A Guidance Law with a Switching Logic for Maintaining Seeker’s Lock-on for Stationary Targets

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Abstract

Modern anti-ship missiles employ complex and sophisticated guidance laws to hit the target and enhance their survivability by executing additional maneuvers. However, such maneuvers may cause the target to move out of the missile seeker’s Field-Of-View (FOV). Maintaining seeker lock-on during an engagement is a critical factor for missile guidance. In this paper, a guidance law switching logic that maintains seeker lock-on and a simple guidance law that keeps the target look angle of the seeker constant is proposed. The proposed method can be used for the terminal homing phase, and can be switched from any kind of guidance laws if a proper switching condition is satisfied. The minimum and maximum flight time calculation method in consideration of the missile maneuver limit and the FOV of the seeker is also provided.

Key Word: missile guidance, seeker’s filed of view limit

Introduction

Since World War II, many missile systems have been developed to satisfy various military demands. Over time, the accompanying missile guidance laws have become more and more precise, robust and sophisticated to meet these demands. The proportional navigation guidance (PNG) system and its variants have successfully satisfied those requirements (Zarchan 2002).

As the air defense systems of modern warships are progressively improved, the guidance method of an anti-ship missile is required not only to hit the target but also to provide additional abilities such as sea skimming, evasive maneuvers, and salvo attacks in order to increase the survivability and kill probability of missiles. Among these additional required guidance functions, the salvo attack strategy, executed by multiple missiles, saturates the air defense systems of the target ship so that the missile’s survivability and kill probability are greatly improved. In most cases, each of the missiles participating in a salvo attack is simply guided according to the prescribed mission parameters defined during the mission-planning phase before missile launch.

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Recently developed Impact Time Control Guidance (ITCG, Jeon, Lee and Tahk 2002) and Impact Time and Angle Control Guidance (ITACG, Lee, Jeon and Tahk 2007) systems are applicable to salvo attacks by anti-ship missiles, assuming that each missile has the time-to-go to their common target. Both of these guidance laws have a structure in which the guidance command consists of a PNG term to hit the target and an augmented term to control impact time (and impact angle in ITACG). Sometimes this method produces a huge maneuver command so that the Line of Sight (LOS) angle from the missile to the target may be located outside the seeker’s Field of View (FOV). In such cases, the seeker fails to maintain a lock on the target and the missile cannot intercept the target. It is therefore necessary to consider means of modifying the missile guidance laws such that it is possible to maintain the seeker lock-on condition during the homing phase.

Xin, Balakrishnan and Ohlmeyer(2006) use the seeker FOV as a constraint of the nonlinear optimal control problem for solving missile guidance law. Alternatively, we focus on the geometric relation between the missile and the target. We develop a guidance-switching concept composed of two guidance laws and switching logic. One guidance law is the original law and the other is termed the constant target look angle law. We switch the guidance law from the original law to the constant target look angle guidance command when the seeker look angle increases to the seeker’s FOV limit. This concept can use any given guidance law with the potential for lock-on failure during engagement as the original guidance law. We also define the original guidance law returning conditions in order to take advantage of the original guidance law during the final homing phase and ensure missile lock on the target.

To verify the performance of the guidance switching logic, we use ITCG as the original guidance law. To use ITCG as an original guidance law, we need the maximum and minimum time to reach the target, so we provide a time to go calculation method that uses only the geometrical relationship between the missile and the target.

This paper is organized as follows: in the first section, we present the background and main idea of this research. The second section contains an explanation of guidance law switching logic, the constant target look angle guidance law and the returning condition. The third section addresses how to estimate the time to go boundary for using ITCG as it applies to our guidance law switching concept. Then, in the fourth section, we show the simulation results. The final section presents the conclusions.

**Guidance Law Switching Logic Design**

Proportional navigation guidance (PNG) is the most popular method used in missile guidance. Using this method in a missile guidance problem, the target look angle measured by a missile’s seeker is decreased to zero as the missile approaches its target. For a stationary target, if a seeker locks on a target positioned in its field of view, the designer does not need to consider the boundary of the seeker’s field of view during the homing phase, because the PNG produces the trajectory to the target with the seeker look angle decreasing to zero. However, when using variations of PNG or other guidance methods in order to achieve additional effects, the guidance law produces a different trajectory that would be produced by PNG. In such cases, the designer should consider the missile’s trajectory and the target position to ensure the missile’s seeker does not lose its target during the homing phase.

