Characteristics of Perturbations in Recent Length of Day and Polar Motion

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Various features of the existing perturbations in the Earth's spin rotation are investigated for the recent and most reliable data by spectral analysis, filtering, and comparison with idealized model. First, theory of Earth's spin rotational perturbation is briefly re-derived in the Earth-fixed coordinate frame. By spectral windowings, different periodic components of the length of day perturbation are separated, and their characters and excitations are discussed. Different periodic components of polar motion are acquired similarly and described with further discussion of their excitations. Causes of the long time trends of both the length of day and polar motion are discussed. Three possible causes are considered for the newly discovered 490-day period component in the polar motion.

Keywords: length of day, polar motion

1. INTRODUCTION

Units of time; ‘hour’, ‘minute’, and ‘second’ were originally defined as the time fractions of one average solar day. Universal time (UT) was originally established by the Earth's spin rotation. Later, perturbation of length of day (LOD) was recognized, and time units have been redefined by more stable oscillation in cesium atom. UT1 and UTC were defined distinctly as follows; UT1 = ‘time based on the Earth's spin rotational angle’ and UTC = ‘time kept by atomic clocks at the mean sea level’. To compensate the accumulation of time delay in UT1 due to Earth's secular deceleration of tidal origin, leap seconds have been applied, so that the difference between UT1 and UTC may not grow bigger than 0.9 second. Seasonal variation of LOD has been clearly recognized since the advent of quartz clock in 1950s. Nowadays, the perturbation in LOD is accurately monitored with sub-microsecond accuracy.

Any position on the Earth's surface can conveniently be represented by its latitude and longitude, which are two angles defined with respect to the Earth's reference ellipsoid. Reference Pole (formerly known as the Conventional International Origin), the north pole of the ellipsoid, was determined by the pole position at the beginning of the last century (1900 - 1905). However, the present Earth's rotational pole is several meters away from the Reference Pole. In fact, true pole position on the Earth varies in complicated way. The relative displacement of the Earth's pole with respect to the Reference Pole is called polar motion, while polar wander is referred to the long time trend of pole drift. Obviously the exact pole position should be known for the coordinate conversions - from Terrestrial Reference Frame to Celestial Reference Frame and for vice versa. It is noted that precession/nutation should be known together with the polar motion for those conversions. While precession and nutation are being predicted with sub-milliarcsec accuracy for next tens years (Wallace & Capitaine 2006), modeling of polar motion cannot be done at such level due mainly to nonlinear effects in the atmosphere and ocean. Formerly, polar motion was monitored by a consortium, called the International Latitude Service (ILS). Nowadays, the International Earth Rotation and References Service (IERS) supports the information of LOD and polar motion on daily basis. After the advent of space geodetic

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techniques in 1980s, the measurement accuracy of Earth’s spin rotational perturbation was enhanced greatly (Cerveira et al. 2007). Single space geodetic technique may yield polar motion estimate, however, combined solution is more accurate and stable. It is noted that VLBI is unique discipline to determine UT1. Recently, Ring Laser Gyroscopes emerged as a promising tool to measure the instantaneous Earth’s angular velocity (Schreiber et al. 2011 and references therein). Followings are two general references for Earth rotational variations; one - introductory and the other - technical catalog (McCarthy & Seidelmann 2009, Petit & Luzum 2010).

In this study, from the IERS datasets of Earth rotation parameters, different periodic components (from secular to sub-weekly) of LOD and polar motion are extracted by spectral windowing; band-pass in frequency domain and consecutive inverse Fourier transformation. The characteristics of each acquired period band signals are described and examined. Further discussions including the possible causes of certain perturbations of them are given.

2. EARTH’S SPIN ROTATIONAL PERTURBATION THEORY: BRIEF REVIEW

In this section, the equation of motion for the Earth’s spin angular velocity perturbation is stated in the body fixed frame. First, the general equation of motion for rigid body rotation and Euler’s equation for wobble are derived. Further considerations for more realistic Earth are given after then.

