EXISTENCE AND NON-UNIQUENESS OF SOLUTION FOR A MIXED CONVECTION FLOW THROUGH A POROUS MEDIUM†

ZAKIA HAMMOUCH* AND MOHAMED GUEDDA

ABSTRACT. In this paper we reconsider the problem of steady mixed convection boundary-layer flow over a vertical flat plate studied in [6],[7] and [13]. Under favorable assumptions, we prove existence of multiple similarity solutions, we study also their asymptotic behavior. Numerical solutions are carried out using a shooting integration scheme.

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1. Introduction

Convective flows in porous media have a wide range of applications in engineering and science. Examples of these applications include solar energy systems, boilers, cooling of electronic devices, compact heat exchangers and the cooling core of nuclear reactors (see [2, 4, 8, 17, 18, 19, 20]. Experimental and numerical investigations have been presented in the recent book [11]. For the author knowledge, Cheng [9] was the first to study the steady mixed convection boundary layer flow about a vertical impermeable surface in a fluid-saturated porous medium when the surface is held at a constant temperature different to that of the ambient fluid. His work was followed by a large number of papers investigating the mixed convection problem under different situations. Merkin [22] (and later Aly et al. [3]) have studied the dual solutions occurring in the problem of steady mixed convection boundary layer flow over a vertical surface in a porous medium with constant and variable surface temperature for the case of the opposing flow. In [23] Merkin and Pop obtained the similarity equations for mixed convection boundary-layer flow over a vertical semi-infinite flat plate in which the free stream velocity is uniform and the wall temperature is inversely

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proportional to the distance along the plate. More recently, Nazar et al. [25] studied numerically (by a Keller-Box method) the steady stagnation point flow through a porous medium bounded by a vertical surface. Where the external velocity, which normally impinges the vertical surface and the surface temperature are assumed to vary linearly with the distance from the stagnation point. In this paper, we are interested in the case of a prescribed power-law function of the distance from the leading edge for the temperature of the plate. Our aim is to show existence of similarity solutions and study their asymptotic behavior at infinity.

First of all, we derive the equations governing the steady two dimensional mixed convection. We consider a heated semi infinite vertical flat plate embedded in a porous medium at the ambient temperature $T_\infty$, and assume that the temperature distribution of the plate is a power function of the height

$$T(x,0) = T_\infty + Ax^\lambda,$$

where $A \neq 0$ and $\lambda$ are prescribed constants, $T = T(x, y)$ is the fluid temperature and $T_\infty$ is the free-stream temperature. Coordinates $(x, y)$ are measured along the plate ($y = 0$) and normal to it, with the origin at the leading edge (See Figure 1).

The governing equations, namely the equation of continuity, the Darcy equation with Boussinesq approximation are of the form (see Cheng and Minkowycz [8])

$$\frac{1}{Pc} \psi_{xx} + \psi_{yy} = \frac{Ra}{Pc} \Theta_y,$$

$$\psi_y \Theta_x - \psi_x \Theta_y = \frac{1}{Pc} \Theta_{xx} + \Theta_{yy}.$$
Where \( \psi \) is the stream-function, \( \Theta \) is the dimensionless temperature defined by \( \Theta = \frac{T - T_\infty}{T_w - T_\infty} \), with \( T_w \) is the reference temperature, \( Ra \) and \( Pe \) are the Rayleigh and the Péclet numbers respectively.

Assume that at the leading edge of the boundary layer, the suction and the stretching velocities are given by \( U_w x^\lambda \) and \( V_w x^{(\lambda-1)/2} \) respectively. Assume that the Péclet number is very large, then we get the boundary layer approximation

\[
\psi_{yy} = \varepsilon \Theta_y, \tag{4}
\]
\[
\psi_y \Theta_x - \psi_x \Theta_y = \Theta_{yy}, \tag{5}
\]
where \( \varepsilon = \frac{Ra}{Pe} \). The appropriate boundary conditions are

\[
\begin{align*}
\psi(x, 0) &= -2 V_w x^{1+\lambda}, \\
\Theta(x, 0) &= A |A| x^{\lambda}, & 0 < x < \infty, \\
\psi_y(x, \infty) &= x^\lambda, & \Theta(x, \infty) = 0, & 0 < x < \infty.
\end{align*}
\tag{6}
\]

