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Investigation of the Thermal Mode-based Thermal Error Prediction for the Multi-heat Sources Model

한준안¹, 김규하¹, 이선규^{1,✉}
Jun An Han¹, Gyu Ha Kim¹, and Sun-Kyu Lee^{1,✉}

¹ 광주과학기술원 기전공학부 (School of Mechatronics, Gwangju Institute of Science & Technology)

✉ Corresponding author: skyee@gist.ac.kr, Tel: +82-62-715-2388

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Thermal displacement is an important issue in machine tool systems. During the last several decades, thermal error compensation technology has significantly reduced thermal distortion error; this success has been attributed to the development of a precise, robust thermal error model. A major advantage of using the thermal error model is instant compensation for the control variables during the modeling process. However, successful application of thermal error modeling requires correct determination of the temperature sensor placement. In this paper, a procedure for predicting thermal-mode-based thermal error is introduced. Based on this thermal analysis, temperature sensors were positioned for multiple heat-source models. The performance of the sensors based on thermal-mode error analysis, was compared with conventional methods through simulation and experiments, for the case of a slide table in a transient state. Our results show that for predicting thermal error the proposed thermal model is more accurate than the conventional model.

Key Words: Machine tool (공작 기계), Slide guide (슬라이드안내면), Thermal modal analysis (열 모드 해석), Thermal error modeling (열 오차 모델링), Temperature sensor placement (온도 측정점)

NOMENCLATURE

$[C_T]$: Heat capacity matrix

$[K_T]$: Heat conductivity matrix

$[Q(t)]$: Nodal thermal load vector

$[T(t)]$: Nodal temperature vector

$[\Phi_T]$: Corresponding eigenvector matrix

$[\Lambda]$: Diagonal matrix composed of all the eigenvalues

$[\Theta(t)]$: Modal temperature

C : Heat capacity

E : Young's modulus

$E(t)$: Thermal errors

h : Heat convection coefficient

k : Thermal conductivity

N : The number of temperature sensors

$P(x)$: The position of the x axis stage.

r^2 : Coefficient of determination in the regression analysis

S_i : Sum of the square of the discrepancy between measured error and mean value of whole measured errors

S_r : Sum of the squares of the discrepancy between measured error and predicted error.

t : Operating time

T_{ext} : External temperature

$T(t_0)$: Initial temperature

$T_i(t)$: Temperature measured from i -th temperature sensor

during time t

w_i : Weighting factor of i -th mode

α : Thermal expansion coefficient

β : Coefficient

λ_i : i -th eigenvalue

ρ : Density

ν : Poisson's ratio

τ_i : Time constant of i -th mode

ζ_i : Modal thermal load

1. Introduction

The importance of improving the positioning accuracy of machine tools due to the increasing demand of products that contain better quality and tight tolerances while maintaining high productivity has been recognized by both industry and academia for the past few decades. The positioning accuracy of the machine tool determines the dimensional accuracy of the machined products. One of the most important factors affecting the positioning accuracy is thermal error which accounts for about 60% of the total error.¹ In order to resolve this thermal error problem, many methods have been adopted such as structural design, heat source interception, etc. However, the most widely studied method is to predict and compensate for the thermal distortion.² The most significant step of the thermal compensation is to create a precise and robust model. Many of the methods mentioned before not only have disadvantages in terms of cost-time but also do not adequately consider the thermal-structural mechanism. For remedying these shortcomings, Zhu suggested the method applying thermal mode concept.³ This method is based on the thermal error model by finding the sensor location using the thermal modes. But there is no consideration about the procedure and quantitative evaluation standard of thermal error prediction and comparison with conventional experience-based method in transient state.

In this paper, the procedure of thermal mode-based thermal error prediction is introduced. Then temperature sensor placement of the spindle model and the slide guide with multiple heat sources were carried out using thermal modal analysis. Then the thermal error modeling using simulation and experiment in transient state were carried out. Finally evaluation and comparison with conventional

experience-based method in transient state were performed.