The governing equation for a rotating body is that the time rate of the angular momentum \( \vec{L} \) of the body equals to the torque \( \vec{\tau} \) acting on it.

\[
\frac{d\vec{L}}{dt} = \frac{d\vec{L}}{dt}_{\text{space}} + \vec{\omega} \times \vec{L} = \vec{\tau}
\]

where \( \vec{\omega} = (\omega_1, \omega_2, \omega_3) \) is the angular velocity of the rotating body. If external torque does not exist \( (\vec{\tau} = 0) \), then \( \omega_3(t) \) should remain constant as \( \omega_3^0 \) and the two components of the governing equation for the rotational perturbation of an axially symmetric body are found as follows.

\[
I_1 \frac{d\omega_1}{dt} + \omega_2 \omega_3 (I_3 - I) = 0,
\]

\[
I_2 \frac{d\omega_3}{dt} + \omega_1 \omega_2 (I - I_3) = 0
\]

where \( I = I_1 = I_2 \) and \( I_3 \) are the principal moments of inertia of the body.

The coupled solution for \( \omega_1 \) and \( \omega_3 \) of Eq. (2) is found as a circular motion as follows.

\[
\omega_1 = m_0 \omega_0 \cos(\Omega t + \theta), \quad \omega_2 = m_0 \omega_0 \sin(\Omega t + \theta)
\]

where \( \Omega \) is defined as \( \Omega = \frac{L_3 - L_1}{I - \omega_3^0} \) and the amplitude \( m_0 \) and phase angle \( \theta \) can be specified as the initial values. This unique motion of rotating body is often called ‘Eulerian free nutation’ or ‘wobble’.

If the Earth were perfectly rigid, it should show this free wobbling motion with a frequency specified as \( \Omega = \frac{c - A \omega_3}{B} \), of which corresponding period is about 304 days. As the Earth rotates daily, an observer on the Earth should see that both the angular momentum and velocity vectors slowly encircle the \( x_3 \)-axis (Fig. 1).

The Earth is not perfectly rigid but undergoes small amount of deformations due to various agitations. Denote the small variations in the inertia tensor, angular velocity, and angular momentum as \( \Delta I_{ij}, \Delta \vec{\omega}, \Delta \vec{m} = m_0 (m_{i1}, m_{i2}, m_{i3}) \), \( \vec{h} = (h_1, h_2, h_3) \), and , then terms for Earth’s spin rotation are defined as follows.

First, the inertia tensor \( I_{ij} \) is written as

\[
\begin{bmatrix}
I_{11} & I_{12} & I_{13} \\
I_{21} & I_{22} & I_{23} \\
I_{31} & I_{32} & I_{33}
\end{bmatrix}
= \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & C \end{bmatrix}
+ \begin{bmatrix} \Delta I_{11} & \Delta I_{12} & \Delta I_{13} \\ \Delta I_{21} & \Delta I_{22} & \Delta I_{23} \\ \Delta I_{31} & \Delta I_{32} & \Delta I_{33} \end{bmatrix}
\]

It is noted that \( I_{ij} = I_{ji} \), i.e., inertia tensor is symmetric. The angular velocity \( \vec{\omega} \) is written as follows.

![Fig. 1. Two illustrations of wobble (free Eulerian nutation). Precession of angular velocity and momentum vectors around principal axis (x_3-axis) observed by an observer on the body (left). To an observer in space, angular momentum vector remains unchanged, while angular velocity vector and x_3-axis rotate around it (right).](image-url)
The angular momentum $\vec{I}$, in tensor form is given as $I_i = \sum \ell_a \omega_a + I_i$, and its matrix representation is given as follows.

\[
\begin{bmatrix}
\omega_1 \\
\omega_2 \\
\omega_3
\end{bmatrix} = \begin{bmatrix}
0 & 0 & m_1 \\
0 & 1 & m_2 \\
1 & m_3 & 1 + m_3
\end{bmatrix} = \begin{bmatrix}
m_1 \\
m_2 \\
1 + m_3
\end{bmatrix}
\]

where the first order terms only are retained with neglecting much smaller second and higher order terms.

