ASSESSMENT OF STABILITY MAPS FOR HEATED CHANNELS WITH SUPERCRITICAL FLUIDS VERSUS THE PREDICTIONS OF A SYSTEM CODE

WALTER AMBROSINI* and MEDHAT BESHIR SHARABI
Università di Pisa, Dipartimento di Ingegneria Meccanica Nucleare e della Produzione
Via Diotisalvi 2, 56126 Pisa, Italy
*Corresponding author. E-mail: walter.ambrosini@ing.unipi.it

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The present work is aimed at further discussing the effectiveness of dimensionless parameters recently proposed for the analysis of flow stability in heated channels with supercritical fluids. In this purpose, after presenting the main motivations for the introduction of these parameters in place of previously proposed ones, additional information on the theoretical bases and on the consequences of this development is provided.

Stability maps, generated by an in-house program adapted from a previous application to boiling channels, are also shown for different combinations of the operating parameters. The maps are obtained as contour plots of an amplification parameter obtained from numerical discretization and subsequent linearization of governing equations; as such, they provide a quantitatively clear perspective of the effect of different boundary conditions on the stability of heated channels with supercritical fluids.

In order to assess the validity of the assumptions at the basis of the in-house model, supporting calculations have been performed making use of the RELAP5/MOD3.3 computer code, detecting the values of the dimensionless parameters at the threshold for the occurrence of instability for a heated channel representative of SCWR proposed core configurations. The obtained results show reasonable agreement with the maps, supporting the applicability of the proposed scaling parameters for describing the dynamic behaviour of heated channels with supercritical fluids.

KEYWORDS: SCWR, Supercritical Fluid, Stability, Heated Channels

1. INTRODUCTION

Stability with respect to different dynamic mechanisms leading to flow oscillations is one of the aspects to be carefully addressed in the design of proposed future supercritical water nuclear reactors [1-6]. In fact, in similarity with boiling channels, heated channels with supercritical fluids are characterised by a greater fluid density at the inlet and a smaller one at the exit, as a consequence of heating; this makes the system to undergo different kinds of instabilities, as the dynamic density-wave oscillations and the static Ledinegg excursion also characterising boiling channels (see e.g., [7-8]).

The potential for unstable behaviour in heated channels with supercritical fluids was recognised in early works on this subject [9], pointing out the possibility of low-frequency oscillations and also of a periodic behaviour in the near critical and supercritical regions. More recent works have addressed the same issue, directly considering proposed plant configurations and assessing their stability by the use of one-dimensional balance equations for a single-phase fluid with variable properties [10-13]. In addition, the classical concepts and computational tools adopted in the analysis of boiling channel stability were adapted to the case of heated channels with supercritical fluids, in the aim to transfer such well assessed analytical machinery to the new space of operating conditions addressed by supercritical light water reactors.

In a recent work [14], a novel dimensionless treatment of one-dimensional balance equations for supercritical fluids was proposed, whose adoption helps in eliminating some inappropriate modelling features that previous formulations inherited by the translation of the stability concepts from the case of boiling channels to the one of supercritical fluids. In particular, the newly proposed formulations have the following characteristics:

- instead of referring to pseudo-saturation conditions,
whose definition is somehow arbitrary and unnatural for a supercritical fluid, the only addressed reference condition is the pseudo-critical one, being the single point discriminating between heavier and lighter fluids:

- the adopted formulations retain meaningful characteristics similar to the ones introduced by the classical dimensionless numbers adopted for boiling channel stability (e.g., the phase-change number and the sub-cooling number), in terms of improved definitions obtained by extrapolating the older concepts (namely, the trans-pseudocritical number and the sub-pseudocritical number);
- the new definitions have a general validity and benefit of a considerable degree of independence with respect to system pressure and to the working fluid.

In a recently published work [15], the analogies between the stability phenomena in boiling channels and heated channels with supercritical fluids were carefully analysed, on the basis of available models, showing that:

- the so-called density-wave oscillations, occurring at sufficiently large heating power and at small subcooling (with respect to saturation or pseudo-critical conditions, in the two cases respectively), have mostly the same dynamic behaviour in the two systems;
- Ledinegg instabilities may occur in both cases, but in the case of supercritical water require so large subcooling (in the previously mentioned sense) that they are practically ruled out in the reference design of SCWR channel considered in the work.