As an example, the ITCG method that consists of PNG and an additional guidance command produces a deformation of a missile’s trajectory so as to control the impact time. When ITCG is used for a stationary target, as the time difference (\( \epsilon_T \)) between the impact time through pure PNG and the designated impact time by ITCG is increased, the target look angle in the homing phase could exceed the FOV of the seeker as the missile approaches the target. In such a case, the designer must ensure that the look angle does not exceed the field of view boundary of the seeker during the homing phase (see Fig. 1 and Fig. 2).
Fig. 1. Trajectory produced by PNG & ITCG  
Fig. 2. Seeker look angle by PNG & ITCG

Our approach is simple and easy to apply to a real missile guidance system. The main idea is to change the missile guidance law from the original law to a second law at the point where the target look angle exceeds a predefined value (the boundary of field of view), and then to return to the original guidance law when the return condition is satisfied. Any guidance law may be chosen as the original guidance method, and the requirement of the second guidance method is that it must constantly maintain the seeker's target look angle during the homing phase.

A. Missile Model

A simple anti-ship missile model is used. The missile motion is considered horizontal plane motion only, because the target (a warship) altitude can be supposed as sea level; and so the guidance channel of the anti-ship missile is divided into lateral and longitudinal direction channels. For subsonic anti-ship missiles, the longitudinal guidance channel focuses on holding the altitude as low as possible to avoid detection by the target's surveillance system, while avoiding the sea surface during the mid-course flight region and, in the terminal homing flight region, guiding the missile so as to hit near the sea level for maximizing the weapon's effect. Therefore, the lateral guidance channel could consider motion in the horizontal plane independently of vertical motion.

The dynamic system we considered in this paper is configured as follows.

$$\dot{X} = V_M \cdot \cos \theta$$

$$\dot{Y} = V_M \cdot \sin \theta$$

Here, $V_M$ is the missile speed and is assumed as constant. $\theta$ is the missile's azimuth (bearing) angle and is affected only by control acceleration input.

$$\dot{\theta} = \frac{a_M}{V_M}$$

It is also assumed that the target position and missile velocity are available, and that the guidance command has no time lag applying to missile motion.

B. Constant target look angle Guidance law

The key concept of the proposed switching logic is observing the seeker's target look angle and changing the guidance law to the second law if look angle exceeds the boundary value. The role of the second guidance law is constantly maintaining the seeker's target look angle. We refer to this guidance law as the constant target look angle guidance law. To drive this guidance law, we use the relationship between the azimuth angle of the missile ($\theta$), the line of sight angle ($\sigma$) and the target look angle of the seeker ($\lambda$), as defined in Fig. 3.
Differentiate Eq. (3)
\[ \dot{\theta} = \dot{\sigma} - \dot{\lambda} \] (4)

The rate of line of sight angle ($\dot{\sigma}$) can be expressed as below.
\[ \dot{\sigma} = \frac{V_M \cdot \sin \lambda}{R_{TM}} \] (5)

Using Eq.(4), Eq.(5) and the relation between control acceleration and turn rate of a missile, Eq.(2), we can find the relationship between the rate of target look angle by the seeker ($\dot{\lambda}$) and guidance command ($a_M$), as shown below.
\[ \dot{\lambda} = \dot{\sigma} - \dot{\theta} = \frac{V_M}{R_{TM}} \sin \lambda - \frac{a_M}{V_M} \] (6)

Using Eq.(6) we obtain the guidance command, which reduces the rate of target look angle by the missile seeker to zero. The constant seeker measuring target look angle guidance law is driven as described below.
\[ a_{M_{\lambda=0}} = \frac{V_M^2 \sin \lambda}{R_{TM}} \] (7)

For a stationary target, if the target look angle ($\lambda$) is constant, Eq.(7) becomes the guidance command that reduces the rate of target look angle ($\dot{\lambda}$) to 0. If we assume the velocity of the missile is constant, this command is affected by the miss distance ($R_{TM}$) only. Though the approaching method and applications are different, Eq.(7) is introduced as the special case of a pure pursuit, a deviated pure pursuit, in Shneydor(1998).