After a little algebra, three components of the governing equation for perturbed spin rotation of the Earth are found as follows.

\[
\begin{align*}
L_1 &= \left[ A + \Delta I_{11} \quad \Delta I_{12} \quad \Delta I_{13} \right] \begin{bmatrix}
m_1 \\
m_2 \\
1 + m_3
\end{bmatrix} + h_1 \\
L_2 &= \left[ \Delta I_{21} \quad A + \Delta I_{22} \quad \Delta I_{23} \right] \begin{bmatrix}
m_1 \\
m_2 \\
1 + m_3
\end{bmatrix} + h_2 \\
L_3 &= \left[ \Delta I_{31} \quad \Delta I_{32} \quad C + \Delta I_{33} \right] \begin{bmatrix}
m_1 \\
m_2 \\
1 + m_3
\end{bmatrix} + h_3
\end{align*}
\]

Divide the first and second equations by $\omega_0 (C - A)$ and rearrange terms, then the following two equations are found.

\[
\frac{1}{\Omega} \frac{d\phi}{dt} + m_2 = -\phi - \frac{1}{\omega_0} \frac{d\phi}{dt}
\]

and

\[
\frac{1}{\Omega} \frac{d\phi}{dt} - m_1 = -\phi - \frac{1}{\omega_0} \frac{d\phi}{dt}
\]

where the two excitation functions $\phi$ and $\phi$ are defined as $\phi = \frac{h_3}{C - A} \Delta I_{33}$ and $\phi = \frac{h_3}{C - A} \Delta I_{33}$.

The third equation can be rewritten as

\[
\frac{dm_3}{dt} = -\frac{d}{dt} \left( \frac{h_3}{C + \Delta I_{33}} \right)
\]

Expression for $\Delta LOD$ due to elastic body tidal deformation is written, after Merriam (1984), as follows.

\[
\Delta LOD = -86400 \times \frac{k_2 M R^2 \Delta r}{C r^4} \left( \frac{5}{\pi} \sin^2 \delta \right)
\]

where $k_2$ is the tidal Love number, $M$ is mass of the tide raising body, $R$ is Earth's radius, $r$ is the distance to the body from the Earth, $\delta$ is the declination angle of the body, and $\Delta r$ is the deviation of $r$ from its average value. In fact, the body tidal $\Delta LOD$ is only some portion of the observed $\Delta LOD$.

In the fore part of this section, the free and forced wobbles of rotating rigid Earth were considered. Calculation of the Chandler wobble period for more realistic Earth model is not a trivial task. Chandler wobble frequency of elastic and oceanless Earth was acquired as $\Omega = \frac{C - A - D}{A + B - C}$, where $D = k_1 R^3 \omega_0 / 5G$ (Smith & Dahlen 1981). Corresponding period was found as 446.2 days. Assuming equilibrium pole tide, their estimate was changed into 425.5 days. More recent theoretical estimate was reported as 423.5 days (Wahr 2005). Among many observational estimates for Chandler period and $Q$ value, 433.0 days and 179 are the most representative ones (Gross 2009).

While Eq. (6) relates the excitation function and polar motion of rigid Earth, similar equation for realistic Earth is acquired by replacing the Chandler frequency with the following one; $\omega = \frac{h_3}{433} \left( \frac{1}{2} + \frac{1}{2} \right)$ (Smith & Dahlen 1981). Formerly, the perturbation in Earth's angular velocity and pole offset were considered to be the same with only difference in direction, i.e., $(m_3, m_i) = (x_i, -y_i)$. In fact,
this relation approximately holds for components of period longer than a few days. Their exact relation was later found as the following: \( m = \rho \frac{\partial}{\partial x} \delta K \), where \( m = m_1 + im_2 \) and \( \rho = x_p - iy_p \) (Gross 2009 and related references therein). Considering this relation between \( m \) and \( \rho \) together, the excitation function for realistic Earth can be expressed in frequency domain as follows.

\[
\Phi(\omega) = \frac{\omega_0}{\omega_0 + \omega} \Omega - \omega M(\omega) = \frac{\Omega - \omega}{\Omega} P(\omega) 
\]  

(9)

where \( \Phi(\omega) \), \( M(\omega) \), and \( P(\omega) \) are the Fourier transform pairs of \( \phi(t) \), \( m(t) \) and \( p(t) \) respectively.