Of course, these conclusions, though supported by reasonable analytical evidence, must be anyway regarded with the caution always applicable to the information obtained by models, concerning phenomena whose real characteristics can be confirmed only by suitable experimental data.

In the mentioned previous work, the Authors performed a limited preliminary comparison of the stability maps obtained by the application of the new formulations with RELAPS/MOD3.3 [17] predictions, mainly aiming to show general coherence of the adopted dimensionless treatment with the dimensional thresholds for stability identified by the code. In the present work, this assessment is performed in a more systematic way, also discussing some interesting features of the adopted formulations in relation to the thermodynamic behaviour of the supercritical fluids.

2. DIMENSIONLESS BALANCE EQUATIONS AND LINEAR STABILITY

2.1 The Pseudo-Critical Conditions as Reference Thermodynamic State

As shown in Refs. 14 and 15, the following dimensionless definitions are introduced:

\[
\rho^* = \frac{\rho}{\rho_{pc}}, \quad h^* = \frac{\beta_{pc}}{C_{p,pc}} (h - h_{pc}) \quad w^* = \frac{w}{w_{in}} \quad i^* = \frac{i_{in}}{L}
\]

\[
z^* = \frac{z}{L}, \quad \Lambda = \frac{f L}{2 D_v}, \quad p^* = \frac{P}{\rho_{pc} w_{in}^2}, \quad G^* = \frac{\rho^* w^*}{\rho_{pc}}, \quad Fr = \frac{w_{in}^2}{g L}
\]

\[
N_{SPC} = \frac{\beta_{pc}}{C_{p,pc}} (h_{pc} - h_{in})
\]

\[
N_{TPC} = \frac{\beta_{pc}}{C_{p,pc}} \frac{\rho_{in} w_{in}}{A C_{p,pc}} = \frac{\dot{Q}_{channel}}{W_{channel}} C_{p,pc} = N_{TPC}^*/\rho_{in}^*
\]

It can be noted that these relationships make use of the pseudocritical conditions at any given pressure to introduce reference values of density, enthalpy, specific heat at constant pressure and isobaric expansion coefficients entering the definition of the dimensionless parameters. This choice represents the main change in perspective with respect to the case of boiling, where the two saturation conditions (liquid and vapour) played a fundamental role in the definition of the dimensionless parameters.

Previously obtained results in relation to these definitions can be summarised as follows:

- the above formulations allow introducing a unique functional dependence of dimensionless density versus dimensionless enthalpy that, at a very good level of approximation, is independent from the value of the (supercritical) pressure [14];
- in addition, the functions \( \rho^* (h^*) \) are very similar for different fluids, as it was shown for water, CO\(_2\) and ammonia, especially beyond the pseudocritical temperature, i.e., in the light fluid region [14];
- the above dimensionless parameters can be obtained from the ones classically introduced for boiling channels by changing the corresponding dimensional groups according to the following relationship [15]

\[
\frac{v_{fg}}{h_{fg}} \rightarrow v_{pc} \frac{\beta_{pc}}{C_{p,pc}}
\]

which clearly expresses the change in perspective from the two saturation states to the single-pseudocritical state.

These results can be considered a consequence of the corresponding state theory and of the observed continuity of the saturation and the pseudocritical lines at the critical point [15], also expressed by the analogy between the
Clausius-Clapeyron relationship and the formulation

\[
\frac{dT_{pc}}{dp} = T_{pc}V_{pc} \frac{\beta_{pc}}{C_{p,pc}},
\]  

(5)

already discussed in a similar form in [9], and can be found to hold reasonably for different fluids.

The trans-pseudocritical number, \( N_{trc} \), and the sub-pseudocritical number, \( N_{spc} \), are the key parameters in the above definitions and, in analogy with the phase-change number and the subcooling number in the case of boiling channels, represent respectively a dimensionless ratio between power and flow rate and the subcooling with respect to pseudocritical conditions.

