Performance of a Turbo-Coded CDMA System in an Indoor Wireless Infrared Channel

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\textbf{ABSTRACT}

Turbo-coded CDMA system is analyzed and simulated in an indoor wireless infrared channel. The indoor infrared channel is modeled as a non-directed diffuse link. From the simulation results, it is confirmed that the turbo coding is very effective in improving BER (bit error rate) performance, and outperforms BCH code.

\textbf{Keyword :} Turbo Coding, CDMA, Infrared Channel

\section{INTRODUCTION}

Since ‘infrared’ has been proposed as a communication medium in 1979, it has attracted much attention for applications to indoor wireless communications [1]. The infrared medium has many advantageous features compared with radio waves, such as robustness to eavesdropping, immunity to multipath fading, unlimited communication resources, and coexistence with other radio systems, etc [2].

The infrared CDMA (code division multiple access) system can allow us to deploy flexible system design in the sense that bit error rate (BER) depends on the number of active users in the system. In order to enhance reliability of transmitted information in an indoor wireless infrared channel, many kinds of forward error correction (FEC) schemes have been proposed. In 1993, ‘turbo code’ has emerged as a promising FEC technique with near Shannon limit performance for a wide range of applications [3,4].

In this letter, the turbo-coded CDMA system is analyzed and simulated in the indoor wireless infrared channel. The indoor wireless infrared channel is modeled as a non-directed diffuse link. Pulse position modulation (PPM) is employed as a modulation format.

\section{SYSTEM MODEL}

1. Infrared CDMA System

The information data bits are first encoded in the turbo encoder, and then modulated by M-ary PPM modulator. In M-ary PPM signaling format, each input symbol is encoded into one of M slot positions.

The encoded and modulated data bits are transmitted through the pulse transmitter. The transmitted signals from each user go through indoor infrared wireless channel.
received signal is correlated with its own optical orthogonal code (OOC) at the correlator. The correlator output passes through photodetector and is then PPM demodulated. At the turbo decoder, the PPM demodulator output is processed to recover the transmitted symbols.

2. Indoor Infrared Channel Model

In the indoor infrared channel environment, line-of-sight path is not always guaranteed due to various kinds of obstacles between transmitter and receiver. In this letter, we employ ceiling bounce model as a kind of the non-directed diffuse link. Then, its impulse response is given by [4]

$$h_i(t) = \gamma_i \frac{6a_i^2}{(t + a_i)^2} u(t),$$  \hspace{1cm} (1)

where $\gamma_i$ is path-loss of the $i$th user, $a_i = 12D_0 \sqrt{11/13}$ is delay spread $D_0$ of the $i$th user, and $u(t)$ is unit impulse response.

III. PERFORMANCE ANALYSIS

1. Uncoded Bit Error Probability

The transmitted signal from the $i$th user is given by

$$s_i(t) = A_i c_i(t) b_i(t),$$  \hspace{1cm} (2)

where $A_i$ is an instantaneous intensity, $c_i(t)$ is OOC with period $F$ and weight $w_i$, and $b_i(t)$ is PPM encoded data sequence.

The received signal from $L$ users is given by

$$r(t) = \sum_{i=1}^{L} s_i(t - \delta_i) \otimes h_i(t) + n(t),$$  \hspace{1cm} (3)

where $\delta_i$ is associated delay for the $i$th user and $n(t)$ is AWGN (additive white Gaussian noise) with one-sided power spectral density $N_0$. In a direct-detection infrared CDMA system, the noise component is additive to the signal component in intensity. The $\delta_i (1 \leq i \leq L)$ is defined by an integer times $T$, and a real number times for the chip synchronous and asynchronous cases, respectively. In this letter, a chip synchronous case is assumed.

The decision statistic of the $i$th slot for the $i$th user is given by

$$Z_{i} = \frac{1}{T} \int_{-T/2}^{T/2} r(t)\tilde{c}_i(t)dt,$$  \hspace{1cm} (4a)

$$= D_{i} + I_{d,i} + I_{n,i} + N_{i},$$  \hspace{1cm} (4b)

$$= Y_{i} + N_{i}, 1 \leq i \leq M,$$  \hspace{1cm} (4c)

where $D_{i}$, $I_{d,i}$, $I_{n,i}$ are desired signal, self-interference, multiple access interference, and noise component, respectively.

The uncoded symbol error probability for the $i$th user is given by

$$P_e = 1 - \Pr(Z_{i} > Z_{i,2} > \cdots > Z_{i,M}),$$  \hspace{1cm} (5a)

$$= 1 - \Pr(Z_{i} > Z_{i,2}) \cdots \Pr(Z_{i} > Z_{i,M}),$$  \hspace{1cm} (5b)

$$= 1 - \prod_{i=2}^{M} \left(1 - \frac{Y_i - Y_{i-1}}{\sqrt{2}\sigma^2} \right),$$  \hspace{1cm} (5c)

where $\sigma^2 = WN_0 / T$ is noise variance of $N_{i}$. The uncoded bit error probability is bounded by

$$P_{s,e} \leq \frac{1}{2} \frac{M}{M-1} P_e.$$

2. Turbo-coded Bit Error Probability

The upper bound on the turbo-coded bit error probability can be obtained by weight distribution of the code.
However, it is difficult to compute the weight distribution of a turbo code with a particular interleaver. Therefore, it is assumed that uniform interleaver is employed.

