs to be explained.

The convergence of the \(M_2\) tide around the narrow section implies quite complicated phenomena. Regarding the \(M_2\) energy consumption, it is likely that this consumption is due to dissipation and/or energy use in relation to other dynamical processes such as generation of the \(M_4\) tide and

<table>
<thead>
<tr>
<th>FCOB Convection terms</th>
<th>T2</th>
<th></th>
<th>T3</th>
<th></th>
<th>Mid point(M)</th>
<th></th>
<th>T4</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>am (cm)</td>
<td>ph (deg)</td>
<td>am (cm)</td>
<td>ph (deg)</td>
<td>am (cm)</td>
<td>ph (deg)</td>
<td>am (cm)</td>
<td>ph (deg)</td>
</tr>
<tr>
<td>(M_2)</td>
<td>N</td>
<td>0.3</td>
<td>205.9</td>
<td>N</td>
<td>0.9</td>
<td>201.7</td>
<td>N</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>Y</td>
<td>0.8</td>
<td>6.7</td>
<td>Y</td>
<td>6.2</td>
<td>359.1</td>
<td>Y</td>
<td>23.2</td>
</tr>
<tr>
<td>(M_2 + M_4)</td>
<td>N</td>
<td>11.1</td>
<td>189.5</td>
<td>N</td>
<td>11.0</td>
<td>186.8</td>
<td>N</td>
<td>10.8</td>
</tr>
<tr>
<td></td>
<td>Y</td>
<td>10.1</td>
<td>190.6</td>
<td>Y</td>
<td>5.0</td>
<td>204.2</td>
<td>Y</td>
<td>13.5</td>
</tr>
<tr>
<td>observed value</td>
<td></td>
<td>9.6</td>
<td>186.4</td>
<td></td>
<td>2.9</td>
<td>205.3</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>

Fig. 7. Co-amplitude line of \(x\) component (cm/s) of the \(M_2\) tidal current by forcing (a) without convection term and (b) with convection term.
Fig. 8. Computed $M_i$ tidal chart for (a) $M_2$ forcing and (b) $(M_2 + M_i)$ forcing.

depressing of the residual surface in the area. These will be described in the following sections.

Results for the $M_i$

The $M_i$ contributions in the channel are investigated for the cases with/without convection terms in the governing equations, and with/without $M_i$ in the boundary forcing. In the case of the English Channel, Pingree and Maddock (1978) calculated the $M_i$ contribution from the individual generating term in governing equations such as friction, advection and non-linear continuity as well as the contribution due to input at open boundaries.

The results for the higher harmonics $M_i$ in terms of various combinations of experimental cases are shown in Table 6 and Fig. 8a and 8b. Fig. 8a shows the $M_i$ tidal chart for only the $M_2$ input condition, and Fig. 8b
represents the $M_4$ tidal chart for $(M_2 + M_4)$ input condition. Fig. 8a shows that the amplitudes are less than 2 cm for the most part except for the region around the narrow region. The amplitude of $M_4$ around the narrowest section is shown to be very large, indicating that the $M_4$ tide is strongly generated around the narrowest section. In the subset of Fig. 8a, the detailed tidal chart is presented with a dotted co-phase line. It is clearly seen that the strongest generation of the $M_4$ tide occurs around the narrowest section where strong current and large current shear occur, as shown in Fig. 7. It is interesting to note that the phase lag around the narrowest section is mostly 0°-60°, showing a smooth variation there, while a relatively sharp gradient occurs outside. However, Fig. 8b shows that the amplitude magnitudes in the domain are about 8-10 cm outside the narrow region. It is clear that this large amplitude outside the narrow section is due to the effect from the external tide propagating from the open boundaries. In the subset of Fig. 8b, it is seen that the amplitude tends to decrease up to about 4 cm, and increases to a large value around the central region. The phase lag also shows some distortion for this case with $(M_2 + M_4)$ forcing, compared with the result with $M_2$ forcing.