### 3. DATA ANALYSIS

For this study, the most recent dataset of LOD and polar motion has been downloaded from the IERS website (http://www.iers.org/IERS/EN/IERSHome/). In Fig. 2, two time series of excessive LOD and UT1-UTC of IERS EOP 08 C04 (simply C04) since 1970 are illustrated. The dataset C04 has been produced as combined solution of VLBI, GPS, and SLR on daily basis and is generally regarded as the most reliable one. From the illustration, annual and decadal perturbations can be recognized easily (Fig. 2). The excessive LOD has been decreasing during the last forty years, which is the opposite trend of secular deceleration of the Earth. The cause of this decrease should be ascribed mainly to the tiny differential rotation in the Earth’s core associated with the geomagnetic field. This anomalous trend of LOD is also shown by leap seconds in the UT1-UTC time series, where the time interval between successive leap seconds has been increasing.

The Fourier amplitude spectrum of the excessive LOD time series was acquired by direct integration of the signal multiplied with sine/cosine waves for each frequency in time domain, and is illustrated in Fig. 3. As a pre-process, linear trend has been eliminated from the time series before spectrum evaluation. No filtering or other further treatment has been applied. Main features revealed on this spectrum are described as follows.

Six conspicuous peaks on the spectrum are marked as T1 - T5 and T9. These six periodicities are 9.13, 13.7, 27.6, 183, 365, and about 6900 days respectively. Other peaks over 95% significance level are marked as T6 - T8 or by arrows. The periods of T6 - T8 are roughly 852 - 893, 1320, 1840 days. Unlike the major peaks of lunar and solar origin, periodicities of the minor peaks cannot be clearly identified. Wide peak of long period marked as T9 is spread between 2830 and 23000 days, and its three split peaks are centered near 3170, 4650, and 13400 days respectively. Because there is no known atmospheric/oceanic perturbation of this period range (T9), main contribution for the wide T9 peak should come from the Earth’s internal process and the 18.6 year lunar orbital precession.

Separate time series of different period bands were acquired by using simple spectral windows in the frequency domain. They are illustrated as four curves together with the total signal (Fig. 4). Each spectral windows were set as unity for specified period only and zero otherwise; (i) T > 2000 days for decadal trend, (ii) 500 < T < 2000 days for long period, (iii) 100 < T < 500 days for seasonal, and (iv) T < 100 days for short period bands respectively.

The Earth’s spin angular velocity and acceleration were calculated from the excessive LOD time series, and are shown in Fig. 5. The angular velocity is directly acquired from the raw data without any filtering, while only the
decadal trend was used to calculate angular acceleration with suppressing high frequency blow up. Due to the large and positive angular acceleration at the two periods; 1981 - 1987 and 1995 - 2003, Earth’s angular velocity has increased recently. Large decadal fluctuations exist in the Earth’s angular acceleration, as can be seen in the figure. Since this perturbation is of period of several years or longer, it cannot be an atmospheric/oceanic effect.

By using Eq. (8), the excessive LOD due to the lunar and solar body tides were calculated. For a time span, between Jan 2010 and Aug 2012, are illustrated in Fig. 6 together with the seasonal and short period components of excessive LOD data. A good part of the monthly perturbation of LOD can be explained by the Earth’s body tidal deformation of lunar origin. However, the seasonal perturbation induced by solar body tide is much smaller than the observed data. The major part of this difference can be explained by seasonal changes in the global atmospheric circulation pattern.