In a turbo code with rate 1/3, the 3 bits at the encoder output can be interpreted as the sum of one uncoded bit and two parity bits. We typically refer to these outputs of rate one from each encoder as code fragments. Thus, the turbo code becomes a parallel concatenation of each code fragment. The transfer function (or generator function) of constituent encoder is given by

$$G(X;I;D) = \sum_{x_1} \sum_{x_2} \sum_{x_3} \sum_{d_1} X_1 D_1 g(x_i, d)$$

where $g(x_i, d)$ is the number of paths of length $x$, input Hamming weight $i$, and output Hamming weight $d$. The conditional probability of producing a codeword fragment of Hamming weight $d$, given a randomly selected input sequence of Hamming weight $i$, is given by

$$p_1(d | i) = \frac{g(N, i, d)}{\sum_d g(N, i, d')}$$

$$= \frac{g(N, i, d)}{\binom{N}{i}}$$

where $N$ is the number of information bits and $\sum_d g(N, i, d)$ is the total number of codewords of Hamming weight $i$.

For a 1/3 turbo encoder, $p_1(d | i)$ is modified into

$$p_2(d_1, d_2 | i) = \frac{g_1(N, i, d_1) g_2(N, i, d_2)}{\binom{N}{i}}$$

where $g_1(N, i, d)$ and $g_2(N, i, d)$ are the $g(N, i, d)$ values for the first and second encoders, respectively. The conditional transfer function for the code fragment with length $N$ is given by [6]

$$G(N, I, D) = \sum_{i} \sum_{d} p_1(d | i) D^i D^j$$

$$= \sum_{i} \sum_{d} p_2(d_1, d_2 | i) D^{d_1} D^{d_2}$$

where $d = d_1 + d_2$ and $g_2(N, i, d_2)$. Then, the turbo-coded bit error probability is upper bounded by

$$P_e \leq \sum_{i=0}^{N} \sum_{d=0}^{N} A(i, d) P_{d},$$

where $A(i, d)$ is the number of codewords generated by information sequence of weight $i$ and a parity bit stream of weight $d$, and given by

$$A(i, d) = \sum_{d_1, d_2} p_2(d_1, d_2 | i)$$

IV. SIMULATION RESULTS

In the infrared system, SNR (signal-to-noise ratio) is defined in a somewhat different manner compared with the RF domain. In a photodetector, the received intensity is converted into a photocurrent which is given by

$$I_p = AC_{p} \mu GB_{p} \eta,$$

where $A$, $C_p$, $G$, $B$, and $\eta$ are instantaneous intensity, channel loss, photodetector responsivity, concentrator gain, actual photodetector area, and optical transmission efficiency, respectively.

Then, the SNR in the infrared domain with PPM modulation is defined by

$$SNR = \frac{1}{\log_2 M} \frac{W}{\sigma^2}.$$

For simulation examples, the following parameters are as-
summed: 1) code length of OOC \( F = 500 \), 2) weight of OOC \( w = 5 \), 3) modulation order of PPM \( M = 8 \), and 4) two identical four-state recursive systematic convolutional codes with code generator polynomials \((1 + D^2, 1 + D + D^2)\), 5) photo detector responsivity \( \mu = 0.5 \), 6) concentrator gain \( G = 2.0 \), 7) photo detector area \( B = 1 \), and 8) chip duration \( T_c = 10^{-7} \).

In Fig. 1, bit error probability vs. SNR is shown for varying number of iterations with MAP algorithm, \( N = 50 \), and \( L = 8 \). It can be noted that if the number of iterations exceeds some number (10, in this case), the more iterations offer only marginal coding gain because the soft information is not available any longer after sufficient number of iterations. The turbo code achieves better BER performance over the \((31, 11)\) BCH code with soft decision decoding. It is evident that performance difference between the turbo code and BCH code becomes more distinct with the number of iterations in the turbo decoding.

![Fig. 1. Bit error probability for varying number of iterations.](image1)

In Fig. 2, bit error probability vs. the number of users in the system is shown for SOVA and MAP decoding algorithms with 10 iterations and SNR = 2 dB. It is confirmed that for a fixed code rate, the performance is gradually improved by increasing the interleaver length. The BER difference between the MAP and SOVA algorithms comes from the basic property that the MAP algorithm minimizes bit error probability while the SOVA algorithm minimizes sequence error probability.

![Fig. 2. Bit error probability for different turbo decoding algorithms.](image2)

V. CONCLUSIONS

We analyze and simulate the turbo-coded CDMA system in the indoor wireless infrared channel. From the simulation results, it is confirmed that the turbo coding has a potential to significantly improve the BER performance of the infrared CDMA system. The performance improvement becomes more distinct with the number of iterations and the interleaver length of turbo code. The results in this paper can be applied to the design of CDMA-based infrared indoor wireless LANs.

**REFERENCE**