The results of the 4 experiments are described in some detail in terms of comparison between observations and model results. In Table 6, the computed $M_4$ harmonic constants for different cases of open boundary forcing as well as for inclusion of convection terms are listed at 4 points along the channel. Again Table 6 indicates that the computed results, for the case with $(M_2 + M_4)$ boundary forcing as well as with convection terms, show a reasonable agreement with the observed values. The computed amplitudes for the case with $M_2$ forcing and without convection are less than 1.2 cm at all stations while the amplitudes at T3 and M are 6.2 and 23.2 cm for cases with $M_2$ boundary forcing and convection terms considered. This indicates that the convection terms play a critical role in the internal generation of the $M_4$ tide. This is comparable to the results of Pingree and Maddock (1978). As mentioned, the locally concentrated high $M_4$ amplitudes both at T3 and M (see Fig. 8a) indicates that the non-linear internal generation of the $M_4$ tide is focused around the narrowest section of the channel. The $M_4$ characteristics at T3 with $M_2$ forcing and advection suggest that this station is under the influence of the strong generation at the narrowest section. However, the results of this case still do not coincide with that of observations, since the computed amplitudes at T2 and T4 are much less than the observed amplitudes, while the computed amplitude at T3 is much larger than the observed value. Furthermore, the difference in phase lag attains a level of more or less 180°.

Experiments with $(M_2 + M_4)$ boundary forcing show quite different results in terms of amplitude distribution, compared with the results with $M_2$ boundary forcing. Fig. 9 represents the $M_4$ tidal chart with $(M_2 + M_4)$ forcing and without convection terms. The amplitudes are more than 8 cm in the whole region, and the amplitudes at four points are larger than 8.8 cm, as listed in Table 6. Considering phase lag, it is seen that the external $M_2$ tidal wave propagates from eastern to western boundaries without any abrupt change in the phase lag. The amplitude and phase lag in this case agrees with those observed except for T3. This means that the $M_4$ tidal characteristics at T2 and T4, where the size of the generated amplitude is less than 1.0 cm, is governed mainly by $M_4$ boundary forcing. Also notice that the computed phase lag (186.8°) at T3 is nearly out of phase with the generated phase lag (359.1°) based on the result obtained as a result of forcing and convection terms.

The $M_4$ tidal chart with $(M_2 + M_4)$ forcing and the advection term is compared with the results with
Fig. 10. Computed co-residual surface (or mean sea level) (cm) for (a) $M_2$ and (b) $(M_2 + M_4)$ boundary forcing.

$(M_2 + M_4)$ forcing and without the advection term. The amplitude and phase lag differences in both results are clear especially around the narrow section. The phase lag in this case does not show an abrupt change around the narrow region where strong $M_2$ generation occurs and the distorted pattern in amplitude from the generated one is also seen. Explicit comparison (Table 6) between the computed result and the observation shows that the computed amplitudes and phase lags at T2, T3 and T4 are in good agreement with the observations. The computed amplitude at T3 shows an abrupt decrease as well as a discontinuous phase lag, a trend that also appears in observed values at T3.

These results indicate that the amplitude and phase lag in the channel are determined by the superposition of the internally generated $M_2$ and the $M_4$ propagated from the
open boundaries. More precisely, at T2 and T4 or in the region apart from the narrowest section, the part due to boundary forcing plays a much larger role in determining the amplitudes than the contribution through generation. However, at point M, the generated amplitude is much larger than the amplitudes induced from boundary forcing. This large amplitude with nearly no change from the generated value in the phase lag, in the results of convection and \((M_2 + M_4)\) forcing, suggest that the \(M_4\) at M is mainly determined by the non-linear generation effect. Meanwhile, at T3 the generated amplitude of the \(M_4\) at T3 (6.2 cm) is comparable to the amplitude 11.0 cm obtained in the case of non-convection and \((M_2 + M_4)\) boundary forcing. This implies that \(M_4\) at T3 is not determined wholly either by the external part or the generated part.