Two graphs describing the polar motion based on the IERS C04 dataset are shown in Fig. 7. The position of the origin (0, 0) of the graphs corresponds to the Reference Pole. The pole denoted by \( x_p, y_p \) is called the Celestial Intermediate Pole. Polar motion is mainly prograde - that is counterclockwise, if seen from above as in Fig. 7. Due to the beating of the two major components; Chandler and annual, the polar motion waxes and wanes with a period of about 6.4 years.

The Fourier spectrum of the polar motion data is shown in Fig. 8. This spectrum was acquired for the polar motion data of time span between Jan. 1, 1981 and Aug. 19, 2012. Fast Fourier transform was applied for the complex time series \( z_p = x_p - iy_p \), which was properly smoothed at both ends and added with long sequence of zeroes for better resolution (total length = 65,536 points). Seven peaks are labeled as p1 – p7 in the spectrum of Fig. 8. Two main peaks p5 and p4 correspond to the Chandler and annual wobbles, and p1, p2, and p3 correspond to the semiannual, semi-Chandler, and 300-day period components, which were formerly identified (Höpfner 2004). There are small side robes between p4 and p5 - caused by two large nearby signals; Chandler and annual wobbles. One might regard the peak p6 as another side robe. More description and discussion about p6 are given below in this section. The peak p7 records over 95%
significance level, and there are seven other peaks marked with question mark (?) with each index number, of which amplitudes are slightly over or below 95% significance level (their amplitude maxima periods are 260-266, 350, 587, 683, 734, 795, and 885 days). No known atmospheric/oceanic phenomena exist for these periodicities.

Five different period bands of the polar motion were acquired by flat spectral window in frequency domain and are shown in Fig. 9. These are the decadal trend (superposed on total component), long period, Chandler, annual, and short period components. The Chandler and annual components are the two dominant ones. However, their amplitudes are varying with time in certain amount. The short period component is composed of different periodic ones as, 300 day period, semi-Chandler, semi-annual, etc. The 490-day period component polar motion marked as p6 in Fig. 8, exists as major constituent in the long period component (Fig. 9).

The linear trend of the polar motion, called polar wander, was identified as 8.1 cm/yr to the direction of 59° W.
shows drift of complicated feature, which is illustrated in Fig. 10 (top). The amplitudes of the four different band polar motions are shown together as four each time series (bottom). Variability of amplitudes of Chandler and annual wobbles can be recognized in Fig. 10, where the occasional oscillatory behaviors of the two main wobbles reflect that the two motions were elliptical at those times.

Two components of polar motion excitation function $\Phi_1$ and $\Phi_2$ calculated by inverse Fourier transform on $\Phi(\omega)$ as defined in Eq. (9) are shown with polar motion for a time span of three years (2009 - 2012) in Fig. 11. High frequency components take relatively larger part in the excitation function than in the polar motion. This is due to low-pass behavior of the Earth in its polar motional response to given excitations. The excitation function evaluated by the above method is often called the geodetic excitation, while excitation function can also be estimated by considering the flow and mass changes in the fluid spheres of the Earth.

4. DISCUSSION AND CONCLUSION

Both the LOD and polar motion are composed of numerous different periodic components. While the overall features of the two were described in the former section, following aspects are discussed in this section; (i) decadal trend of LOD time series, and (ii) possible origin of 490-day period component of polar motion. Further studies are suggested as well.

Currently, average LOD is longer by a small amount than its nominal value, and has been decreasing through the last few decades. As noted earlier, positive angular acceleration existed at two time spans; 1981 - 1987 and 1995 - 2003, and recent Earth’s angular velocity has been increased accordingly. Average excessive angular velocity of the Earth since year 2000 is $\Delta \omega_0 = -0.54\pm0.47\times10^{-12}$ (rad/s). Taking this rate as constant yields one leap second for every 4.3 year, which is longer than the average interval of successive leap seconds ever made (1.6 year) or that of last century (1.3 year). Therefore, for the near future, introduction of leap second will not be necessary as often as last century. Earth’s spin angular acceleration is affected by several causes including the change in the Earth’s dynamical form factor $J_2$ and the core motion. Accurate monitoring of $J_2$ has been done by the satellite orbit analysis since 1960s. However, large uncertainty lies in the estimation of angular momentum of core/mantle motion. More realistic simulation of the geomagnetic field generation would greatly improve this situation. The contribution from elastic/viscoelastic deformation of the Earth due to fast glacier melting of these days should be considered.