C. Definition of Guidance Law Returning Point

To define the point when the guidance law returns to the original guidance law, we construct a Lyapunov function with the seeker measuring target look angle as in Eq.(8).
\[ V = \frac{1}{2} \lambda^2 \] (8)

If we differentiate Eq.(8), we get
\[ \dot{V} = \lambda \cdot \dot{\lambda} = \lambda \cdot \left( \frac{V_M}{R_{TM}} \sin \lambda - \frac{a_M}{V_M} \right) \]  \hspace{1cm} (9) 

Here, we find the condition that Eq.(9) makes negative definite to ensure this Lyapunov function converges to zero. That condition is as summarized below. Usually, target look angle direction sign from the seeker is defined maximum look angle( \( \lambda_{\text{max}} \) ) as plus(\( + \)) and minimum look angle( \( \lambda_{\text{min}} \) ) as minus(\( - \)).

If \( \lambda \) becomes the maximum boundary limit of the seeker field of view (\( \lambda_{\text{max}} \)),

\[ \lambda_{\text{max}} \cdot \left( \frac{V_M}{R_{TM}} \sin \lambda_{\text{max}} - \frac{a_M}{V_M} \right) < 0 \]
\[ \frac{V_M}{R_{TM}} \sin \lambda_{\text{max}} - \frac{a_M}{V_M} < 0 \]
\[ \lambda_{\text{max}} > \frac{V_M^2}{R_{TM}^2} \sin \lambda_{\text{max}} \]  \hspace{1cm} (11)

If \( \lambda \) approaches the minimum boundary limit of the seeker field of view (\( \lambda_{\text{min}} \)), then

\[ \lambda_{\text{min}} \cdot \left( \frac{V_M}{R_{TM}} \sin \lambda_{\text{min}} - \frac{a_M}{V_M} \right) < 0 \]
\[ \frac{V_M}{R_{TM}} \sin \lambda_{\text{min}} - \frac{a_M}{V_M} > 0 \]
\[ \lambda_{\text{min}} < \frac{V_M^2}{R_{TM}^2} \sin \lambda_{\text{min}} \]  \hspace{1cm} (12)

\[ a_{M_{\text{max}}} > \frac{V_M}{R_{TM}} \sin \lambda_{\text{max}} \]
\[ a_{M_{\text{min}}} < \frac{V_M}{R_{TM}} \sin \lambda_{\text{min}} \]  \hspace{1cm} (13)

Eq.(11) and Eq.(13) satisfies the necessity condition of decreasing the target look angle to 0.

The missile guidance law reverts from the constant look angle law to the original law when the original guidance command satisfies the conditions below, Eq.(14). At this point, the original guidance command decreases the target look angle to zero.

\[ |a_{M_{\text{max}}}| \leq |a_M| \leq |a_{M_{\text{min}}}| \]  \hspace{1cm} (14)

**Estimating Time to Go**

The guidance law switching logic in this paper can use any guidance law as the original guidance law. As an example, we apply ITCG2 as the original guidance law. The ITCG method can designate the impact time. If the designated impact time is the same as or smaller than the minimum flight time to the target, then the guidance law behaves as PNG. However, if the designated impact time is larger than the calculated impact time, then the additional guidance command is added to its output so that the trajectory of the missile is transformed. Like the seeker’s field of view, all systems have physical limitations. All missile systems have limits to their maneuvering acceleration and the field of view of their sensors. Before we use a trajectory deformable guidance law such as the ITCG, we must know how much we can change the trajectory of a missile without exceeding the seeker’s field of view limit during the homing phase.
When the limits of the maneuvering acceleration and the seeker’s field of view are defined, we assume the target is fixed and the velocity of the missile is constant. Then the minimum and maximum times to go to the target can be calculated geographically.