Note that from (4) and (5) we deduce

\[
\psi_y = x^\lambda + \varepsilon \Theta. \tag{7}
\]

We are looking for similarity solutions of (4)–(6) in the standard form

\[
\psi(x, y) = (2x^{1+\lambda})^{1/2} f(\eta), \quad \Theta(x, y) = sx^\lambda \theta(t), \tag{8}
\]
where \( \eta \) denotes the similarity variable defined by \( \eta = \frac{y}{(2x^{1+\lambda})^{1/2}} \) and \( s = \frac{A}{|A|} = \pm 1 \). Let us note that \( s = 1 \) for aiding flows and \( s = -1 \) for opposing flows. From (4)–(8) we get the BVP

\[
\begin{align*}
f''' + f''f' + \delta(1 - f')f'' &= 0, \\
f(0) &= \alpha, \quad f'(0) = 1 + \varepsilon, \quad f''(0) = \gamma, \tag{9}
\end{align*}
\]
where \( \alpha = -\sqrt{2 |V_w| / \lambda+1} \) and \( \delta = \frac{2 \lambda}{\lambda+1} \) measures the pressure gradient in the stream direction.

Very recently, Brighi and Hoernel [6] studied a similar problem

\[
\begin{align*}
f''' + \frac{\lambda+1}{\lambda+2} ff'' + 2 \lambda(1 - f')f' &= 0, \\
f(0) &= \alpha, \quad f'(0) = \beta, \quad f''(0) = \gamma. \tag{10}
\end{align*}
\]
Where \( \alpha, \delta, \gamma \) are real numbers. Their investigations led to prove existence and uniqueness of concave and convex solution for positive values of the power-law exponent \( \lambda \). In [13], the author considered problem (10) for \( \alpha = 0 \) and \( \lambda \in (-1, 0) \) and \( \beta \in (0, \frac{1}{2}) \). He showed the existence of infinitely many solutions which are unbounded at infinity. The aim of this work is to extend the results of [13], to the case of a permeable plate (\( \alpha \neq 0 \)) and also to study more precisely the asymptotic behavior of the solutions.

Thus, the present work investigates the effects of \( f''(0) \) and \( \varepsilon \) on the existence of solutions of (9). In this work, we shall adopt the same approach as in [13] to explore a relationship between \( f''(0) \) and \( \varepsilon \) such that problem has multiple solutions. General references to this technique concerning this family of problems, include [12, 14] and [16].
2. Existence of solutions

In this section, we are interested in the existence of solutions of the problem (9) for $-1 < \delta < 0$. The method used to solve the boundary value problem (9) is the shooting. For that, let $f_\gamma$ denote the solution of the initial value problem:

$$\begin{cases}
    f''' + f f'' + \delta (1 - f') f' = 0, \\
    f(0) = \alpha, \quad f'(0) = 1 + \varepsilon, \quad f''(0) = \gamma,
\end{cases}
$$

where $\alpha$ is any fixed real number and $\gamma$ is the shooting parameter, recall that the local Nusselt number $Nu_x$ is related to $\gamma$ by the following formula

$$Nu_x = \left( \frac{x^{3\lambda+1}}{2} \right)^{\frac{1}{2}} (Pe)^{\frac{1}{2}} \frac{f''(0)}{\varepsilon}.
$$

Let $[0, \eta_\gamma)$ be the right maximal interval of existence of $f_\gamma$. Our purpose is to find the range of $\gamma$ which satisfies $\eta_\gamma = \infty$ and $f'_\gamma \to 1$ as $\eta \to \infty$. We need the following tools to prove the main results.

(i) Integrating equation (11) on $[0, \eta)$ gives

$$f''' + f_\gamma f'_\gamma + \delta f_\gamma = \gamma + \alpha (1 + \varepsilon) + (\delta + 1) \int_0^\eta f'_\gamma(s)^2 ds.
$$

(ii) If $\eta_\gamma$ is finite then $\lim_{\eta \to \eta_\gamma} |f_\gamma(\eta)| = \infty$ (see [10]).

Our main result is the following

**Theorem 2.1.** Let $\alpha \in \mathbb{R}$, $\varepsilon \in (-1, \frac{1}{2})$ and $-1 < \delta < 0$, for each $\gamma \in \mathbb{R}$ such that

$$\gamma^2 \leq -\frac{\delta}{3} (1 + \varepsilon)^2 (1 - 2\varepsilon)
$$

$f_\gamma$ is global and satisfies $f'_\gamma(\infty) = 1$.

