Performance of All-Optical Multihop RoFSO Communication System over Gamma-Gamma Atmospheric Turbulence Channels

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(Received May 7, 2015 : revised August 3, 2015 : accepted August 3, 2015)

In this paper, we analyze the performance of the all-optical multihop radio over a free space optical (RoFSO) communication system with amplify-and-forward (AF) relays under varying weather conditions. The proposed channel model considers the propagation loss, attenuation and atmospheric fading modeled by the Gamma-Gamma (GG) distribution. Both the amplified spontaneous emission (ASE) noise in the all-optical relays and the background noise projected onto receiver apertures have been considered in the analysis. The lower bound analytical expressions for the end-to-end bit error rate (BER) and outage probability are derived for the multihop system employing the all-optical relays with the full channel state information (CSI). Meanwhile, the exact results for BER and outage probability are obtained via Monte Carlo simulation. Results indicate the performance of the proposed system will be improved by the multihop transmission technology. For a fixed number of relays, the BER and outage probability will be increased with the deterioration of the weather conditions.

Keywords: All-optical relays, Radio over free-space optical communication, Atmospheric channel, Amplified spontaneous emission noise

OCIS codes: (010.1330) Atmospheric turbulence; (200.2605) Free-space optical communication; (060.1155) All-optical networks; (060.4510) Optical communications

I. INTRODUCTION

Recently, transmission of radio frequency (RF) signals over free space optical (FSO) link, which is also referred to as radio over FSO, has gained a lot of research attention. It has been proposed as one of the solutions for last-mile applications [1, 2]. Compared with the RF system, FSO is less affected by snow and rain, but can be severely affected by atmospheric turbulence and fog. RoFSO communication systems can transfer radio signals with high transmission capacity, unlicensed spectrum, ease of deployment, low cost and high security [3]. However, the performance of RoFSO can be degraded by various atmospheric conditions such as fog, snow, smoke, and turbulence, which result in attenuation and intensity fluctuation of the laser beam at the receiver side. Experimental results showed that dense fog can cause an extinction coefficient of up to 270 dB/km [4]. Additionally, the optical power attenuation due to thick fog, snow and haze is wavelength independent [5]. To achieve the over 99.9% availability requirements of the telecom industry in a dense foggy environment, the FSO link is limited to about 500 m, where sufficient link margin is available [6]. For longer transmission link range under dense fog, a complementary RF link at lower data rate can be used to back up the FSO link.

In clear atmosphere with a typical attenuation coefficient of 0.43 dB/km, an FSO link of over 1 km is achievable. However, the major challenge for FSO in clear atmosphere is the turbulence induced irradiance fluctuation, especially for the link range beyond 1 km [7]. The irradiance fluctuation, also referred as scintillation, is caused by the inhomogeneities in the temperature and pressure of the atmosphere along the propagation path, which causes random variations of refractive-index [8]. Scintillation could result in deep signal fades that lasts for ~ 1-100 µs [9]. For example, a link operating at 1 Gbps could result in a loss of up to 10^7 consecutive bits. To fully understand and predict FSO performance, a number of models have been developed to describe the statistical distribution of atmospheric turbulence. The most
widely reported is the lognormal channel model, which is mathematically convenient and tractable [7]. For longer link range where multiple scatterings exist, the incident wave becomes increasingly incoherent and the lognormal model becomes invalid. The GG turbulence model is based on the assumption that the irradiance fluctuation of a light beam propagating through the turbulent atmosphere consists of small scale (scattering) and large scale (refraction) effects [7]. The GG statistic distribution of irradiance fluctuation is valid for all turbulence scenarios from weak to strong.