2.2 Balance Equations in Dimensionless Form

Making use of the definitions adopted in the previous section, the mass, momentum and energy balance equations can be written in dimensionless form as follows

\[
\frac{\partial \rho^*}{\partial t^*} + \frac{\partial G^*}{\partial z^*} = 0
\]  

(6)

\[
\frac{\partial G^*}{\partial t^*} + \frac{\partial}{\partial z^*} \left( \frac{G^2}{\rho^*} \right) + \frac{\partial p^*}{\partial z^*} = 0
\]  

(7)

\[
-\frac{\rho^*}{Fr} \left[ \Lambda + K_{in} \delta^* (z^*) + K_{out} \delta^* (z^* - 1) \right] \frac{G^2}{\rho^*} = \frac{\partial p^*}{\partial z^*}
\]

(8)

in which flow work is neglected in the energy balance. The boundary conditions are:

\[
G^*_in = p^*_in \quad \text{or} \quad p^*_in = p^*_in (t^*)
\]  

(9)

\[
h^*_in = -N_{spc}
\]  

(10)

\[
p^*_out = p^*_out (t^*)
\]  

(11)

The presence of the apparent trans-pseudocritical number, defined by the last of Eqs. (3), at the RHS in the energy equation can be also noted.

The state equation adopted for \( \rho^* (h^*) \) is the same piecewise continuous formulation used in previous work [14] and devised on the basis of the general trends obtained for water. This trend was obtained by making use of the NIST property package [16] and, as shown in Figure 1, fits reasonably well the unique trend obtained at different pressures for dimensionless density as a function of dimensionless enthalpy.

2.3 Analysis of Linear Stability in Heated Channels with Supercritical Fluids

As in previous works [14-15], the above described
dimensionless balance equations are discretised in space and time by a semi-implicit numerical scheme [18] and then linearised by perturbation to determine the stability conditions for a single heated channel with imposed pressure drop. The algorithm adopted in this purpose is the same previously utilised for similar studies in boiling channels [18].

Linearisation and calculation of the spectral radius of the matrix embedding the linear dynamics of the discretised systems is used to determine an amplification factor, $Z$, whose sign allows discriminating between stable conditions (negative $Z$) and unstable ones (positive $Z$). The stability maps are conveniently set up as contour plots of $Z$, clearly highlighting the different degrees of stability in different operating regions.

In relation to the choice to discretise the balance equations in space and time, before proceeding to linearization, the following considerations can be made:

- if sufficient detail is used in discretisation, in order to minimise the effects of truncation error, this procedure represents a convenient way to avoid the relatively cumbersome developments needed to find the roots of characteristics equations involved in the classical direct linearisation of partial differential equations;
- in the case in which the numerical method adopted for discretisation is representative of a class of methods whose effect on stability prediction is the objective of the study, this procedure is able to clearly show the changes that stability maps undergo due to truncation error with different discretisation parameters.

As a matter of fact, the first applications of this linear stability methodology were aimed just at the latter of the above two objectives [19].

In previous applications [14], the friction factor was also independently specified, obtaining a value of $A=22$ used for setting up the stability maps. This value can be considered a realistic one, considering a roughness of $2.5 \times 10^{-4}$ m and the specified hydraulic diameter but, after noting the slight discrepancies between the maps obtained by this value with RELAP5 data, $A$ will be taken equal to 14, in order to get a basis for a closer comparison in relation to other effects. This change in the value of $A$ must be considered just as a convenient way to correct uncertainties coming from different sources in this comparison among codes, postponing a more careful examination of their nature to further analyses.

In similarity with previous work, the addressed physical system is a single channel with imposed external pressure drop. In both the RELAP5 and the simplified model, 48 nodes are used to discretise the channel. The nodalization used in this purpose is schematically reported in Figure 2. In the case of the simplified model, a maximum Courant number of 0.9 has been assumed in the calculations; in RELAP5 analyses, a maximum time-step of 0.1 s was used and the usual Courant limitation is active in the code, as required by the adopted semi-implicit numerical scheme. Negligible thermal inertia and resistance is assumed in the heating rods, to simulate as close as possible imposed heat flux conditions.

In order to get the stability boundaries for different operating conditions by RELAP5, the following procedure was used, similar to the one already adopted in Ref. 14 for supercritical water and in Ref. 18 for boiling water:

- the channel has initially no heating and uniform pressure (i.e., no flow);
- pressure drop is firstly increased during a short time period.

