FUZZY $r$-MINIMAL $\beta$-OPEN SETS ON FUZZY MINIMAL SPACES

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Abstract. We introduce the concept of fuzzy $r$-minimal $\beta$-open set on a fuzzy minimal space and basic some properties. We also introduce the concept of fuzzy $r$-$M$ $\beta$-continuous mapping which is a generalization of fuzzy $r$-$M$ continuous mapping and fuzzy $r$-$M$ semicontinuous mapping, and investigate characterization for the continuity.

1. Introduction

The concept of fuzzy set was introduced by Zadeh [5]. Chang [1] defined fuzzy topological spaces using fuzzy sets. In [2], Ramadan introduced the concept of smooth topological space, which is a generalization of fuzzy topological space. We introduced the concept of fuzzy $r$-minimal space [4] which is an extension of the smooth fuzzy topological space. The concepts of fuzzy $r$-open sets and fuzzy $r$-$M$ continuous mappings are also introduced and studied. We introduced the concepts of fuzzy $r$-minimal semiopen sets [3] and fuzzy $r$-$M$ semicontinuous mappings, and investigate properties of such concepts. In this paper, we introduce the concept of fuzzy $r$-minimal $\beta$-open set on a fuzzy minimal space and basic some properties. We also introduce the concept of fuzzy $r$-$M$ $\beta$-continuous mapping which is a generalization of fuzzy $r$-$M$ continuous mapping and fuzzy $r$-$M$ semicontinuous mapping, and investigate characterization for the continuity.

2. Preliminaries

Let $I$ be the unit interval $[0,1]$ of the real line. A member $A$ of $I^X$ is called a fuzzy set [5] of $X$. By $\tilde{0}$ and $\tilde{1}$, we denote constant maps on $X$ with value $0$ and $1,$

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respectively. For any $A \in I^X$, $A^c$ denotes the complement $\tilde{1} - A$. All other notations are standard notations of fuzzy set theory.

An fuzzy point $x_\alpha$ in $X$ is a fuzzy set $x_\alpha$ defined as follows

$$x_\alpha(y) = \begin{cases} 
\alpha & \text{if } y = x \\
0 & \text{if } y \neq x.
\end{cases}$$

A smooth topology $[2]$ on $X$ is a map $T : I^X \to I$ which satisfies the following properties:

1. $T(\tilde{0}) = T(\tilde{1}) = 1$.
2. $T(A_1 \cap A_2) \geq T(A_1) \land T(A_2)$.
3. $T(\cup A_i) \geq \land T(A_i)$.

The pair $(X, T)$ is called a smooth topological space.

Let $X$ be a nonempty set and $r \in (0, 1] = I_0$. A fuzzy family $M : I^X \to I$ on $X$ is said to have a fuzzy $r$-minimal structure $[4]$ if the family

$$M_r = \{ A \in I^X | M(A) \geq r \}$$

contains $\tilde{0}$ and $\tilde{1}$.

Then the $(X, M)$ is called a fuzzy $r$-minimal space $[4]$ (simply $r$-FMS). Every member of $M_r$ is called a fuzzy $r$-minimal open set. A fuzzy set $A$ is called a fuzzy $r$-minimal closed set if the complement of $A$ (simply, $A^c$) is a fuzzy $r$-minimal open set.

Let $(X, M)$ be an $r$-FMS and $r \in I_0$. The fuzzy $r$-minimal closure of $A$, denoted by $mC(A, r)$, is defined as

$$mC(A, r) = \cap \{ B \in I^X : B^c \in M_r \text{ and } A \subseteq B \}.$$

The fuzzy $r$-minimal interior of $A$, denoted by $mI(A, r)$, is defined as

$$mI(A, r) = \cup \{ B \in I^X : B \in M_r \text{ and } B \subseteq A \}.$$

**Theorem 2.1** ($[4]$). Let $(X, M)$ be an $r$-FMS and $A, B \in I^X$.

1. $mI(A, r) \subseteq A$ and if $A$ is a fuzzy $r$-minimal open set, then $mI(A, r) = A$.
2. $A \subseteq mC(A, r)$ and if $A$ is a fuzzy $r$-minimal closed set, then $mC(A, r) = A$.
3. If $A \subseteq B$, then $mI(A, r) \subseteq mI(B, r)$ and $mC(A, r) \subseteq mC(B, r)$.
4. $mI(A, r) \cap mI(B, r) \supseteq mI(A \cap B, r)$ and $mC(A, r) \cup mC(B, r) \subseteq mC(A \cup B, r)$.
5. $mI(mI(A, r), r) = mI(A, r)$ and $mC(mC(A, r), r) = mC(A, r)$.
6. $\tilde{1} - mC(A, r) = mI(\tilde{1} - A, r)$ and $\tilde{1} - mI(A, r) = mC(\tilde{1} - A, r)$. 