To mitigate the weather effects, a number of schemes have been proposed, such as coding techniques [10, 11], adaptive techniques [12-14] and space diversity techniques [8, 15, 16]. Additionally, multihop FSO systems employing both amplify-and-forward (AF) and decode-and-forward (DF) relaying schemes have been studied [17]. Unlike in RF communications, the small-scale fading statistics of FSO communications are distance dependent. Therefore, the multihop technique not only improves the path-loss compared to direct transmission, but also mitigates the small-scale fading statistics [18]. Most of the existing work reported multihop FSO systems employing electrical amplification, where the received optical signal at each relay was converted into an electrical signal by a photodetector, amplified and then converted back into an optical signal by a laser [17, 19]. In [17], the outage probability of multihop FSO link using AF and DF relaying was investigated in lognormal fading, where the noise at the relays and the destination is dominated by the shot noise modeled as an additive white Gaussian noise (AWGN) model. The closed-form expressions for the outage probability and the average BER of binary modulation schemes for multihop FSO employing AF relaying under a GG turbulence channel have been presented [19].

The main objective of this paper is to use all-optical AF relays employing erbium-doped fiber amplifiers (EDFAs) for amplification, which avoids the complexities associated with optical-to-electrical (OE) and electrical-to-optical (EO) conversion. EDFAs are a mature technology, which has a wide-spread use in optical fiber communications as a preamplifier or an in-line amplifier in the 1550 nm range [18]. The application of EDFAs in FSO AF relays has been recently proposed in [20-22]. However, results in [22] only included the amplified spontaneous emission (ASE) noise and shot noise caused by background radiation without considering the weather conditions in the FSO system. In this paper, we consider an all-optical multihop RoFSO communication system with AF relays, particularly the CSI technique. The lower bounds for BER and outage probability have been derived considering the ASE noise and the background noise at each set of relays under different channel conditions. The proposed channel model takes into account the propagation loss, atmospheric attenuation and turbulence induced fading. The GG statistic distribution has been used to describe the atmospheric turbulence.

The rest of this paper is organized as follows. In Section II, the all-optical multihop RoFSO communication system is introduced. The end-to-end SNR and lower bound expressions for BER and outage probability of the multihop FSO system are derived in Section III. Results discussions are presented in Section IV. Finally, some conclusions are given in Section V.

II. SYSTEM AND CHANNEL MODEL

2.1. System Model

We consider an all-optical multihop RoFSO communication system employing the IM/DD, as shown in Fig. 1 [17]. The source node S transmits the optical signal modulated by RF signals to the destination node D via N-1 all-optical relays in series. So, let L_{0}, i = 1,..., N and h_{i}(t), i = 1,..., N be the distance and channel gain between R_{i-1} and R_{i} where R_{0} and R_{N} represent the source node S and the destination node D. The received optical signals are filtered and amplified by relays employing the EDFAs without OE or EO converters, as shown in Fig. 2 [21]. Therefore, the background noise and ASE noise are filtered by the optical filter. The amplification gain of the relays is variable by assuming the full CSI is available.

In this paper, we assume the noise sources at relays are mainly the ASE noise and the background radiation. The background noise projected onto the ith receiver aperture is expressed as n_{d}(t), i = 1,..., N. We assume G_{i} and n_{i}(t) for i = 1,..., N-1 to be the amplification gain and the ASE noise of the ith relay.

In this work, we assume the source node S transmits the optical signal modulated by s(t) where s(t) is binary phase shift keying (BPSK) signal. So, the received light intensity of the relay R_{i} can be expressed as I_{i}(t) = I_{0}h_{i}(t)(1+\xi s(t))^{2}+n_{d}(t) where I_{0} represents the intensity of optical signal transmitted by the source node S and \xi is the modulation index [8]. In order to ensure that the optical transmitter operates within its dynamic range and avoids over-modulation induced clipping, the condition |\xi s(t)| \leq 1 must be satisfied. For the ith (i = 2,..., N) relay, the received light intensity is expressed as I_{i}(t) = h_{i}(t)(G_{i}I_{i-1}(t) + n_{d}(t)) + n_{d}(t). Here, the intensity of optical signal at the destination node D can be written as:

FIG. 1. An all-optical multihop RoFSO communication system with AF relays.