3. ADDRESSED STABILITY PROBLEM AND METHODOLOGY OF ANALYSIS

As in the previous work [14], the parameters taken as reference for the study were freely inspired to those of a typical subchannel as considered in literature analyses [11-13]. In particular, the subchannel has the following nominal characteristics:

- fuel rod diameter: 10.2 mm;
- lattice pitch: 11.2 mm;
- rod-to-wall distance: 1 mm;
- flow area: $5.49 \times 10^{-4}$ m$^2$;
- hydraulic diameter: 3.4 mm;
- channel length: 4.2672 m (14 ft);
- system pressure: 25 MPa;
- inlet temperature: 280 °C;
- individual rod coolant flow: 0.055 kg/s;
- linear power: 25 kW/m;
- inlet orifice pressure loss coefficient: 20; outlet pressure loss coefficient: 1.

![Fig. 2. Nodalization of the Heated Channel for RELAP5.](image)
and subsequently kept constant;
- heating of the channel is then started at different rates, initially larger and smaller later on, in order to reduce the calculation times, still approaching unstable conditions in a reasonably quasi-static transient conditions;
- the calculation is continued with increasing power up to 3000 s, unless the highly unstable conditions encountered cause its stop because of numerical reasons.

Analyses are performed for different constant values of subcooling with respect to pseudocritical conditions at each operating pressure; a FORTRAN program is used to automatically prepare the several input decks at different subcoolings starting from a base input deck for each addressed condition.

Postprocessing of the obtained transient evolutions has the purpose to estimate the value of the trans-pseudocritical number, \( \text{N}_{\text{TPC}} \), at which oscillations firstly occur at a given sub-pseudocritical number, \( \text{N}_{\text{PSC}} \). In this aim, an algorithm was set up and checked for accuracy, in order to detect the conditions at which the onset of density-wave oscillations occurs, thus calculating the values of the relevant dimensionless parameters at that stage. The algorithm is presently incapable of detecting excursive instabilities; therefore, its use is here limited to the density-wave region of the stability maps.

4. ASSESSMENT OF THE MAPS

Data from different calculations performed by RELAP5 are compared in Figure 3 to Figure 9 with the stability maps obtained by the simplified model. In comparing the results, attention must be paid to the different definitions of the inlet and outlet pressure drop coefficients that in the model must be nearly halved with respect to the values adopted in the code and include also acceleration effects due to contraction and expansion at the inlet and the outlet.

Figure 3 compares the stability boundaries obtained by RELAP5 at the nominal pressure of 25 MPa and uniform power profile with the map obtained for the corresponding conditions and, in particular, for \( \text{Fr} = 22 \). The observed discrepancy between the two marginal stability loci represents the overall degree of approximation of the present adopted simplified model with respect to the system code predictions.

On the other hand, in Figure 4, whose map was obtained with \( \text{Fr} = 14 \), an excellent match of the marginal stability line is observed with the stability boundaries obtained by RELAP5 at three different pressures for uniform heating. It must be observed that the nearly perfect agreement with the data at different pressures testifies for the effectiveness of the proposed dimensionless numbers in providing information on stability mostly independent of the operating pressure.

The effect of axial power distribution can be observed in Figure 5, where the stability map and the RELAP5 results are reported for a sinusoidal power profile. Though the match among RELAP5 data at different pressures is less accurate than in the case of uniform heating, the considerable stabilising effect introduced by the sinusoidal profile, especially at low subcooling, is predicted nearly in the same way by the system code and the simplified model.

A limiting case of the effect of the Froude number is
Fig. 4. Comparison Between RELAP5 Stability Boundaries for the Nominal Conditions at 25, 30 and 40 MPa and the Stability Map by the Simplified Model with 48 Nodes, \( \Delta=14, \) Fr=0.030, \( K_i=10.5, K_{out}=0, C_{in}=0.9 \)

Fig. 5. Comparison Between RELAP5 Stability Boundaries for Sinusoidal Power Axial Distribution at 25, 30 and 40 MPa and the Stability Map by the Simplified Model with 48 Nodes, \( \Delta=14, \) Fr=0.030, \( K_i=10.5, K_{out}=0, C_{in}=0.9 \)

reported in Figure 6, where results of RELAP5 for vertical and horizontal channels are compared with each other and with a stability map obtained for Fr = 10^4 (note that for a perfectly horizontal channel the Froude number should be infinite). As already noted in the past for boiling channels, this dimensionless group has mostly negligible effects on stability against density wave oscillations in this range of operating parameters, being mainly responsible of a change in the location of the Ledinegg instability region (upper lobe in the map). In this connection, it must be noted that the nominal value of 0.03 adopted in most the maps for the Froude number represents a rough reference value, since Fr varies with \( N_\text{SPC} \) and \( N_\text{TPC} \) and should not be assumed constant.