In order to quantitatively investigate the superposition of each contribution at T3, we do simple complex summation of both parts. The result of \(M_2\) forcing and convection is considered the case for pure generation effect of the \(M_4\), while \((M_2 + M_4)\) the result with and without convection can be counted as a constituent determined by boundary forcing, since other non-linear generation effects such as friction is expected to be minor. Summation from both contributions at T3 yields

\[
6.2e^{359.1i} + 11.0e^{196.8i} = 4.9e^{196.5i}
\]

Amplitude of 4.9 cm and phase lag of 196.5° nearly reach the observed values at T3, 2.9 cm and 205.3°. Interestingly, this also explains the discontinuity in the phase lag. Therefore, from the results it can be said that the abrupt decrease in \(M_4\) amplitude at T3 is due to the locally generated component, being nearly out of phase with the propagated component arising from boundary forcing. Similarly, the propagation of the \(M_4\) tidal waves from the one open boundary (T5) to the other boundary (T1) is distorted, as shown in Fig. 8b, by the existence of an out-of-phase \(M_4\) tide around the narrowest section of the channel, which is generated by strong \(M_2 - M_4\) non-linear interaction. This distortion appears as a discontinuity of the \(M_4\) phase lag in T3 and M where the magnitudes of the generated amplitude are little less or much larger than the propagated one from open boundaries.

The above experiments have shown that the generation of the higher harmonic \(M_4\) is quite strong around the narrowest section of the channel and it is expected that the other shallow water constituents such as \(M2\) and \(M4\), whose characteristics are dominantly governed by the \(M_2\) tide, could be explained by similar physical processes as for the \(M_4\) tide.

Results for mean sea level

Tide-induced residual surfaces or mean sea levels are often presented in model works (Pingree and Griffiths 1978; Foreman et al. 1995; Glorioso and Simpson 1994). However, a comparison between observed and computed results is rarely made probably due to the lack of observed data and the complexities involved. Due to the geographical and coastal characteristics of Uldolmok waterway, the observation of tidal levelling became possible, as seen in Figs. 1 and 2. This enables us to attempt to verify the computed residual surface against observations.

The computed result for the residual sea level (tide-induced mean sea level) is shown in Fig. 10a for \(M_2\) boundary forcing and Fig. 10b for \((M_2 + M_4)\) forcing with convection terms. (The results without the advection term are not presented due to its negligible order of magnitude less than 1.0 cm.) The detailed structures of the residual sea surface around the narrow region are presented in each subset of the figures. The difference between results for \(M_2\) and \((M_2 + M_4)\) boundary forcings is nearly unnoticeable, since input boundary forcing of mean sea level was not included in either case. These figures illustrate that there is a clear trend of amplitude decreasing toward the narrow section, even though some local variabilities of the computed mean sea level are found around the corner area. The decrease of mean sea level at T3 is about 8.5 cm, which coincides with the magnitude of the observed mean sea level shown in Fig. 2 where the mean sea level at 86T2 is 10 cm lower than at 86T1 and 7 cm lower than at 86T3. The minimum decreasing of mean sea level is seen to reach up to 30 cm in the narrowest central section as illustrated in the figure.

This result demonstrates that the change in the local mean sea level is possible, and the prediction can be made with reasonable accuracy. The verification of the \(M_4\) distribution reinforces the modelled variation for the mean sea level, since this variation can be explained in a similar manner to the \(M_4\) variation, as discussed in the next section.

