On Asymptotic Properties of a Maximum Likelihood Estimator of Stochastically Ordered Distribution Function

Myongsik Oh

Abstract

Kiefer (1961) studied asymptotic behavior of empirical distribution using the law of the iterated logarithm. Robertson and Wright (1974a) discussed whether this type of result would hold for a maximum likelihood estimator of a stochastically ordered distribution function; however, we show that this cannot be achieved. We provide only a partial answer to this problem. The result is applicable to both estimation and testing problems under the restriction of stochastic ordering.

Keywords: Law of the iterated logarithm, maximum likelihood estimation, stochastically ordered distribution function.

1. Introduction

Kiefer (1961) showed that if a distribution function $F$ is absolutely continuous with respect to a Lebesgue measure, then

$$P \left[ \lim_{n \to \infty} \left( \frac{n}{\ln \ln n} \right)^{1/2} \sup_x |\hat{F}_n(x) - F(x)| = \sqrt{2} \right] = 1,$$

where $\hat{F}_n$ is an empirical distribution of $F$. Robertson and Wright (1974a) discussed about if such an iterated logarithmic results holds for a maximum likelihood estimator of distribution functions under stochastic ordering.

This paper investigates if the constrained estimator of multinomial parameter, $p_i$, under stochastic ordering has a Kiefer type asymptotic property, i.e., the asymptotic behavior of $\max_{1 \leq i \leq k} |\hat{p}_i^* - p_i|$, where $\hat{p}_i^*$ is a constrained estimator of a multinomial parameter under stochastic ordering.

In a multinomial setting with parameter $\mathbf{p} = (p_1, p_2, \ldots, p_k)$, a maximum likelihood estimator, $\hat{p}_i^*$, of $p_i$ under a certain order restriction satisfies the following properties:

$$P \left[ \lim_{n \to \infty} \left( \frac{n}{\ln \ln n} \right)^{1/2} \max_{1 \leq i \leq k} |\hat{p}_i^* - p_i| = \max_{1 \leq i \leq k} \left[ 2p_i(1 - p_i) \right]^{1/2} \right] = 1.$$

The proof of this result can be seen in Robertson et al. (1988). Part of this proof depends on the property (reduction of error) i.e.,

$$\max_{1 \leq i \leq k} |\hat{p}_i^* - p_i| \leq \max_{1 \leq i \leq k} |\hat{p}_i - p_i|.$$
See Robertson and Wright (1974b) for details.

However this type of property does not hold for stochastic ordering problem as can be seen in the following. Suppose \( p \) is observed to be \((0.35, 0.11, 0.18, 0.12, 0.24) \) and \( q \) is \((0.2, 0.2, 0.2, 0.2, 0.2) \). Then the constrained MLE of \( p \) is

\[
\frac{1}{380}(140, 44, 72, 48, 76),
\]

and hence

\[
0.16842 = \max_{1 \leq i \leq 3} |\hat{p}^* - q_i| \geq \max_{1 \leq i \leq 3} |\hat{p} - q_i| = 0.15.
\]

This means that we cannot expect that the conjecture given by Robertson and Wright (1974) will come true. However we are able to give a partial answer to this problem. This fact is quite useful in some testing problems under stochastic ordering.

2. Some Issues in Estimation and Test

In this section, we discuss the one-sample problem for a stochastic ordering between two multinomial parameters.

Now let \( p = (p_1, p_2, \ldots, p_k) \) and \( q = (q_1, q_2, \ldots, q_k) \) be two multinomial parameters. We assume that both \( p \) and \( q \) are in \( \{x \in \mathbb{R}^k : x_i > 0, \sum_{i=1}^k x_i = 1\} \) and \( q \) is known. The stochastic ordering between \( p \) and \( q \) can be expressed as

\[
\sum_{j=1}^i p_j \geq \sum_{j=1}^i q_j, \quad i = 1, 2, \ldots, k - 1,
\]

\[
\sum_{j=1}^k p_j = \sum_{j=1}^k q_j.
\]