**Proof.** First we deal with the case $\alpha > 0$.

Define the energy function

$$E(\eta) = \frac{1}{2} f''_\gamma(\eta)^2 - \frac{\delta}{3} f'_\gamma(\eta)^3 + \frac{\delta}{2} f''_\gamma(\eta)^2
$$

for all $\eta < \eta_\gamma$, hence $E(0) \leq 0$, and from the equation (11) we get

$$E'(\eta) = -f_\gamma(\eta) f'''_\gamma(\eta),$$

as soon as $f_\gamma$ exists. can see that if $\eta_\gamma$ is finite $f_\gamma(\eta)$ is not bounded and goes to infinity as $\eta$ approaches $\eta_\gamma$.

In what follows, we shall see that $f_\gamma$ is positive on $(0, \eta_\gamma)$ and $\eta_\gamma = \infty$; that $f_\gamma$ is global. First, since $\varepsilon > -1$, we may assume that $f'_\gamma > 0$ on some interval $(0, \eta_0)$, where $0 < \eta_0 < \eta_\gamma$. Because $f_\gamma(0) \geq 0$, we get $f_\gamma > 0$ on $(0, \eta_0)$. Assume that there exists $\eta_1$ such that $\eta_0 \leq \eta_1 \leq \eta_\gamma$ verifying

$$f'_\gamma(\eta_1) = 0 \quad \text{and} \quad f'_\gamma > 0 \quad \text{on} \quad (0, \eta_1).$$

It follows from the positivity of $f_\gamma$ that the function $E$ is decreasing, hence $E(\eta_1) \leq E(0)$. 

The left hand side of the above inequality is non-negative while the right hand side is non-positive thus \( E(\eta_1) = E(0) = 0 \). Consequently \( E(\eta) = 0 \) for all \( 0 \leq \eta \leq \eta_1 \). Hence, we deduce that

\[
f''_\gamma = 0 \quad \text{on} \quad (0, \eta_1),
\]

and this implies that

\[
\gamma = 0 \quad \text{and} \quad (1 + \varepsilon)^2(1 - 2\varepsilon) = 0,
\]

this is a contradiction. Therefore \( f' \) is positive, then \( f \gamma \) is also positive. To show that \( f \gamma \) is global, we use again the function \( E \) to deduce

\[
\frac{1}{2} f''_\gamma^2 - \frac{\delta}{3} f'_{\gamma}^3 + \frac{\delta}{2} f'_{\gamma}^2 \leq \frac{1}{2} \gamma^2 - \frac{\delta}{3}(1 + \varepsilon)^3 + \frac{\delta}{2}(1 + \varepsilon)^2.
\]

Therefore \( f'' \) and \( f' \) are bounded on \((0, \eta_\gamma)\). Thus, if \( \eta_\gamma \) is finite, \( f \gamma \) is bounded on \((0, \eta_\gamma)\), and this implies that \( f \gamma(\eta) \) cannot tend to infinity as \( \eta \) approaches \( \eta_\gamma \), a contradiction with (ii). Consequently, \( \eta_\gamma = \infty \) and \( f \gamma \) has a limit, say \( L \in (0, \infty) \), at infinity, since \( f' \) is positive. To prove that \( L \) is infinite, we assume for the sake of contradiction that \( f \gamma \) is bounded on \((0, \infty)\). Namely \( L \) is finite, so there exists a sequence \((\eta_k)\) converging to infinity with \( k \), such that \( f'_{\gamma}(\eta_k) \) tends to zero as \( k \) goes to infinity.

\[
-\frac{\delta}{3} f'_{\gamma}^3(\eta_k) + \frac{\delta}{2} f'_{\gamma}^2(\eta_k) \leq E(\eta_k) \leq E(0),
\]

for all \( k \in \mathbb{N} \), which implies

\[
0 \leq E(\infty) \leq E(0).
\]

As above we get, a contradiction. Therefore \( f \gamma \) is unbounded at infinity. Now we show that \( f'' \) goes to zero at infinity.