Let \((X, M)\) be an \(r\)-FMS and \(A \in I^X\). Then a fuzzy set \(A\) is called a fuzzy \(r\)-minimal semiopen set \([3]\) in \(X\) if
\[ A \subseteq mC(mI(A, r), r). \]
A fuzzy set \(A\) is called a fuzzy \(r\)-minimal semiclosed set if the complement of \(A\) is fuzzy \(r\)-minimal semiopen.

Let \((X, M)\) and \((Y, N)\) be two \(r\)-FMS's. Then \(f : X \rightarrow Y\) is said to be fuzzy \(r\)-M continuous function if for every \(A \in N_r\), \(f^{-1}(A)\) is in \(M_r\).

3. Fuzzy \(r\)-minimal \(\beta\)-open Sets

In this section, we introduce and study the concept of fuzzy \(r\)-minimal \(\beta\)-open sets. The two operators \(m\beta C(A, r)\) and \(m\beta I(A, r)\) are introduced and investigated.

**Definition 3.1.** Let \((X, M)\) be an \(r\)-FMS and \(A \in I^X\). Then a fuzzy set \(A\) is called a fuzzy \(r\)-minimal \(\beta\)-open set in \(X\) if
\[ A \subseteq mC(mI(mC(A, r), r), r). \]
A fuzzy set \(A\) is called a fuzzy \(r\)-minimal \(\beta\)-closed set if the complement of \(A\) is fuzzy \(r\)-minimal \(\beta\)-open.

**Remark 3.2.** From definitions of fuzzy \(r\)-minimal semiopen set and fuzzy \(r\)-minimal \(\beta\)-open set, the following implications are obtained but the converses are not true in general.

fuzzy \(r\)-minimal open \(\Rightarrow\) fuzzy \(r\)-minimal semiopen \(\Rightarrow\) fuzzy \(r\)-minimal \(\beta\)-open

**Example 3.3.** Let \(X = I = [0, 1]\) and let \(A\) and \(B\) be fuzzy sets defined as follows
\[ A(x) = \begin{cases} -x + \frac{1}{2}, & \text{if } 0 \leq x \leq \frac{1}{4}, \\ \frac{1}{3}(x - 1) + \frac{1}{2}, & \text{if } \frac{1}{4} \leq x \leq 1; \end{cases} \]
\[ B(x) = \frac{1}{4}(x + 3), & \text{if } 0 \leq x \leq 1. \]
Let us consider a fuzzy minimal structure
\[ M(\mu) = \begin{cases} \frac{2}{3}, & \text{if } \mu = \tilde{0}, \tilde{1}, A, \\ 0, & \text{otherwise.} \end{cases} \]
Then the fuzzy set \(B\) is a fuzzy \(\frac{2}{3}\)-minimal \(\beta\)-open set but not fuzzy \(\frac{2}{3}\)-minimal semiopen.
Lemma 3.4. Let $(X, M)$ be an $r$-FMS. Then a fuzzy set $A$ is fuzzy $r$-minimal $\beta$-closed if and only if $mI(mC(mI(A, r), r), r) \subseteq A$.

Theorem 3.5. Let $(X, M)$ be an $r$-FMS. Any union of fuzzy $r$-minimal $\beta$-open sets is fuzzy $r$-minimal $\beta$-open.

Proof. Let $A_i$ be a fuzzy $r$-minimal $\beta$-open set for $i \in J$. Then from Theorem 2.1,

\[ A_i \subseteq mI(mC(A_i, r), r) \subseteq mI(mC(\bigcup A_i, r), r). \]

This implies $\bigcup A_i \subseteq mI(mC(\bigcup A_i, r), r)$ and so $\bigcup A_i$ is fuzzy $r$-minimal $\beta$-open. □

Remark 3.6. In general, the intersection of two fuzzy $r$-minimal $\beta$-open sets may not be fuzzy $r$-minimal $\beta$-open as shown in the next example.