4. Discussions

The numerical model has shown that the advection term plays a critical role in generating the \(M_4\) tide and depressing the mean sea level. In addition, an analytical consideration for the advection term is made in order to investigate qualitatively the generation effect of the \(M_4\) tide, the tide-induced change in the mean sea level, and the regeneration of the \(M_4\) tide through \(M_2\) and \(M_4\) interaction in the
channel. For simplicity, the channel is assumed to be a one-dimensional channel. The flow is assumed to consist of two tidal constituents with angular frequencies of \( \omega_1 \) and \( \omega_2 \) and current amplitudes of \( u_1(x) \) and \( u_2(x) \), and phase lags of \( \theta_1 \) and \( \theta_2 \). Then the flow is represented as follows.

\[
 u = u_1(x) \sin(\omega_1 t - \theta_1) + u_2(x) \sin(\omega_2 t - \theta_2) \]  

(12)

The advection terms become

\[
 \frac{\partial u}{\partial x} = \frac{1}{2} \left[ u_1 u_1' \{1 - \cos(2\omega_1 t - 2\theta_1)\} - u_1^2 \theta_1' \sin(2\omega_1 t - 2\theta_1) \right] \\
+ u_2 u_2' \{1 - \cos(2\omega_2 t - 2\theta_2)\} - u_2^2 \theta_2' \sin(2\omega_2 t - 2\theta_2) \\
- \frac{1}{2} u_1 u_2 \left[ \cos((\omega_1 + \omega_2) t - (\theta_1 + \theta_2)) - \cos((\omega_1 - \omega_2) t - (\theta_1 - \theta_2)) \right] \\
- \frac{1}{2} u_2 u_1 \left[ \cos((\omega_1 + \omega_2) t - (\theta_1 + \theta_2)) + \cos((\omega_1 - \omega_2) t - (\theta_1 - \theta_2)) \right] \\
+ \frac{1}{2} u_1 u_2 \left[ \sin((\omega_1 + \omega_2) t - (\theta_1 + \theta_2)) - \sin((\omega_1 - \omega_2) t - (\theta_1 - \theta_2)) \right] \\
- \frac{1}{2} u_2 u_1 \left[ \sin((\omega_1 + \omega_2) t - (\theta_1 + \theta_2)) + \sin((\omega_1 - \omega_2) t - (\theta_1 - \theta_2)) \right] \\
\] 

(13)

where primes in \( u_{1,2}' \) and \( \theta_{1,2}' \) denotes partial differentiation with respect to \( x \). Eq. (13) indicates that the advection term with input tidal forcing with two frequencies of \( \omega_1 \) and \( \omega_2 \) generates new tidal oscillations having the frequencies of \( 2\omega_1 \), \( 2\omega_2 \), \( \omega_1 + \omega_2 \), \( \omega_1 - \omega_2 \), and 0. The motion with 0 frequency denotes the variation of mean water level with spatially varying amplitude \( \frac{1}{2} \{ u_1(x) u_1(x) + u_2(x) u_2(x) \} \). This value explains the existence of the varying local mean sea level shown in Fig. 10. The strong current speed and large shear in the speed may explain the large amplitude of the mean sea level in the narrowest section of the channel.

If instead of two constituents only one constituent from the open boundary is given, that is, \( u_1 = 0 \), the advection term becomes

\[
 \frac{\partial u}{\partial x} = \frac{1}{2} \left[ u_1 u_1' \cos(2\omega_1 t - 2\theta_1) - u_1^2 \theta_1' (2\omega_1 t - 2\theta_1) \right] \\
- \frac{1}{2} \left\{ u_1 u_1' + \sqrt{u_1^2 + \frac{u_1^2}{2}} \right\} \sin(2\omega_1 t - 2\theta_1 + \alpha) \right), \\
\tan \alpha = \frac{u_1'}{u_1 \theta_1} 
\]

(14)

This means that additional constituents with frequencies of 0 and \( 2\omega_1 \) are generated and the phase lag of generated motion is \( 2(\theta_1 - \alpha) \). The motion with 0 frequency now induces the variation of mean water level with spatially varying amplitude of \( \frac{1}{2} u_1(x) u_1(x) \) and spatially varying \( M_4 \) amplitude of \( \frac{1}{2} u_1(x) u_1(x) + u_2(x) \theta(x) \).