If there is no limitation to the acceleration of the missile, the minimum time to go to target is expressed as dividing miss distance between the missile and the target by the velocity of the missile.

$$ T_{g0_{\text{max}}} = \frac{R_0}{V_M} \quad (15) $$

If a field of view limitation for the missile seeker is defined, the maximum time to go can be defined geometrically. Homing with maximum target look angle makes the longest trajectory (Fig. 4). To maintain maximum seeker look angle during the homing phase, the guidance command may have the form of Eq.(7). In this case the closing speed takes the form below.

$$ V_c = - V_M \cos \lambda_{\text{max}} \quad (16) $$

So, if the initial miss distance between the missile and target is defined, then for a stationary target the maximum time to go with the seeker field of view limit, $\lambda_{\text{max}}$, is defined as below.

$$ T_{g0_{\text{max}}} = \frac{R_0}{|V_M \cos \lambda_{\text{max}}|} \quad (17) $$

In the real world, a missile has limits to both its seeker field of view and its maneuvering capabilities. So, the above results, the maximum and minimum time to go to target, Eq.(15) and Eq.(17), are not enough to explain the longest or shortest trajectory of missile.

The shortest trajectory is composed of an initial turning for normalizing heading error, and a straight trajectory to the target. As shown at Figure 5, we can calculate the minimum trajectory length as initial turning zone and straight flight zone. To calculate initial turning angle ($\theta_{s1}$), we need the minimum turning radius ($r_{\text{min}}$) and initial heading error ($\lambda_0$). The minimum turning radius is defined by missile speed and maximum maneuvering acceleration.

$$ r_{\text{min}} = \frac{V_M^2}{a_{M_{\text{max}}}} \quad (18) $$

In Fig. 5, we can define two lengths, as shown below.

![Diagram showing the missile trajectory following constant target look angle](image_url)
Fig. 5. The shortest trajectory of missile with the limit of maneuvering acceleration and seeker field of view

\[
\eta_f = R_0 \sin \lambda_0 - r \\
\xi_f = R_0 \cos \lambda_0
\]

(19)

Using Eq.(19) and Fig. 5, we can get the length relation as shown below.

\[
r_{\text{min}} + \eta_f \cos \theta_{s1} = \xi_f \sin \theta_{s1}
\]

(20)

We can transform Eq.(20) to quadratic form as follows.

\[
(\xi_f^2 + \eta_f^2) \sin^2 \theta_{s1} - 2r_{\text{min}} \xi_f \sin \theta_{s1} + (r_{\text{min}}^2 - \eta_f^2) = 0
\]

(21)

Using the solutions of the quadratic equation we can get the sine value of the initial turn angle as below.

\[
\sin \theta_{s1} = \frac{\xi_f r_{\text{min}} \pm \eta_f \sqrt{\xi_f^2 + \eta_f^2 - r_{\text{min}}^2}}{\xi_f^2 + \eta_f^2}
\]

(22)

As shown here, if the missile needs to turn right the sine value becomes bigger than 0, we use the following equation

\[
\sin \theta_{s1} = \frac{\xi_f r_{\text{min}} + \eta_f \sqrt{\xi_f^2 + \eta_f^2 - r_{\text{min}}^2}}{\xi_f^2 + \eta_f^2}
\]

(23)

Otherwise, if the missile needs the left turn the sine value becomes minus one. we use the equation below.

\[
\sin \theta_{s1} = \frac{\xi_f r_{\text{min}} - \eta_f \sqrt{\xi_f^2 + \eta_f^2 - r_{\text{min}}^2}}{\xi_f^2 + \eta_f^2}
\]

(24)

Using Eqs.(23) or (24), we can obtain the initial turning angle and the length of the turning trajectory.

\[
s_{s1} = r_{\text{min}} \cdot \theta_{s1}
\]

(25)

The length of the straight trajectory from the end of the initial turning to the target is obtained geometrically, as below.
\[ s_{n2} = \eta_f \sin \theta_{s1} + \xi_f \cos \theta_{s1} \]  

Using Eq.(25) and Eq.(26), we obtain the length of the trajectory from the initial homing point to the target.