2. Thermal modal analysis and sensor placement

2.1 Thermal modal analysis

In order to carry out the thermal modal analysis, the finite element heat transfer problem should be calculated which requires the integration of the coupled differential equations.³

$$[C_T] \cdot [\dot{T}(t)] + [K_T] \cdot [T(t)] = [Q(t)] \quad (1)$$

The Eigen equation related with Eq. (1) is

$$[K_T][\Phi_T] = [C_T][\Phi_T][\Lambda] \quad (2)$$

Theoretically, the relationship between λ_i and its corresponding time constant τ_i is reciprocal.

$$\lambda_i = \frac{1}{\tau_i} \quad (3)$$

The modal temperature can transform to nodal temperature by multiplying eigenvector matrix.

$$[T(t)] = [\Phi_T] \cdot [\theta(t)] \quad (4)$$

Nodal thermal load vector is transformed into modal thermal load,

$$[\Phi_T]^T [Q(t)] = \xi_i \quad (5)$$

To substitute Eq. (4) into Eq. (1) then the elements of each matrix is represented by first derivative equation of a single variable as follows.

$$\dot{\theta}_i(t) + \frac{\theta_i(t)}{\tau_i} = \xi_i(t) \quad (6)$$

When step heat input is applied, the Eq. (6) can be expressed as follows:

$$\theta(t) = \xi_i \tau_i (1 - e^{-\frac{t}{\tau_i}}) \quad (7)$$

The summation of entire modal temperature data at specific point is the temperature data at the same point. Modal temperature of each mode contributes to the temperature data at specific point as much as the

weighting factor.

The variation of the thermal load of the machine tool can be approximated as the superposition of a step input. The whole process of thermal deformation of the machine tools can be expressed by the couple of the dominant thermal mode. To accomplish this task, the weighting factor of each mode is defined as follows:

$$w_i = |\xi_i \tau_i| \quad (8)$$

The magnitude of the weighting factor is to quantify the importance of each mode. Generally a small numbers of modes account for a large proportion of the total weighting factor.

2.2 Temperature sensor placement

It is desirable to place a temperature sensor in the anti-node point of the targeted Eigen mode. If the sensor is misplaced, useless information can be collected. By placing the temperature sensor near the extreme values of the dominant field, the information about the dominant thermal mode can be obtained. If there are not enough sensors, thermal modes will not express the whole thermal deformation. The more sensors that are attached, the more thermal modes are captured. Hence, the temperature prediction becomes much more accurate.

3. Thermal error modeling and its evaluation

Thermal error of machine tool is divided into position-independent thermal error such as displacement of spindle and position-dependent thermal error such as positioning error of slide table.

3.1 Thermal error modeling

The position independent thermal error formula can be given by

$$E(t) = \sum_{i=1}^N \beta_i T_i(t) \quad (9)$$

The formula of position dependent thermal error is given by

$$E(x, t) = \sum_{i=1}^N (\beta_{0i} + \beta_{1i} P(x) + \beta_{2i} P^2(x) + \dots) T_i(t) \quad (10)$$

3.2 Evaluation of the thermal error model

To evaluate the performance of the thermal error model, the coefficient of determination is used.⁴ This coefficient of determination is usually used to evaluate the regression model by quantifying the residual error of the regression line. In this paper, error is defined as a thermal expansion error.

$$r^2 = \frac{S_t - S_r}{S_t} \quad (11)$$

$$S_t = \sum (E - \bar{E})^2 \quad (12)$$

$$S_r = \sum (E - E_{predict})^2 \quad (13)$$

4. Thermal displacement of spindle

Fig. 1 shows the thermal error modeling process of spindle system. There are 3 processes: first, temperature sensor placement; second, thermal error modeling; third, the model evaluation.

In the temperature sensor placement process, making spindle geometry and setting the boundary condition are carried out in Eigen value analysis of FEA then each mode shape and weighting factor are calculated by using extracted result data. Temperature sensors are placed by applying the method which is mentioned in chapter 2.2.

In thermal error modeling process, spindle geometry and boundary condition are set in time-dependent thermal-structural analysis in FEA from 0 to 2 hours then

Fig. 1 Thermal error modeling process of spindle system

the temperature and thermal displacement data can be extracted. The model is completed after substituting result data of 1 hour into Eq. (9) for finding beta.

In evaluation process, next 1 hour displacement and predicted displacement which are calculated by substituting next 1 hour temperature data into the thermal error model are compared.

Fig. 2 and Table 2 represent a simplified spindle system and its heat source. The material properties of the spindle system are expressed in Table 1. The input heat flux, Q_1 and Q_2 , are assumed to be generated by the spindle motor and spindle end from both ends. Heat convection between the spindle surface and the atmosphere is assumed to exist.