If \( f'' \) is monotonic on \((\eta_1, \infty)\), for \( \eta_1 \) large enough. Then we have easily \( f'' \to 0 \), since \( f' \) and \( f'' \) are bounded. Assume now that \( f'' \) is not monotonic on any interval \([\eta_2, \infty)\). Then, there exists a sequence \((\eta_k)\) going to infinity with \( k \) such that:

- \( f'''(\eta_k) = 0 \),
- \( f''(\eta_{2k}) \) is a local minimum,
- \( f''(\eta_{2k+1}) \) is a local maximum.

From (9), we have

\[
f''(\eta_k) = \frac{\delta(1 - f'_{\gamma}(\eta_k)) f'_{\gamma}(\eta_k)}{f_{\gamma}(\eta_k)}.
\]

Since \( f'_{\gamma} \) is bounded and \( f_{\gamma} \) goes to infinity with \( \eta_k \), we get easily from the above that \( f''(\eta_k) \) goes to zero with \( \eta \).

It remains to show that \( f'_{\gamma}(\infty) = 1 \). Since \( f'' \) goes to zero and \( E \) has a finite limit at infinity, because \( E^3 \leq 0 \) and \( E \) is bounded, we deduce that

\[
\lim_{\eta \to \infty} (-\frac{\delta}{3} f'_{\gamma}(\eta)^3 + \frac{\delta}{2} f'_{\gamma}(\eta)^2).
\]
exists, say \( l \in \left[ \frac{\delta}{3}, 0 \right] \).

Let \( l_1 = \liminf_{\eta \to \infty} f'_\gamma(\eta) \) and \( l_2 = \limsup_{\eta \to \infty} f'_\gamma(\eta) \). Suppose that \( l_1 \neq l_2 \) and fix \( l_3 \) such that \( l_1 < l_3 < l_2 \). Then there exists a sequence \( \tau_k \) converging to infinity with \( k \) and

\[
\lim_{k \to \infty} f'_\gamma(\tau_k) = l_3.
\]

Using the energy function \( E \), we get

\[
l = -\frac{\delta}{3} + \frac{\delta}{2} l_3^2 \quad \forall \ l_3 \in (l_1, l_2),
\]

which is impossible. Therefore, \( l_1 = l_2 \), that is \( f'_\gamma \) has a finite limit at infinity. Let us note this limit by \( l^* \), which is positive. Otherwise we would have \( E(\eta) = 0 \), for all \( \eta \geq 0 \) and get a contradiction. It remains to prove that \( l^* = 1 \). From \( (i) \), we get for \( \eta \) large

\[
f''_\gamma(\eta) = \delta \eta l^*(l^* - 1) + o(\eta)
\]

and this is only possible if \( l^* = 1 \). Consequently \( f_\gamma \) is the desired solution satisfying \( (9) \).

Now, we treat the case \( \alpha < 0 \).

Because \( \alpha \) is negative, the function \( f_\gamma \) still negative on a some \( (0, \eta_0) \), \( \eta_0 \) small. Following \( (12) \), this function cannot have a local maximum, it follows that \( f \) remains negative for all the time, or it vanishes at some point \( \eta_* > 0 \) and becomes positive for all \( \eta > \eta_* \). If we assume that the first possibility holds we get

\[
\alpha < \lim_{\eta \to \infty} f_\gamma(\eta) \leq 0,
\]

which leads to \( \lim_{\eta \to \infty} f'_\gamma(\eta) = 0 \), but this is impossible since \( f'_\gamma \) is positive and by \( (12) \), \( f'' \) is also positive. Consequently the second possibility holds. Define the function

\[
g : \eta \mapsto g(\eta) = f(\eta + \eta_*).
\]

This function satisfies equation \( (11)_1 \) with the conditions

\[
g(0) \geq 0 \quad g'(0) = 1 + \varepsilon \quad \text{and} \quad g''(0) = \gamma,
\]

using the same arguments as the above, we conclude that \( g \) is a solution of \( (9) \). The proof is finished. \( \square \)

**Remark 2.1.** The above result indicates that \( \gamma \) is arbitrary in the interval

\[
\left[ -(1 + \varepsilon) \sqrt{\frac{\delta - (1 - 2\varepsilon)}{3}}, (1 + \varepsilon) \sqrt{\frac{\delta - (1 - 2\varepsilon)}{3}} \right].
\]

Hence the problem \( (9) \) has an infinite number of solutions for any \( \alpha \in \mathbb{R} \) and \( \varepsilon \in (-1, \frac{1}{2}) \).