Figure 7 and Figure 8 allow discussing the results of increasing the inlet and outlet pressure drop coefficients,
Fig. 6. Comparison Between RELAP5 Stability Boundaries for Vertical and Horizontal Channel at 25 MPa and the Stability Map by the Simplified Model with 48 Nodes, $A=14$, $Fr=105$, $K_{in}=10.5$, $K_{out}=0$, $C_{max}=0.9$

Fig. 7. Comparison Between RELAP5 Stability Boundaries for Nominal and Increased Inlet Pressure Drop ($K_{in}=30$) at 25 MPa and the Stability Map by the Simplified Model with 48 Nodes, $A=14$, $Fr=0.03$, $K_{in}=15.5$, $K_{out}=0$, $C_{max}=0.9$

showing the expected stabilising and destabilising effects respectively; the agreement between RELAP5 stability boundaries and the marginal stability line from the simplified model is somehow better in the case of larger inlet than larger outlet pressure losses. Finally, Figure 9 shows the effect of doubling the hydraulic diameter of the channel, which mainly affects the parameter $A$, whose consequent decrease results in a map showing greater stability, consistently with RELAP5 data.

5. CONCLUSIONS

The results obtained in this work support the correctness of the assumptions made in proposing dimensionless parameters for discussing the stability in heated channels with supercritical fluids. In particular, a good agreement was found between the dimensionless predictions of the simplified model and the data obtained in dimensional terms by RELAP5 for a realistic channel.
This model-to-model comparison was performed in the belief that the system code and the simplified model, though completely independent from each other except for some basic modelling assumptions, contain enough information about the physics of the addressed phenomena to grasp the overall stability behaviour. Needless to say, a more interesting and meaningful assessment should involve a comparison of model predictions with experiments, which are presently rather scarce or lacking (see e.g., [20-21]), to be performed whenever sufficient data will be made available for a thorough validation.
NOMENCLATURE

Roman Letters

$A$ cross section area [m$^2$]
$C_{\text{max}}$ maximum Courant number, i.e. maximum of $w \Delta t / \Delta x$
$C_p$ specific heat at constant pressure [J/(kgK)]
$D_h$ hydraulic diameter [m]
$f$ friction factor
$f(z)$ normalized distribution of heat flux
$Fr$ Froude number
$g$ gravity [m/s$^2$]
$G$ junction mass flux [kg/(m$^2$s)]
$h$ fluid specific enthalpy [J/kg]
$j$ volumetric flux [m$^3$/s]
$K_{\text{in}}$ localized pressure drop coefficient at the channel inlet
$K_{\text{out}}$ localized pressure drop coefficient at the channel outlet
$L$ channel length [m]
$N_{\text{pec}}$ phase change number
$N_{\text{sub}}$ subcooling number
$N_{\text{spc}}$ sub-pseudo-critical number
$N_{\text{trc}}$ apparent trans-pseudo-critical number
$N_{\text{rpc}}$ true trans-pseudo-critical numbers
$p$ pressure [Pa]
$q'''$ channel power [W]
$q''$ heat flux [W/m$^2$]
$t$ time [s]
$v$ specific volume [m$^3$/kg]
$w$ velocity [m/s]
$\dot{W}_{\text{channel}}$ channel flow rate [kg/s]
$z$ axial coordinate along the channel [m]
$Z_r$ real part of a complex exponent

Greek Letters

$\beta$ isobaric thermal expansion coefficient [K$^{-1}$]
$\delta_1$ dimensional Dirac delta function [m$^{-1}$]
$\delta^r$ dimensionless Dirac delta function
$\Delta$ friction dimensionless group (Euler number)
$\Pi_h$ heated perimeter [m]
$\rho$ density [kg/m$^3$]
$\Delta t'$ dimensionless time step
$\Delta z'$ dimensionless spatial increment

Subscripts

$f$ saturated liquid
$g$ saturated vapour (steam)
$fg$ difference between saturated vapour and saturated liquid
$in$ inlet
$out$ outlet
$p$ constant pressure
$pc$ pseudo-critical
$0$ reference value

Superscripts

* starred variables indicate dimensionless value

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