\[
s_{\min} = s_{n1} + s_{n2} = r_{\min} \cdot \theta_{s1} + (\eta_f \sin \theta_{s1} + \xi_f \cos \theta_{s1})
\]

After that, we find the minimum time to go to target given the limitations of the missile's maneuvering acceleration and seeker field of view.

\[
T_{go, \min} = \frac{s_{\min}}{V_m}
\]

The maximum time to go with the acceleration limit and field of view limit can be obtained from the longest trajectory. The longest trajectory can be organized in three stages. The first stage is the initial turning phase, where the missile turns its maximum acceleration to achieve the maximum target look angle \(\lambda_{\text{max}}\) as fast as possible the next stage is the maximum target look angle maintenance phase, when the missile homes to the target while constantly retaining the maximum look angle. During the final stage of the trajectory, the missile follows the circular navigation guidance law (Manchester and Savkin, 2006), which guarantees the look angle will decrease to zero and can use predefined maximum acceleration by defining its turning curvature (Fig. 7).

At the first trajectory stage, to get maximum length of trajectory the initial turning angle is necessary to achieve maximum target look angle as fast as possible. So we need the initial turning angle \(\theta_{s1}\) which makes the maximum target look angle.

\[
\theta_{s1} = \theta_{\text{max}} - \theta_{\text{ref}}
\]

Here, the reference bearing angle \(\theta_{\text{ref}}\) is defined as the turning angle from the virtual position where the target look angle is zero and the maximum bearing angle \(\theta_{\text{max}}\) is defined from the initial start point to the maximum look angle position by the maximum available maneuver. The relation of each angle and length is defined as below(Fig. 6).

![Fig. 6. The relationship between initial target look angle and the necessary initial turning angle to the maximum look angle](image)
\[ \overline{AB} = r_{\text{min}} \sin \theta_{\text{ref}} + R_0 \cos \lambda_0 = R_{\text{ref}} \cos \theta_{\text{ref}} \]
\[ \overline{CD} = r_{\text{min}} \sin \theta_{\text{max}} + R_1 \cos \lambda_0 = R_{\text{ref}} \cos \theta_{\text{max}} \]

Using the same sequence from Eq.(20) to Eq.(22), each angle can be obtained as below.

\[ \sin \theta_{\text{ref}} = \frac{R_{\text{ref}} \sqrt{r_{\text{min}}^2 + R_{\text{ref}}^2 - R_0^2 \cos^2 \lambda_0} - R_0 r_{\text{min}} \cos \lambda_0}{r_{\text{min}}^2 + R_{\text{ref}}^2} \]
\[ \sin \theta_{\text{max}} = \frac{R_{\text{ref}} \sqrt{r_{\text{min}}^2 + R_{\text{ref}}^2 - R_1^2 \cos^2 \lambda_{\text{max}} - R_1 r_{\text{min}} \cos \lambda_{\text{max}}}}{r_{\text{min}}^2 + R_{\text{ref}}^2} \]

Thus, the initial turning time that requires the maximum look angle can be obtained.

\[ T_{s1} = \frac{\theta_{s1}}{a_{\text{max}} / V_M} = \frac{V_M \cdot \theta_{s1}}{a_{\text{max}}} \]

The miss distance defined as $R_1$ and $R_{\text{ref}}$ can be obtained using the law of cosines.

The second phase of the trajectory is maintaining the maximum target look angle, during which the missile uses the guidance law shown in Eq.(7), and the reducing rate of miss distance is shown in Eq.(16). This phase is continued until the miss distance approaches the minimum range, $R_2$, at which point hitting the target by the Circular Navigation Guidance (CNG) methods (Manchester and Savkin, 2006).

In the third flight phase, the missile guidance command becomes it maximum available values and maintained until hit the target so the trajectory of missile becomes a arc with radius is $R_2 = \frac{V_M^2}{a_{\text{max}}}$. the range to go in this phase becomes the length of a chord of this arc(Fig 7).

\[ R_2 = 2 \frac{V_M^2}{a_{\text{max}}} \sin \lambda_{\text{max}} \]

in the second phase, the missile maintains the maximum target look angle. the closing velocity of missile to target can be defined as $V_{\text{close}} = V_M \cos \lambda_{\text{max}}$. we assume the missile speed is constant. the time to go of this phase can be formulated as the function of two ranges.