**Remark 2.2.** The forced convection case \( (\varepsilon = 0) \) has been recently treated by Magyari and Aly [21].
3. Asymptotic behavior

Here we are interested in the large \( \eta \) behavior of solutions to (11). For large \( \eta \) the dimensionless velocity goes to one, hence we can use the approximation \( f \sim \eta \).

Setting \( h = f' - 1 \). Equation (11) reads

\[
h'' + \eta h' - \delta h(h + 1) = 0.
\]

We claim first that \( h \) is monotonic. Actually from the above analysis, we have \( f' \) is monotonic on \((\eta_*, \infty)\) for \( \eta_* \) large enough. Then we have the following result

**Theorem 3.1.** Let \( h \) be a global solution of (16), then we have either

\[
h(\eta) \sim \eta^\delta
\]

or

\[
h(\eta) \sim e^{-\eta^2/2}
\]

as \( \eta \) approaches infinity.

To prove the above estimates, we use an elementary method as in [5] and [26]. First, we show that we have the following

**Lemma 3.1.** Assume that \( h > 0 \) for large \( \eta \). Then

- we have

\[
\lim_{\eta \to \infty} \frac{h' h}{h} = 0, \quad (a) \quad \text{or} \quad \lim_{\eta \to \infty} \frac{h'}{h} = -\infty \quad (b).
\]

- If (b) holds, we have \( \lim_{\eta \to \infty} \eta h(\eta) = 0 \)

**Proof.** According to section 2, we have \( \frac{h'}{h} < 0 \) for \( \eta \) large, then

\[
\lim_{\eta \to \infty} \frac{h'}{h} \in [-\infty, 0].
\]

Assume that \( L := \lim_{\eta \to \infty} \frac{h'}{h} \neq 0 \) and \( L \) is finite. According to the preceding section, we know that

\[
\lim_{\eta \to \infty} h(\eta) = \lim_{\eta \to \infty} h'(\eta) = 0.
\]

By the l'Hôpital rule we get with the help of equation (16), that

\[
L := \lim_{\eta \to \infty} \frac{-\eta h' + \delta(h + 1)h}{h'} = \lim_{\eta \to \infty} \frac{-\eta h' + \delta(h + 1)h}{h'}
\]

\[
L := \lim_{\eta \to \infty} -\eta + \frac{\delta}{L}.
\]
This is impossible unless $L = -\infty$.

Assume that (b) holds. For any $a > 0$, there exists a number $\eta_a > 0$ such that

$$\frac{h'(\eta)}{h(\eta)} \leq -a \quad \text{for} \quad \eta \geq \eta_a$$

and then $h(\eta) \leq c_a e^{-a\eta}$ for $\eta \geq \eta_a$, where $c_a = h(\eta_a)e^{a\eta_a}$. Thanks to the positivity of $h$ we deduce $\lim_{\eta \to \infty} \eta h(\eta) = 0$.

**Proof of Theorem 3.1**

Now, assume that (a) holds. Set $\phi = \frac{h'}{h}$. Then using (16) for $h$, we get

$$\phi' + \eta \phi = \chi(\eta),$$

where $\chi := \delta(h + 1) - \phi^2$. Since $(h, h')$ is a slow orbit, we find that $\chi(\eta)$ goes to $\delta$ at infinity. On the other hand, (20) can be written as

$$(\phi e^{\frac{\phi^2}{2}})' = \chi(\eta)e^{\frac{\phi^2}{2}},$$

from which we deduce that

$$\phi(\eta) = \int_{\eta_0}^{\eta} \frac{\chi(\tau)e^{\frac{\phi^2(\tau)}{2}}}{e^{\frac{\phi^2(\tau)}{2}}} d\tau$$

Making recourse to the l'Hôpital rule we deduce

$$\lim_{\eta \to \infty} \eta \phi(\eta) = \delta$$

Taking into account that $h > 0$ for large $\eta$, we get (17) by a simple integration.

Now, assume that (b) holds ($(h, h')$ is a fast orbit). Using again the l'Hôpital rule and the second item of Lemma 3.1 to deduce

$$\lim_{\eta \to \infty} \frac{h'}{\eta h} = \lim_{\eta \to \infty} \frac{-\eta h' + \delta(h + 1)h}{\eta h' + h} = -1.$$

Hence (18) follows easily by integration.