Example 3.7. Let $X = I = [0, 1]$ and let $A, B$ and $C$ be fuzzy sets defined as follows

\[
A(x) = -\frac{1}{2}(x - 1), \quad \text{if } x \in I; \\
B(x) = \frac{1}{2}x, \quad \text{if } x \in I; \\
C(x) = \frac{3}{4}x, \quad x \in I.
\]

Let us consider a fuzzy minimal structure

\[
N(\mu) = \begin{cases} 
\frac{2}{3}, & \text{if } \mu = 0, 1, A, B, A \cup B \\
0, & \text{otherwise}.
\end{cases}
\]

Then the fuzzy sets $A$ and $B$ are fuzzy $\frac{2}{3}$-minimal $\beta$-open. But $A \cap B$ is not fuzzy $\frac{2}{3}$-minimal $\beta$-open, because of $mI(mC(A \cap B, \frac{2}{3}), \frac{2}{3}) = 0$.

Definition 3.8. Let $(X, M)$ be an $r$-FMS. For $A \in I^X$, $m\beta C(A, r)$ and $m\beta I(A, r)$, respectively, are defined as the following:

\[
m\beta C(A, r) = \cap\{F \in I^X : A \subseteq F, \ F \text{ is fuzzy } r\text{-minimal } \beta\text{-closed}\} \\
m\beta I(A, r) = \cup\{U \in I^X : U \subseteq A, U \text{ is fuzzy } r\text{-minimal } \beta\text{-open}\}.
\]

Theorem 3.9. Let $(X, M)$ be an $r$-FMS and $A \in I^X$. Then

1. $m\beta I(A, r) \subseteq A$.
2. If $A \subseteq B$, then $m\beta I(A, r) \subseteq m\beta I(B, r)$.
3. $A$ is $r$-minimal $\beta$-open iff $m\beta I(A, r) = A$.
4. $m\beta I(\beta mI(A, r), r) = m\beta I(A, r)$.
5. $m\beta C(\bar{1} - A, r) = \bar{1} - m\beta I(A, r)$ and $m\beta I(\bar{1} - A, r) = \bar{1} - m\beta C(A, r)$.
Proof. (1), (2), (3) and (4) are clear from Theorem 3.5.

(5) For $A \in I^X$,
$$\tilde{1} - m\beta I(A, r) = \tilde{1} - \bigcup\{\tilde{1} - U : U \subseteq A, U \text{ is fuzzy } r\text{-minimal } \beta\text{-open}\}$$
$$= \bigcap\{\tilde{1} - U : U \subseteq A, U \text{ is fuzzy } r\text{-minimal } \beta\text{-open}\}$$
$$= \bigcap\{\tilde{1} - U : \tilde{1} - A \subseteq \tilde{1} - U, U \text{ is fuzzy } r\text{-minimal } \beta\text{-open}\}$$
$$= m\beta C(\tilde{1} - A, r).$$

Similarly, we can show that $m\beta I(\tilde{1} - A, r) = \tilde{1} - m\beta C(A, r)$. $\square$

Theorem 3.10. Let $(X, M)$ be an $r$-FMS and $A \in I^X$. Then

(1) $A \subseteq m\beta C(A, r)$.

(2) If $A \subseteq B$, then $m\beta C(A, r) \subseteq m\beta C(B, r)$.

(3) $F$ is $r$-minimal $\beta$-closed iff $m\beta C(F, r) = F$.

(4) $m\beta C(m\beta C(A, r), r) = m\beta C(A, r)$.

Proof. It is similar to the proof of Theorem 3.9. $\square$

Lemma 3.11. Let $(X, M)$ be an $r$-FMS and $A \in I^X$. Then

(1) $x_\alpha \in m\beta C(A, r)$ if and only if $A \cap V \neq \tilde{0}$ for every $r$-minimal $\beta$-open set $V$ containing $x_\alpha$.

(2) $x_\alpha \in m\beta I(A, r)$ if and only if there exists a fuzzy $r$-minimal $\beta$-open set $G$ such that $G \subseteq A$.