Fig. 7. The maximum trajectory (solid) and minimum trajectory (dotted) with maneuvering limit and seeker field of view for a stationary target
\[ T_{s2} = \frac{R_1 - R_2}{V_M \cos \lambda_{\text{max}}} \]  

(34)

The final part of the trajectory is defined by the arc of a circle with a radius \( r_{\text{min}} \) and an internal angle of \( 2\lambda_{\text{max}} \). The time to go in this final section is calculated as follows.

\[ T_{s3} = \frac{2r_{\text{min}} \lambda_{\text{max}}}{V_M} \]  

(35)

The minimum and maximum time to go for a stationary target with the maneuvering limitation and seeker field of view can be summarized as below using Eq.(28), Eq.(32), Eq.(34) and Eq.(35).

\[
T_{g0_{\text{min}}} = \frac{r \cdot \theta_{s1} + (\eta_f \cdot \sin t \theta_{s1} + \xi_f \cdot \cos \theta_{s1})}{V_M} \\
T_{g0_{\text{max}}} = \frac{V_M(\theta_{\text{max}} - \theta_{\text{ref}})}{a_{\text{max}}} + \frac{R_1 - 2 \cdot r \cdot \sin \lambda_{\text{max}}}{V_M \cos \lambda_{\text{max}}} + \frac{2 \cdot r \cdot \lambda_{\text{max}}}{V_M} 
\]  

(36)

We can identify the maximum and minimum trajectory for a stationary target in Fig. 7.

**Simulation**

We performed a simulation to verify the efficacy of our guidance changing logic for missiles with a limited seeker’s field of view and maneuvering acceleration.

The target was stationary and the position of the target was defined as (10,000m, 0m). The starting point of the missile was given at (0m, 0m). The speed of the missile was constant (300m/sec). The motion of the missile was considered to be planar motion only. The limit of maneuvering acceleration was 5g and the seeker’s field of view was defined as ±45 degrees. The initial heading error was defined as 30 degrees, so the initial target look angle by the missile seeker was ±30 degrees.

Using initial conditions, we calculated the available impact time boundary. In this case, the minimum time to go was 33.5 sec and the maximum time to go was 43.8 sec, from Eq.(36). We used the ITCG method as the original guidance law, and so assigned the impact time as 36 sec and 42 sec. The results are plotted below in Figs. 8 and 9.

In the case of the impact time assigned as 36 sec (dash–double dotted line in Fig. 8 and Fig. 9), the target look angle did not exceed the limit of the field of view, so the guidance law changing logic was not activated and the missile followed only ITCG law throughout the homing

![Fig. 8. Simulation (target look angles)](image1)

![Fig. 9. Simulation (trajectory)](image2)
simulation. But when the impact time is assigned as 46sec, if the guidance law is not changed to the constant target look angle law, defined in Eq.(7), the seeker look angle exceeds its boundary limit (the dashed line in Fig. 8 and Fig. 9). The logic for switching guidance laws and the constant target look angle guidance law help to demonstrate that keeping the seeker's target look angle in the seeker field of view limit, along with providing the original guidance law return condition, works well to make the target look angle not to exceed seeker field of view during the final homing phase.

**Conclusion**

In this paper, we proposed a guidance law switching logic for maintaining the seeker lock-on condition. This logic consists two parts. One is the constant target look angle guidance law, using a geometrical relation between the missile and target, which makes the target look angle constant during the homing phase. The other is the guidance law returning condition, which is defined by the Lyapunov function constructed with the seeker measurement, and which makes the target look angle measured by the seeker always decrease to zero. These concepts were applied to the guidance law switching logic. Using the guidance switching logic, we can use any guidance law without the possibility of lock-on failure during engagement due to a seeker’s field of view limitation. As an example, the proposed scheme was applied to the ITCG method in the plane engagement condition with a fixed target, and appropriate results were obtained.

**References**