Fig. 2 Schematic of spindle system

Table 1 Material properties of the spindle system

Properties	Value	Unit
Initial temperature	20	°C
Thermal conductivity	42.6	W/m ² ·K
Heat capacity	473	J/kg·K
Density	7850	kg/m ³
External Temperature	20	°C
Heat convection coefficient	10	W/(m ² ·K)
Young's modulus	205	GPa
Poisson's ratio	0.29	
Thermal expansion coefficient	12.2	μm/mK

Table 2 Various form of heat source

Case	Q_1 [W/m ²]	Q_2 [W/m ²]
Case 1	1000	0
Case 2	0	1000
Case 3	1000	1000
Case 4	1000	500
Case 5	1000	2000

4.1 Sensor placement through thermal modal analysis

Fig. 3 shows thermal mode shape of spindle system. 1, 2, 3, and 6th mode are selected in order of weighting factor; the sum of weighting factor of these 4 modes is 99% of the total weighting factor. As shown in Fig. 4, Based on the method described in Section 2.2, temperature sensor 1, 2, 3, and 4 are set 5, 261, 386, and 495mm, respectively.

4.2 Modeling and evaluation

Fig. 5 shows a heat flux input which is applied to the spindle system for modeling. The β_1 to β_4 values through Eq. (9) are calculated by using the relationship between the temperature data that are gathered from temperature sensors and the thermal expansion data of spindle end.

Thermal error models of 5 Cases are represented as follows.

Fig. 3 Thermal mode shape of spindle system

Fig. 4 Weighting factor distribution

$$E_1(t)=0.79T_1(t)+17.73T_2(t)-42.04T_3(t)+33.57T_4(t) \quad (14)$$

$$E_2(t)=6.69T_1(t)-0.65T_2(t)+2.62T_3(t)+0.47T_4(t) \quad (15)$$

$$E_3(t)=3.59T_1(t)-1.40T_2(t)+6.46T_3(t)-2.64T_4(t) \quad (16)$$

$$E_4(t)=3.78T_1(t)-2.95T_2(t)+10.89T_3(t)-6.15T_4(t) \quad (17)$$

$$E_5(t)=3.33T_1(t)-0.50T_2(t)+4.44T_3(t)+0.94T_4(t) \quad (18)$$

Table 3 shows the evaluation of spindle model for each case. All of the r^2 value is over 96% and most of the residual data are under $1\mu\text{m}$. Although the shapes of the thermal error models using thermal mode analysis are different, the models predict the spindle thermal expansion well. But it is found that the r^2 value and the residual data are relatively increased according to the difference of the thermal load input like case 1.

5. Positioning error of slide table

The material properties of the slide guide are indicated in Table 4. The heat flux inputs are assumed to be generated at the guide surface (Dash line), the support

Fig. 5 Heat input for modeling

Table 3 Evaluation of spindle model

Case	Average of Residual	σ of Residual	r^2
1	$1.58\mu\text{m}$	$2.46\mu\text{m}$	96.32 %
2	$0.42\mu\text{m}$	$0.46\mu\text{m}$	98.86 %
3	$-0.43\mu\text{m}$	$0.49\mu\text{m}$	99.85 %
4	$-0.50\mu\text{m}$	$0.79\mu\text{m}$	99.62 %
5	$0.00\mu\text{m}$	$0.07\mu\text{m}$	100.00 %

bearing and the motor (Dash-dot line) as shown in Fig. 7. Heat convection between the slide guide surface and the atmosphere is assumed to exist.

5.1 Sensor placement through thermal modal analysis

Fig. 6 represents thermal mode shape of the slide guide. 1, 2, 12, and 3rd mode are selected in order of weighting factor, the sum of the weighting factor of these 4 modes is 97% of the total weighting factor. 4 temperature sensors are selected by using the above mentioned method of temperature sensor placement.