Now let \( \hat{p} \) be the vector of relative frequencies of a sample of size \( m \) from the \( p \) population. Robertson and Wright (1981) provides the restricted MLE of \( p \) under \( H_1 \) as follows; If \( \hat{p}_i > 0, i = 1, 2, \ldots, k \), then the restricted MLE, \( \hat{p} \), of \( p \) is

\[
\hat{p} = \hat{p}_E(C|p).
\]

where \( C = \{x \in \mathbb{R}^k : x_1 \geq x_2 \geq \cdots \geq x_k\} \) and, for \( x, y \in \mathbb{R}^k \), \( xy \) denotes the vector \((x_1/y_1, x_2/y_2, \ldots, x_k/y_k)\) and \( x/y \) = \((x_1/y_1, \ldots, x_k/y_k)\). They also prove that \( P(\lim_{m \to \infty} \hat{p} = p) = 1 \) provided \( p \gg q \). If some of \( \hat{p}_i \)'s are equal to zero, the restricted MLE cannot be obtained by the above. See Lee (1987) for this case. It provides a restricted MLE of \( p \) when some of \( \hat{p}_i \)'s are zero.

Suppose \( p \gg q \). Let \( D_{pq} = \{\eta_1, \eta_2, \ldots, \eta_k\} \) with \( 0 = \eta_0 < \eta_1 < \cdots < \eta_k = k \) and

\[
p_1 + \cdots + p_i = q_1 + \cdots + q_i, \quad \text{for } i = \eta_1, \eta_2, \ldots, \eta_k,
\]

\[
p_1 + \cdots + p_i > q_1 + \cdots + q_i, \quad \text{for } i \neq \eta_1, \eta_2, \ldots, \eta_k.
\]

Note that \( D_{pq} \) is nonempty. This is quite important to explain the asymptotic behavior in the estimation and testing problem. We briefly state the application to the testing problem. Consider the likelihood ratio test procedure to test stochastic ordering against all alternatives. Let \( H_1 \) be the hypothesis associated to stochastic ordering and \( H_2 \) be all alternatives. Robertson and Wright (1981) studied the
likelihood ratio test and gave the limiting distribution of the test statistic under $H_1$. Following their notation the test rejects $H_1$ for large value of

$$ S_{12} = -2m \sum_{i=1}^{k} \hat{p}_i (\ln \hat{p}_i - \ln \hat{p}_j). \tag{2.1} $$

Then for all $t$, $\lim_{m \to \infty} P[S_{12} \geq t] = \sum_{\ell=1}^{A} P(\ell; A; \mathbf{q}^*) P[\chi^2_{\ell-1} \geq t]$, where $\mathbf{q}^* = (q_1^*, q_2^*, \ldots, q_A^*)$. The limiting null distribution depends on $p$ through $A$, the $\eta_i$’s and $\mathbf{q}^*$. To approximate the null distribution one needs to estimate $A$, $\eta_i$’s and $\mathbf{q}^*$. Since $A$ and $\mathbf{q}^*$ are determined according to $\eta_i$’s one only needs to estimate $\eta_i$’s.

**3. The Main Result**

**Theorem 1.** For each $\eta \in D_{pq}$,

$$ P \left[ \limsup_{m \to \infty} \left( \frac{m}{\ln \ln m} \right)^{1/2} \left| \sum_{j=1}^{\eta} \bar{P}_j - \sum_{j=1}^{\eta} q_j \right| \leq \sqrt{2} \right] = 1. $$

To prove theorem we need the following two lemmas. Before we mention the lemmas we briefly describe the computation of $E_w(x|C)$. For $S$, a nonempty subset of $\{1, 2, \ldots, k\}$, set

$$ Av(S) = \sum_{i \in S} w_i x_i / \sum_{i \in S} w_i. $$

Set $i_0 = 0$ and choose $i_1$ the largest positive integer $i$ that maximizes $Av((i_0 + 1, \ldots, i))$. Next choose $i_2$ the largest integer $i$ greater than $i_1$ that maximizes $Av((i_1 + 1, \ldots, i))$. Continuing this process, we obtain $0 = i_0 < i_1 < \cdots < i_k = k$ and the projection

$$ E_w(x|C) = Av\left(\{i_{j-1} + 1, \ldots, i_j\}\right), \quad \text{for } i \in \{i_{j-1} + 1, \ldots, i_j\} \text{ and } j = 1, 2, \ldots, \ell. $$

The sets $\{i_{j-1} + 1, \ldots, i_j\}$ are called the level sets. Details regarding level sets are discussed in Robertson et al. (1988).