The observed recent change in the linear trend of
pole drift is also noticeable, however it cannot be clearly interpreted either. Compared with the drift of last century; 10.9 cm/yr to 79° W (Gross 2009), recent drift 8.1 cm/yr to 59° W is slower by 2.8 cm/yr and tilted by 20°. No single cause may be sufficient for this change, and both the internal and external processes of the Earth should all together be responsible. Again assessments of each processes including core/mantle motion and surface reformation of the Earth would be desirable. The recent pole drift might be affected by the recent vast glacier reduction in Greenland. Knowledge of elastic/viscoelastic response of the Earth due to loading/unloading is necessary in this regard.

Numerical experiment led positive evidence of the 490-day period signal in the Earth's polar motion being a real component (Na et al. 2012). It is interesting that almost identical periodicity exists in different phenomena such as auroral occurrence, solar wind, and geomagnetic field (Silverman & Shapiro 1983, Richardson et al. 1994, and Střešík 2009). It should be rather surprising coincidence, if none of these 490 day or 1.3 - 1.4 year periodicity of the four aforementioned phenomena are not related with others. Here, the reality and its origin of the 490-day period polar motion are considered. First, it is claimed that the 490-day component is not another side robe such as those can be seen between the annual and Chandler peaks in the polar motion Fourier spectrum (Fig. 8). After comparison of calculated polar motion spectra by maximum entropy method both for the data and synthesized, they suggested 490-day period signal is more likely real signal than an artifact (Na et al. 2012). As can be seen in both the spectrum (Fig. 8) and the time series (long period component in Fig. 7), the amplitude of 490-day period signal is about 20 milliarcsec and bigger than other formerly recognized minor components; semiannual, 300-day period, semi-Chandler component. Inner-core wobble might be a candidate for the mode associated with the 490-day polar motion. However, it is discouraging to see that theoretical estimates of inner-core wobble period are longer as between 900 and 2500 days (Dehant et al. 1993, Rochester & Crossley 2009). On the contrary, those theoretical estimates could have been misled due to difficulties involving boundary condition or others. One may resort to the possibility of it being a driven oscillation by solar wind. This possibility is not large, because the mechanical momentum/angular momentum carried by solar wind flux passing near the Earth is not sufficiently strong. Finally, there may be an unknown mode of oscillation in the Earth associated with the anomalous region near the core-mantle boundary. More concrete and quantitative assessments about the aforementioned possibilities are desirable.

By using accurate ocean tide model and atmospheric circulation model, it is possible to calculate their excitations in LOD and polar motion. Fortnightly ocean tide and seasonal atmospheric wind pattern change should be the main causes of the excessive LOD of corresponding period ranges. Comparison of these excitation functions with the ones acquired directly from Earth's rotation data as shown in Fig. 11 will enable further investigations. Some of this type researches have been reported in literatures, and more thorough ones will be accomplished, if more accurate and extensive observations on the geophysical processes be available.

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REFERENCES


Höpfner J, Low-Frequency Variation, Chander and Annual Wobbles of Polar Motion as observed over one century, Sur Geophys, 25, 1-54 (2004).

McCarthy DD, Seidelmann PK, TIME from Earth Rotation to Atomic Physics (Wiley-VCH, Weinheim, 2009), 1-94.


Richardson JD, Paularena KL, Belcher JW, Lazarus AJ, Solar
wind oscillations with 1.3 year period, GRL, 21, 1559-1560 (1994). http://dx.doi.org/10.1029/94GL01076