Fig. 7(a) shows temperature sensor position on the slide guide by employing thermal modal analysis. Fig. 7(b) shows temperature sensor positions on the basis of

Table 4 Material properties of the slide guide

Properties	Value	Unit
Initial Temperature	20	$^{\circ}\text{C}$
Thermal conductivity(Rail)	49.8	$\text{W/m}^2\cdot\text{K}$
Heat capacity(Rail)	486	$\text{J/kg}\cdot\text{K}$
Density(Rail)	7850	kg/m^3
External Temperature	20	$^{\circ}\text{C}$
Thermal conductivity(Bed)	55.8	$\text{W/m}^2\cdot\text{K}$
Heat capacity(Bed)	565	$\text{J/kg}\cdot\text{K}$
Density(Bed)	7300	kg/m^3
Heat convection coeff.	7.3	$\text{W/m}^2\cdot\text{K}$
Heat Input Q_1 (Rail)	200	W/m^2
Heat Input Q_2 (Motor+Bearing)	150	W/m^2

Fig. 6 Thermal mode shape of the slide guide

conventional experience-based method of which propose the temperature sensor placement nearby heat source.

5.2 Modeling and evaluation

The slide table was operated at 7.2m/min for about 4 hours. The temperature was measured by thermocouples

Fig. 7 Temperature sensor position on the slide guide: (a) Using thermal modal analysis, (b) Conventional method

(a) Using thermal modal analysis

(b) Conventional experience-based method

Fig. 8 Thermal error modeling

which were attached to the above mentioned places. The positioning error of the table was measured by a laser interferometer. The temperature and positioning error was measured every 20 minutes (12 times).

The β values through Eq. (10) are calculated using the relationship between the temperature data that are gathered from thermocouples and the positioning error of the slide table.

In the Fig. 8, the dot and solid line express the modeled and measured positioning error, respectively. The solid lines are the measured positioning error and these have a linear relationship to position changes. Then Eq. (10) can be expressed as follows

$$E(x, t) = \sum_{i=1}^N (\beta_{0i} + \beta_{1i} P(x)) T_i(t) \quad (19)$$

The temperature and distance data at 20, 40, 80 and 160 minutes are used for thermal error modeling.

Based on Fig. 8(a), Eq. (20) represents thermal error models using thermal modal analysis.

$$E(x, t) = [-0.89T_1(t) + 2.80T_2(t) + 1.61T_3(t) - 3.44T_4(t)] + [0.09T_1(t) + 0.01T_2(t) - 0.08T_3(t) - 0.02T_4(t)]P(x). \quad (20)$$

Based on Fig. 8(b), thermal error models using conventional experience-based method is represented as follows.

$$E(x, t) = [0.63T_1(t) - 0.89T_2(t) + 0.38T_3(t)] + [-0.01T_2(t) + 0.01T_3(t)]P(x). \quad (21)$$

The distance data at 0, 60, 120 and 220 minutes are predicted by suggested model and conventional model, respectively.

In the Fig. 9, the dot line is predicted positioning error and the solid line express the measured positioning error.

Table 5 and 6 show the evaluation of thermal modal analysis model and conventional experience-based method model, respectively. It can be seen that all of the r^2 value and the residual data of suggested model are better than conventional model. Eventually, it is verified that the suggested model predicts well at any time.

(a) Using thermal modal analysis model

(b) Conventional experience-based method model

Fig. 9 Thermal error prediction

Table 5 Evaluation of thermal modal analysis model

	Average of Residual	σ of Residual	r^2
0min	-1.47 μm	1.73 μm	94.35 %
60min	0.00 μm	0.52 μm	98.93 %
120min	-0.00 μm	0.65 μm	97.52 %
220min	0.29 μm	0.46 μm	98.90 %
Total	-0.32 μm	1.01 μm	97.39 %

Table 6 Evaluation of conventional experience-based method model

	Average of Residual	σ of Residual	r^2
0min	3.76 μm	4.15 μm	67.47 %
60min	-0.03 μm	0.52 μm	98.92 %
120min	0.09 μm	0.66 μm	97.47 %
220min	0.45 μm	0.56 μm	98.42 %
Total	0.68 μm	2.22 μm	87.44 %

6. Conclusion

In this paper, the thermal mode-based thermal error modeling for the spindle and the slide table accompanying with multi-heat source is investigated. Temperature sensor placements of the spindle and the slide guide were performed by adopting thermal modal analysis. Also, the quantitative evaluation standards are applied. Then the comparison and evaluation with the conventional temperature sensor placement method in transient state was carried out through simulation and experiment.

As a result, it can be seen that the temperature sensor placements of suggested method are still effective regardless of multi-heat sources. Also, the thermal error models of the spindle and the slide guide by using suggested method predict well at any time. Using the suggest method, the time and cost for making thermal error model are significantly reduced.

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