Lemma 1 provides the general form of the level sets to compute $E_p(q/\mathbf{p}|C)$ when the sample size $m$ is sufficiently large.

**Lemma 1.** For almost all $\omega$ (in the underlying probability space) there exists an $m_0(\omega)$ such that if $m \geq m_0(\omega)$ then the level sets to compute the projection $E_p(q/\mathbf{p}|C)$ are of the form $\{\eta_1 + 1, \ldots, \eta_k\}$ with $0 \leq j < \ell \leq A$.

**Lemma 2.** Suppose $i_0 \in D_{pq}$ so that there is an $\ell_0$ such that $i_0 = \eta_{\ell_0}$ and $1 \leq \ell_0 \leq A$. Then for almost all $\omega$ (in the underlying probability space) there exists an $m_0(\omega)$ such that if $m \geq m_0(\omega)$ then

$$ \left| \sum_{j=1}^{\eta_1} \hat{p}_j - \sum_{j=1}^{\eta_2} q_j \right| \leq \max_{0 \leq \eta_1 \leq \ell_0} \left( \sum_{j=\eta_2+1}^{\eta_3} \hat{p}_j - \sum_{j=\eta_3+1}^{\eta_4} p_j \right) + \max_{0 \leq \eta_2 \leq \ell_0} \left( \sum_{j=\eta_3+1}^{\eta_4} \hat{p}_j - \sum_{j=\eta_4+1}^{\eta_5} p_j \right). $$

**Proof of Theorem 1:** First we assume that $i_0 \in D_{pq}$ so that there exists an $\ell_0$ such that $i_0 = \eta_{\ell_0}$ and $1 \leq \ell_0 \leq A$. Then by Lemma 2, there exists a set $E_1$ such that $P(E_1) = 1$ and $\omega \in E_1$ implies that there
exists an \( m_0(\omega) \) such that \( m \geq m_0(\omega) \) implies that
\[
\left| \sum_{j=1}^{l_0} \hat{p}_j - \sum_{j=1}^{l_0} q_j \right| \leq \max_{0 \leq a \leq A \leq \beta} \left( \sum_{j=j_0+1}^{\eta_j} \hat{p}_j - \sum_{j=j_0+1}^{\eta_j} p_j \right) + \max_{0 \leq a \leq \ell_0} \left( \sum_{j=j_0+1}^{l_0} \hat{p}_j - \sum_{j=j_0+1}^{l_0} p_j \right). \tag{3.1}
\]

By multiplying both sides by \( \sqrt{m / \ln m} \) and taking \( \limsup_{m \to \infty} \) on both sides of (3.1) we have
\[
\limsup_{m \to \infty} \left( \frac{m}{\ln m} \right)^{1/2} \left| \sum_{j=1}^{l_0} \hat{p}_j - \sum_{j=1}^{l_0} q_j \right| \leq \max_{0 \leq a \leq \ell_0} \limsup_{m \to \infty} \left( \frac{m}{\ln m} \right)^{1/2} \left| \sum_{j=j_0+1}^{\eta_j} \hat{p}_j - \sum_{j=j_0+1}^{\eta_j} p_j \right|
+ \max_{0 \leq a \leq \ell_0} \limsup_{m \to \infty} \left( \frac{m}{\ln m} \right)^{1/2} \left| \sum_{j=j_0+1}^{l_0} \hat{p}_j - \sum_{j=j_0+1}^{l_0} p_j \right|. \tag{3.2}
\]

The inequality is because any real sequences \([a_n]\) and \([b_n]\) \( \limsup_{n \to \infty} a_n + \limsup_{n \to \infty} b_n \) and \( \limsup_{n \to \infty} a_n \lor \limsup_{n \to \infty} b_n \), where \( \lor \) denotes the larger of the two numbers. See Section 4 of Billingsley (1986) for details.