Proof. (1) If there is a fuzzy $r$-minimal $\beta$-open set $V$ containing $x_\alpha$ such that $A \cap V = \tilde{0}$, then $\tilde{1} - V$ is a fuzzy $r$-minimal $\beta$-closed set such that $A \subseteq \tilde{1} - V, x_\alpha \notin \tilde{1} - V$. From this fact, $x_\alpha \notin m\beta C(A, r)$.

The converse is easily proved by definition of the operator of $m\beta C(A, r)$.

(2) Obvious. $\square$

4. FUZZY $r$-$M$ $\beta$-CONTINUITY AND FUZZY $r$-$M(M^*)$ $\beta$-OPEN MAPPINGS

In this section, we introduce the concepts of fuzzy $r$-$M$ $\beta$-continuous mapping, fuzzy $r$-$M$ $\beta$-open mapping and fuzzy $r$-$M^*$ $\beta$-open mapping, and investigate characterization for such mappings.

Definition 4.1. Let $(X, M)$ and $(Y, N)$ be $r$-FMS’s. Then a mapping $f : (X, M) \rightarrow (Y, N)$ is said to be fuzzy $r$-$M$ $\beta$-continuous if for each point $x_\alpha$ and each fuzzy $r$-minimal open set $V$ containing $f(x_\alpha)$, there exists a fuzzy $r$-minimal $\beta$-open set $U$ containing $x_\alpha$ such that $f(U) \subseteq V$. 
Let \((X, M)\) and \((Y, N)\) be \(r\)-FMS’s. Then a mapping \(f : (X, M) \rightarrow (Y, N)\) is said to be fuzzy \(r\)-\(M\) semicontinuous \([3]\) if for each point \(x_\alpha\) and each fuzzy \(r\)-minimal open set \(V\) containing \(f(x_\alpha)\), there exists a fuzzy \(r\)-minimal semiopen set \(U\) containing \(x_\alpha\) such that \(f(U) \subseteq V\).

**Remark 4.2.** It is obvious that every fuzzy \(r\)-\(M\) semicontinuous mapping is fuzzy \(r\)-\(M\) \(\beta\)-continuous but the converse may not be true as shown in the next example.

**fuzzy \(r\)-\(M\) continuous \(\Rightarrow\) fuzzy \(r\)-\(M\) semicontinuous \(\Rightarrow\) fuzzy \(r\)-\(M\) \(\beta\)-continuous**

**Example 4.3.** For \(X = [0, 1]\), consider two fuzzy minimal structures \(M\) and \(N\) defined in Example 3.3 and Example 3.7, respectively. The identity mapping \(f : (X, M) \rightarrow (X, N)\) is fuzzy \(r\)-\(M\) \(\beta\)-continuous but not fuzzy \(r\)-\(M\) semicontinuous.

**Theorem 4.4.** Let \(f : (X, M) \rightarrow (Y, N)\) be a mapping on \(r\)-FMS’s \((X, M)\) and \((Y, N)\). Then the following statements are equivalent:

1. \(f\) is fuzzy \(r\)-\(M\) \(\beta\)-continuous.
2. \(f^{-1}(V)\) is a fuzzy \(r\)-minimal \(\beta\)-open set for each fuzzy \(r\)-minimal open set \(V\) in \(Y\).
3. \(f^{-1}(B)\) is a fuzzy \(r\)-minimal \(\beta\)-closed set for each fuzzy \(r\)-minimal closed set \(B\) in \(Y\).
4. \(f(m_\beta C(A, r)) \subseteq m C(f(A), r)\) for \(A \subseteq X\).
5. \(m_\beta C(f^{-1}(B), r) \subseteq f^{-1}(m C(B, r))\) for \(B \in I^Y\).
6. \(f^{-1}(m I(B, r)) \subseteq m I(f^{-1}(B), r)\) for \(B \in I^Y\).

**Proof.**

(1) \(\Rightarrow\) (2) Let \(V\) be any fuzzy \(r\)-minimal open set in \(Y\) and \(x_\alpha \in f^{-1}(V)\). By hypothesis, there exists a fuzzy \(r\)-minimal \(\beta\)-open set \(U\) containing \(x_\alpha\) such that \(f(U) \subseteq V\). This implies that \(\cup U = f^{-1}(V)\) and hence from Theorem 3.5, \(f^{-1}(V)\) is fuzzy \(r\)-minimal \(\beta\)-open.