4. **Numerical solutions**

Problem (11) were solved numerically using a shooting algorithm. In table 1 we give some selected values of the mixed convection parameter $\varepsilon$, related values of $\gamma_{sh}$ found via the shooting argument and $\gamma(13)$ computed by with formula (13).

We find that the results plotted for various value of the shooting parameter are in a good agreement with the results of Theorem 1 (see Figures 2-8).

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>$\gamma_{sh}$</th>
<th>$\gamma(13)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.50</td>
<td>0.2355</td>
<td>$\gamma \in (-0.2887, 0.2887)$</td>
</tr>
<tr>
<td>-0.25</td>
<td>0.1284</td>
<td>$\gamma \in (-0.3750, 0.3750)$</td>
</tr>
<tr>
<td>0.00</td>
<td>-0.1260</td>
<td>$\gamma \in (-0.4082, 0.4082)$</td>
</tr>
<tr>
<td>0.25</td>
<td>-0.1468</td>
<td>$\gamma \in (-0.3608, 0.3608)$</td>
</tr>
</tbody>
</table>
Figure 2. Profiles of the dimensionless temperature. Full lines for various value of $\gamma_{(13)}$. (a) Dotted line for $\gamma_{sh} = 0.1098$ and $\varepsilon = 0.25$. (b) Dotted line for $\gamma_{sh} = 0.1978$ and $\varepsilon = -0.5$.

Figure 3. Profiles of the dimensionless velocity. Full lines for various value of $\gamma_{(13)}$. (a) Dotted line for $\gamma_{sh} = 0.1098$ and $\varepsilon = 0.25$. (b) Dotted line for $\gamma_{sh} = 0.1978$ and $\varepsilon = -0.5$.

5. Conclusion

The present paper gives the similarity solutions for steady mixed convection boundary layer flow and heat transfer over a vertical plate embedded in a porous medium with a power-law velocity distribution. The analysis is devoted to study the existence, non-uniqueness and the asymptotic behavior at infinity of solutions. By using a shooting argument the following conclusion can be drawn:

- Equation (11) has multiple solutions for any fixed value of the suction/injection parameter, $\alpha \in (-\infty, +\infty)$ in and any $\delta \in (-1, 0)$. Such solution is positive on $\infty(0, \infty)$ and parameterized by
  \[
  \gamma := f''(0) \in \left[ -(1 + \varepsilon) \sqrt{-\frac{\delta(1 - 2\varepsilon)}{3}}, (1 + \varepsilon) \sqrt{-\frac{\delta(1 - 2\varepsilon)}{3}} \right].
  \]
- These multiple solutions have two types of possible asymptotic behavior, namely
  \[
  \varepsilon \theta(\eta) \sim \eta^\delta \quad \text{and} \quad \varepsilon \theta(\eta) \sim e^{-\frac{\eta^2}{2}}.
  \]
Figure 4. Profiles of the dimensionless shear-stress. Full lines for various value of $\gamma_{(13)}$. (a) Dotted line for $\gamma_{sh} = 0.1098$ and $\varepsilon = 0.25$. (b) Dotted line for $\gamma_{sh} = 0.1978$ and $\varepsilon = -0.5$.

Figure 5. Profiles of the dimensionless stream-function. Full lines for various value of $\gamma_{(13)}$ corresponding to $\varepsilon = 0.25$. Dotted line for $\gamma_{sh} = 0.1098$.

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References

Zakia Hammouch received M.Sc. from Fes Sais Faculty of Science and Techniques Sidi Med Ben Abdellah University Morocco and Ph.D at University of Picardie Jules Verne Amiens France. Since 2009 she has been at Moulay Ismail University FST Errachidia as an assistant professor. Her research interests include: Computational methods, Fluids Dynamics and Fractional Calculus.

Department of Mathematics, Moulay Ismail University FST Errachidia BP.509, Boutalamine Morocco.
e-mail: hammouch.zakia@gmail.com

Mohamed Guedda received M.Sc. and Ph.D. from François Rabelais University of Tours France. He is currently a professor at University of Picardie Jules Verne Amiens France. His research interests are : Calculus of variations, Fluid mechanics, Ordinary differential equations and Partial differential equations.

LAMFA, CNRS UMR 6140, Université de Picardie Jules Verne, Faculté de Mathématiques et d’Informatique, 33 Rue Saint-Leu 80039 Amiens France.
e-mail: guedda@u-picardie.fr