Now by Kolmogorov’s law of the iterated logarithm, for each \( \alpha \) and \( \beta \) such that \( 0 \leq \alpha < \ell_0 \leq \beta \leq A \) and \( l_0 \), we have, with probability one,
\[
\limsup_{m \to \infty} \sqrt{\frac{m}{\ln m}} \left| \sum_{j=j_0+1}^{\eta_j} \hat{p}_j - \sum_{j=j_0+1}^{\eta_j} p_j \right| = \sqrt{2} \sum_{j=j_0+1}^{\eta_j} p_j \left( 1 - \frac{\eta_j}{\eta_j} p_j \right), \tag{3.3}
\]
\[
\limsup_{m \to \infty} \sqrt{\frac{m}{\ln m}} \left| \sum_{j=j_0+1}^{l_0} \hat{p}_j - \sum_{j=j_0+1}^{l_0} p_j \right| = \sqrt{2} \sum_{j=j_0+1}^{l_0} p_j \left( 1 - \frac{l_0}{l_0} p_j \right). \tag{3.4}
\]

We may assume that (3.3) and (3.4) are true for such \( \omega \). Hence (3.2) becomes
\[
\limsup_{m \to \infty} \left( \frac{m}{\ln m} \right)^{1/2} \left| \sum_{j=1}^{l_0} \hat{p}_j - \sum_{j=1}^{l_0} q_j \right| \leq \max_{0 \leq a \leq \ell_0} \sqrt{2} \sum_{j=j_0+1}^{\eta_j} p_j \left( 1 - \frac{\eta_j}{\eta_j} p_j \right) + \max_{0 \leq a \leq \ell_0} \sqrt{2} \sum_{j=j_0+1}^{l_0} p_j \left( 1 - \frac{l_0}{l_0} p_j \right)
\leq 2 \sqrt{\frac{1}{2}} = \sqrt{2}.
\]

The last inequality follows from the fact that \( 2p(1-p) \leq 1/2 \) for \( 0 \leq p \leq 1 \). \( \square \)

4. Remarks

For the case of two-sample problem, \( i.e., \) both \( p \) and \( q \) are unknown, we can also obtain the similar result as Theorem 1.

Using Theorem 1 of previous section we can find a strongly consistent estimator of \( D_{pq} \). An example of such estimator is
\[
D_{pq}(c) = \left\{ i \in \{1, 2, \ldots, k\} : \sum_{j=1}^{i} \hat{p}_j - \sum_{j=1}^{i} q_j \leq c \right\}.
\]
for suitable choice of \( c = (c_1, c_2, \ldots, c_k), c_1 > 0 \). Note that \( D_{pq}(c) \) is nonempty because it contains \( k \). Based on this fact we can approximate the asymptotic distribution of \( S_{12} \). Details regarding this approximation will appear elsewhere.

**Appendix: Proofs**

**Proof of Lemma 1**: By the strong law of large numbers there exists a set \( E \) such that \( P(E) = 1 \) and \( \omega \in E \) implies that there exists an \( m_0(\omega) \) and \( \epsilon > 0 \) for which

\[
\frac{q_{j+1} + \cdots + q_i}{\tilde{p}_{j+1} + \cdots + \tilde{p}_i} < 1 - \epsilon, \quad \text{(A.1)}
\]

for each \( j = 0, \ldots, A - 1 \) and \( i > \eta_j \) with \( i \neq \eta_{j+1}, \ldots, \eta_A \), and

\[
\frac{q_{\eta+1} + \cdots + q_1}{\tilde{p}_{\eta+1} + \cdots + \tilde{p}_1} > 1 - \epsilon, \quad \text{(A.2)}
\]

for each \( 0 \leq j < \ell \leq A \). By (A.3) and (A.2), for each \( j = 0, \ldots, A - 1 \) and \( \eta_j + 1 \leq i < \eta_{j+1}, \)

\[
\frac{q_{\eta+1} + \cdots + q_i}{\tilde{p}_{\eta+1} + \cdots + \tilde{p}_i} < 1 - \epsilon < \frac{q_{\eta+1} + \cdots + q_{\eta_{i+1}}}{\tilde{p}_{\eta+1} + \cdots + \tilde{p}_{\eta_{i+1}}}. \quad \text{(A.3)}
\]

By the strict Cauchy mean value function property, which is shown in Robertson and Wright (1974b), this implies

\[
\frac{q_{\eta+1} + \cdots + q_i}{\tilde{p}_{\eta+1} + \cdots + \tilde{p}_i} < \frac{q_{\eta+1} + \cdots + q_{\eta_{i+1}}}{\tilde{p}_{\eta+1} + \cdots + \tilde{p}_{\eta_{i+1}}},
\]

and

\[
\frac{q_{\eta+1} + \cdots + q_i}{\tilde{p}_{\eta+1} + \cdots + \tilde{p}_i} < 1 - \epsilon < \frac{q_{\eta+1} + \cdots + q_{\eta_{i+1}}}{\tilde{p}_{\eta+1} + \cdots + \tilde{p}_{\eta_{i+1}}}. \quad \text{(A.3)}
\]