(2) \(\Rightarrow\) (3) Obvious.

(3) \(\Rightarrow\) (4) For \(A \in I^X\),

\[
\begin{align*}
f^{-1}(m C(f(A), r)) &= f^{-1}(\cap \{ F \in I^Y : f(A) \subseteq F \text{ and } F \text{ is fuzzy } r\text{-minimal closed} \}) \\
&= \cap \{ f^{-1}(F) \in I^X : A \subseteq f^{-1}(F) \text{ and } f^{-1}(F) \text{ is fuzzy } r\text{-minimal } \beta\text{-closed} \} \\
&\supseteq \cap \{ K \in I^X : A \subseteq K \text{ and } K \text{ is fuzzy } r\text{-minimal } \beta\text{-closed} \} \\
&= m_\beta C(A, r).
\end{align*}
\]

Hence \(f(m_\beta C(A, r)) \subseteq m C(f(A), r)\).
(4) ⇒ (5) For $B \in I^Y$, 
\[ f(m_{\beta C}(f^{-1}(B), r)) \subseteq m_{C}(f(f^{-1}(B)), r) \subseteq m_{C}(B, r). \]
So $m_{\beta C}(f^{-1}(B), r) \subseteq f^{-1}(m_{C}(B, r))$.

(5) ⇒ (6) For $B \subseteq Y$, from Theorem 2.1 and Theorem 3.9, it follows
\[ f^{-1}(m_{I}(B, r)) = f^{-1}(\tilde{1} - m_{C}(\tilde{1} - B, r)) = \tilde{1} - f^{-1}(m_{C}(\tilde{1} - B, r)) \subseteq \tilde{1} - m_{\beta C}(f^{-1}(\tilde{1} - B), r) = m_{\beta I}(f^{-1}(B), r). \]
This implies $f^{-1}(m_{I}(B, r)) \subseteq m_{\beta I}(f^{-1}(B), r)$.

(6) ⇒ (1) Let $V$ be any fuzzy $r$-minimal open set containing $f(x_\alpha)$ for a fuzzy point $x_\alpha$. By hypothesis, $x_\alpha \in f^{-1}(V) = f^{-1}(m_{I}(V, r)) \subseteq m_{\beta I}(f^{-1}(V), r)$. Since $x_\alpha \in m_{\beta I}(f^{-1}(V), r)$, by Lemma 3.11, there exists a fuzzy $r$-minimal $\beta$-open set $U$ containing $x_\alpha$ such that $U \subseteq f^{-1}(V)$. This implies $f^{-1}(V)$ is fuzzy $r$-minimal $\beta$-open, and hence $f$ is fuzzy $r$-M $\beta$-continuous. \qed

**Definition 4.5.** Let $f : (X, \mathcal{M}) \rightarrow (Y, \mathcal{N})$ be a mapping on $r$-FMS’s $(X, \mathcal{M})$ and $(Y, \mathcal{N})$. Then $f$ is said to be fuzzy $r$-$M^*\beta$-open if for every fuzzy $r$-minimal $\beta$-open set $A$ in $X$, $f(A)$ is fuzzy $r$-minimal open in $Y$.

**Theorem 4.6.** Let $f : (X, \mathcal{M}) \rightarrow (Y, \mathcal{N})$ be a mapping on $r$-FMS’s $(X, \mathcal{M})$ and $(Y, \mathcal{N})$.

(1) $f$ is fuzzy $r$-$M^*\beta$-open.

(2) $f(m_{\beta I}(A, r)) \subseteq m_{I}(f(A), r)$ for $A \in I^X$.

(3) $m_{\beta I}(f^{-1}(B), r) \subseteq f^{-1}(m_{I}(B, r))$ for $B \in I^Y$.

Then (1) ⇒ (2) ⇒ (3).

**Proof.** (1) ⇒ (2) For $A \in I^X$,
\[ f(m_{\beta I}(A, r)) = f(\bigcup\{B \in I^X : B \subseteq A, B \text{ is fuzzy } r\text{-minimal } \beta\text{-open}\}) = \bigcup\{f(B) \in I^Y : f(B) \subseteq f(A), f(B) \text{ is fuzzy } r\text{-minimal open}\} \subseteq \bigcup\{U \in I^Y : U \subseteq f(A), U \text{ is fuzzy } r\text{-minimal open}\} = m_{I}(f(A), r) \]
Hence $f(m_{\beta I}(A, r)) \subseteq m_{I}(f(A), r)$.