It is interesting to note that the spatial distribution of the amplitude of the residual surface (Fig. 10a) is actually more or less similar to that of the generated \( M_4 \) amplitude (Fig. 8a) in magnitude around the narrow section of the waterway. Considering the amplitude distribution of the residual surface is explained by \( \frac{1}{2} u_1(x) u_1(x) \), such a similarity could be explained in terms of the dominance of \( u_1(x) u_1(x) \) over \( u(x) \theta(x) \) in the generation of \( M_4 \). Additionally, as described in the results for the mean sea level, it is likely that the verification of the \( M_4 \) partly supports the modeled result of the mean sea level.

We proceed to estimate the generation of \( M_2 \) through \( M_2 \) and \( M_4 \) interaction. If \( \omega_1 = M_2 \) and \( \omega_2 = M_4 \) in eq. (13), the generated constituents from the advection term are 0, \( M_4(=2\omega_1) \), \( M_2(=2\omega_2) \), \( M_6=(\omega_1 + \omega_2) \), \( M_8=(\omega_1 - \omega_2) \). The regeneration of the \( M_4 \) by the non-linear interaction between \( M_2 \) and \( M_4 \) by the terms with frequency \( (\omega_1 - \omega_2) \) is natural and interesting. As shown in the model result of Table 4 and Fig. 8a and 8b, the magnitude of the \( M_4 \) tide appears differently depending upon whether \( M_2 \) or \( (M_2 + M_4) \) is prescribed in the open boundary. Eventually, the new generation of the \( M_2 \) through \( M_2 \) and \( M_4 \) interaction may explain this \( M_2 \) variation.

5. Summary

In this study, shallow-water tides are described along the Uldolmok waterway, located on the southwestern tip of Korean Peninsula, where strong tidal flows up to more than 10 knots exist at the narrowest part. Harmonic analyses of the observed water levels reveal that there are rapid changes in the amplitudes and phase lags of shallow water constituents, \( M_4, M_6, M_8, M_{12}, M_{14} \) at the station located near the narrowest part. The abrupt changes of harmonic constants of higher harmonics, \( M_4, M_6 \) and \( M_{14} \) take place in a reverse pattern of amplitude distribution, compared with those of lower harmonics, \( M_{12} \) and \( M_{14} \). Also, it was observed that the amplitude of semi-diurnal constituents, \( M_2 \) and \( S_2 \), are obviously diminishing toward the narrowest part from both the open boundaries.

A non-linear two-dimensional numerical model has shown that the directions of \( M_2 \) energy flux are toward the narrow section from both boundaries and about two-thirds that of the incoming energy flux toward the region coming
from the eastern boundary. In particular, energy roughly equal to the energy flux from the eastern boundary (average of those in sections 1 and 2) is expected to be utilized for the mean sea level depressing and generating the $M_2$ tide there, since frictional damping does not account for the whole energy consumption. Also, the regeneration effect of the $M_2$ constituent through the interaction between the major $M_2$ forced from the boundaries and quarter-diurnal constituent $M_4$ was found to exist.

This study also demonstrates that the $M_4$ generation is very strong around the narrowest section of the channel, and the characteristics of the $M_4$ along the waterway are explained by the superposition of the external tidal oscillations from both boundaries and the internally generated oscillation through the advection effect. This superposition also explains the abrupt change of $M_4$ tide and discontinuity of phase lag around the narrow region. This process is expected to explain the discontinuity of the other higher harmonic constituents such as $MS$ and $MN$, governed mainly by the $M_4$ tide. The results for other shallow water constituents will be reported in the next stage.

In this paper, it is also shown that there is a clear trend for the mean sea level to decrease toward the narrow section, and this agrees well with observations. Furthermore, the verification of the $M_4$ distribution against observation reinforces the modelled variation in the mean sea level, since the amplitude distribution of the residual surface can be explained similarly to that of the $M_4$ tide.

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