By (A.3) and (A.1) we have

\[
\frac{q_{\eta+1} + \cdots + q_i}{\tilde{p}_{\eta+1} + \cdots + \tilde{p}_i} < 1 - \epsilon < \frac{q_{\eta+1} + \cdots + q_{\eta_{i+1}}}{\tilde{p}_{\eta+1} + \cdots + \tilde{p}_{\eta_{i+1}}},
\]

for \( \eta_j + 1 \leq i_1 < \eta_{j+1} \) and \( \eta_j \leq i_2 \leq \eta_{j+1} \). Hence in using the PAVA to compute the projection, \( q_{\eta+1}/\tilde{p}_{\eta+1}, \ldots, q_{\eta_{i+1}}/\tilde{p}_{\eta_{i+1}} \) will be pooled.

Now the projection can be obtained as follows.

\[
E_{\tilde{p}} \left( \frac{q}{\tilde{p}} \right) | C = E_{\tilde{p}^{*}} \left( \frac{q^{*}}{\tilde{p}^{*}} \right) | C^{*} \right)
\]

for \( j = 0, \ldots, A - 1 \) and \( \eta_j + 1 \leq i \leq \eta_{j+1} \), where

\[
\hat{p}^* = \left( \sum_{j=0}^{n_j} \hat{p}_j \right), \quad \text{and}
\]

\[
\frac{q^{*}}{\hat{p}^{*}} = \left( \sum_{j=0}^{n_j} q_j \right) \left( \sum_{j=0}^{n_j} \hat{p}_j \right) = \left( \sum_{j=0}^{n_j} q_j \right) \left( \sum_{j=0}^{n_j} \hat{p}_j \right),
\]

for suitable choice of \( c = (c_1, c_2, \ldots, c_k), c_1 > 0 \). Note that \( D_{pq}(c) \) is nonempty because it contains \( k \). Based on this fact we can approximate the asymptotic distribution of \( S_{12} \). Details regarding this approximation will appear elsewhere.
By Lemma 1, there exists a set \( E \) such that \( P(E) = 1 \) and \( \omega \in E \) implies that there exists an \( m_0(\omega) \) such that \( m \geq m_0(\omega) \) implies that the level sets in computing \( E_p(\mathbf{q}/\mathbf{p}|C) \) are of the form \( [\eta_j + 1, \ldots, \eta_\ell] \) with \( 0 \leq j < \ell \leq A \). This completes the proof.

**Proof of Lemma 2:** By Lemma 1, there exists a set \( E \) such that \( P(E) = 1 \) and \( \omega \in E \) implies that there exists an \( m_0(\omega) \) such that \( m \geq m_0(\omega) \) implies that the level sets in computing \( E_p(\mathbf{q}/\mathbf{p}|C) \) are of the form \( [\eta_j + 1, \ldots, \eta_\ell] \) with \( 0 \leq j < \ell \leq A \).

Now we fix \( \omega \) and \( m \geq m_0(\omega) \). Suppose the level sets for such \( \omega \) and \( m \) are \( \{\xi_\ell + 1, \ldots, \xi_{\ell+1}\} \) for \( \ell = 0, \ldots, L \leq A - 1 \) with \( \xi_0 = 0 \) and \( \xi_{\ell+1} = \eta_\ell = k \). Note that \( \{\xi_1, \ldots, \xi_{L+1}\} \subseteq [\eta_1, \eta_2, \ldots, \eta_k] \) and \( \xi \)'s depend on \( \omega \) as well as \( m \). Then for \( i = 1, 2, \ldots, k \) and \( \ell = 0, \ldots, L \)

\[
E_p\left(\frac{\mathbf{q}}{\mathbf{p}}|C\right) = \frac{\sum_{j=\xi_\ell+1}^{\xi_{\ell+1}} q_j}{\sum_{j=\xi_\ell+1}^{\xi_{\ell+1}} p_j} \quad \text{with} \quad \xi_\ell + 1 \leq i \leq \xi_{\ell+1}.
\]