(2) ⇒ (3)

For $B \in I^Y$, from (3),
\[ f(m\beta I(f^{-1}(B), r)) \subseteq m I(f(f^{-1}(B)), r) \subseteq m I(B, r). \]

Similarly, we have the implication (3) \(\Rightarrow\) (2).

Let \( X \) be a nonempty set and \( M : I^X \to I \) a fuzzy family on \( X \). The fuzzy \( r \)-minimal structure \( M_r \) is said to have the property (\( U \)) [4] if for \( A_i \in M_r (i \in J) \),

\[ M_r(\cup A_i) \geq \wedge M_r(A_i). \]

**Theorem 4.7** ([4]). Let \((X, M)\) be an \( r \)-FMS with the property (\( U \)). Then \( m I(A, r) = A \) if and only if \( A \) is fuzzy \( r \)-minimal open for \( A \in I^X \).

From the above Theorem 4.7, obviously the following corollary is obtained:

**Corollary 4.8.** Let \( f : (X, M) \to (Y, N) \) be a mapping on \( r \)-FMS’s \((X, M)\) and \((Y, N)\). If \((Y, N)\) has the property (\( U \)), then the following are equivalent:

1. \( f \) is fuzzy \( r \)-\( M^* \)-\( \beta \)-open.
2. \( f(m\beta I(A, r)) \subseteq m\beta I(f(A), r) \) for \( A \in I^X \).
3. \( m\beta I(f^{-1}(B), r) \subseteq f^{-1}(m\beta I(B, r)) \) for \( B \in I^Y \).

**Definition 4.9.** Let \( f : (X, M) \to (Y, N) \) be a mapping on \( r \)-FMS’s \((X, M)\) and \((Y, N)\). Then \( f \) is said to be fuzzy \( r \)-\( M \)-\( \beta \)-open if for fuzzy \( r \)-minimal open set \( A \) in \( X \), \( f(A) \) is fuzzy \( r \)-minimal \( \beta \)-open in \( Y \).

**Theorem 4.10.** Let \( f : (X, M) \to (Y, N) \) be a mapping on \( r \)-FMS’s \((X, M)\) and \((Y, N)\). Then the following are equivalent:

1. \( f \) is fuzzy \( r \)-\( M \)-\( \beta \)-open.
2. \( f(m I(A, r)) \subseteq m I(f(A), r) \) for \( A \in I^X \).
3. \( m I(f^{-1}(B), r) \subseteq f^{-1}(m I(B, r)) \) for \( B \in I^Y \).

**Proof.**

1. \( \Rightarrow \) (2) For \( A \in I^X \),

\[
\begin{align*}
f(m I(A, r)) &= f(\cup \{ B \in I^X : B \subseteq A, B \text{ is fuzzy } r \text{-minimal open} \}) \\
&= \cup \{ f(B) \in I^Y : f(B) \subseteq f(A), f(B) \text{ is fuzzy } r \text{-minimal } \beta \text{-open} \} \\
&\subseteq \cup \{ U \in I^X : U \subseteq f(A), U \text{ is fuzzy } r \text{-minimal } \beta \text{-open} \} \\
&= m \beta I(f(A), r)
\end{align*}
\]

Hence \( f(m I(A, r)) \subseteq m \beta I(f(A), r) \).

2. \( \Rightarrow \) (3)

For \( B \in I^Y \), from (3) it follows that
\[ f(mI(f^{-1}(B), r)) \subseteq m\beta I(f(f^{-1}(B)), r) \subseteq m\beta I(B, r). \]

Hence we get (3).

(3) ⇒ (2) It is similar to the proof of the implication (2) ⇒ (3).

(2) ⇒ (1) Let \( A \) be a fuzzy \( r \)-minimal open set in \( X \). Then \( A = mI(A, r) \). By (2), \( f(A) = m\beta I(f(A), r) \) and hence by Theorem 3.9 (3), \( f(A) \) is fuzzy \( r \)-minimal \( \beta \)-open.

\[ \square \]

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