Hence we have

\[
\hat{p}_i = \hat{p}_i \sum_{j=\xi_\ell+1}^{\xi_{\ell+1}} q_j / \sum_{j=\xi_\ell+1}^{\xi_{\ell+1}} \hat{p}_j \quad \text{with} \quad \xi_\ell + 1 \leq i \leq \xi_{\ell+1}.
\]

Consider a level set containing \( i_0 \). Since \( i_0 \in D_{pq} = [\eta_1, \eta_2, \ldots, \eta_k] \), we can choose an \( \ell_1 \) such that \( \xi_{\ell_1} + 1 \leq i_0 \leq \xi_{\ell_{1+1}}, \) i.e., the level set containing \( i_0 \) is \( [\xi_{\ell_1} + 1, \xi_{\ell_{1+1}}] \). Then we have

\[
\sum_{j=1}^{i_0} \hat{p}_j - \sum_{j=1}^{i_0} q_j = \sum_{\ell=0}^{\ell_{1-1}} \left( \sum_{j=\xi_\ell+1}^{\xi_{\ell+1}} \hat{p}_j - \sum_{j=\xi_\ell+1}^{\xi_{\ell+1}} q_j \right) + \sum_{\ell=0}^{\ell_1} \sum_{j=\xi_\ell+1}^{\xi_{\ell+1}} \hat{p}_j - \sum_{j=\xi_\ell+1}^{\xi_{\ell+1}} q_j
\]

\[
= \sum_{\ell=0}^{\ell_{1-1}} \left( \sum_{j=\xi_\ell+1}^{\xi_{\ell+1}} \hat{p}_j - \sum_{j=\xi_\ell+1}^{\xi_{\ell+1}} q_j \right) + \sum_{j=\xi_\ell_1+1}^{i_0} \hat{p}_j - \sum_{j=\xi_\ell_1+1}^{i_0} q_j
\]

Now we are going to find the upper bound of fluctuations when the sequence \( \{\sum_{j=1}^{i_0} \hat{p}_j\} \) converges to \( \sum_{j=1}^{i_0} q_j \) as the sample size \( m \) approaches infinity.
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Note that \( \ell_1 \) depends on \( m \) as well as \( i_0 \). This means that level set containing \( i_0 \) may change as \( m \) varies. Now we consider all possible level sets containing \( i = \eta \) for \( m \geq m_0(\omega) \). Such level sets are of form

\[
\{\eta_0 + 1, \ldots, \eta_b\}, \quad \text{for } 0 \leq \alpha < \eta_0 \leq \beta \leq A.
\]

Hence we have

\[
\sum_{j=\xi_1+1}^{\ell_1+1} \hat{\ell}_j - \sum_{j=\xi_1+1}^{\ell_1+1} p_j \leq \max_{0 \leq \alpha < \xi_0 \leq \beta \leq A} \left| \sum_{j=\xi_0+1}^{\eta_0} \hat{\ell}_j - \sum_{j=\xi_0+1}^{\eta_0} p_j \right| \quad \text{and}
\]

\[
\sum_{j=\xi_1+1}^{\ell_0} \hat{\ell}_j - \sum_{j=\xi_1+1}^{\ell_0} p_j \leq \max_{0 \leq \alpha < \xi_0 \leq \beta \leq A} \left| \sum_{j=\xi_0+1}^{\eta_0} \hat{\ell}_j - \sum_{j=\xi_0+1}^{\eta_0} p_j \right|.
\]

Then (A.4) becomes

\[
\sum_{j=1}^{\ell_0} \hat{\ell}_j - \sum_{j=1}^{\ell_0} q_j \leq \max_{0 \leq \xi_0 < \xi_0 \leq \beta \leq A} \left| \sum_{j=\xi_0+1}^{\eta_0} \hat{\ell}_j - \sum_{j=\xi_0+1}^{\eta_0} p_j \right| + \max_{0 \leq \alpha < \xi_0 \leq \beta \leq A} \left| \sum_{j=\xi_0+1}^{\eta_0} \hat{\ell}_j - \sum_{j=\xi_0+1}^{\eta_0} p_j \right|.
\]

This completes the proof. \( \square \)

### References


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