FIG. 2. Block diagram of an all-optical variable gain relay.
the same distance L. The atmospheric attenuation is given as:

\[ h_p = e^{-\alpha L^2}, \]  

where \( \alpha \) is the atmospheric attenuation coefficient. Here,

\[ \alpha = \frac{3.91}{V(Km)} \left( \frac{\lambda}{550nm} \right)^q \]

where \( V \) is the visibility, \( \lambda \) is the wavelength of the optical signal and the exponent \( q \) is given by

\[
q= \begin{cases} 
1.6 & V > 50 \\
1.3 & 6 < V < 50 \\
0.16V^2+0.34 & 1 < V < 6 . \\
V-0.5 & 0.5 < V < 1 \\
0 & V < 0.5 
\end{cases}
\]

The fading \( h_n(t) \) caused by atmospheric turbulence is a random variable which can be described by the GG distribution. Its probability density function (PDF) can be expressed as [7]:

\[
p(h_n) = \frac{2(\alpha \beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} h_n^{\alpha-1} \cdot \frac{(\alpha+\beta)}{2\sqrt{\pi \alpha \beta h_n}} \cdot K_{\alpha-\beta}(2\sqrt{\alpha \beta h_n}), \quad h_n > 0
\]

where \( \Gamma(\cdot) \) is the Gamma function, \( K_{\cdot}(\cdot) \) is the modified Bessel function of the second kind of order \( n \). Assuming plane wave propagation and considering the impact of the aperture-average, \( \alpha \) and \( \beta \) are given by

\[
\alpha = \left[ \exp \left( \frac{0.49\sigma_{k}^2}{(1+0.65d^2+1.11\sigma_{k}^2)^{5/6}} \right) \right]^{-1}, \\
\beta = \left[ \exp \left( \frac{0.51\sigma_{k}^2(1+0.69\sigma_{k}^{12/5})^{5/6}}{(1+0.90d^2+0.62d^2\sigma_{k}^{12/5})^{5/6}} \right) \right]^{-1}.
\]

Here, \( d = \sqrt{k D_{T}^2 / 4L_p} \) where \( L_p \) is the propagation path length. \( \sigma_{k}^2 = 1.23C_n^2 k^2 L_p^{11/6} \) is the Rytov variance where \( C_n^2(m^{-2}) \) is the refractive index structure parameter, \( k = 2\pi/\lambda \) is the optical wave number and \( \lambda \) is the wavelength of the optical signal.

### III. PERFORMANCE ANALYSIS

#### 3.1. End-to-end SNR

The BPSK modulation scheme will be considered in this
paper. The BPSK signal can be transferred without changing its format by the RoFSO system, which avoids the adaptive threshold required by the OOK-modulated FSO systems [23]. The end-to-end electrical SNR before the RF demodulator according Eq. (3) can be expressed as:

$$\text{SNR}_e = \frac{\prod_{i=1}^{N} G_i^2 h_i^2(t) \frac{I_{b0}^2}{2}}{\sum_{i=1}^{N} P_{ni}(t) \prod_{j=i+1}^{N} G_j^2 h_j^2(t) + \prod_{i=1}^{N} \prod_{j=i+1}^{N} G_j^2 h_j^2(t)}, \quad (12)$$

where $P_{ni}(t)$ and $P_{bi}$ represent the power of the ASE noise and the background noise at the $i$th relay, respectively. $A$ is the amplitude of the BPSK signal $s(t)$. $P_n$ can be expressed as [24]:

$$P_{ni}(t) = h \alpha_0 n_{sp} B_0 (G_1^2 - 1), \quad (13)$$

where $\hbar$ is the Planck’s constant, $\alpha_0$ is the center frequency of the optical signal, $n_{sp}$ is the ASE parameter and $B_0$ is the bandwidth of the optical filter. In this paper, we can assume that $P_{bi} = N_i B_{in}$, $i = 1, \ldots, N$, where $N_i$ is the power density of the background noise radiation which follows an AWGN distribution.

Assuming the maximum output power $P_i$ of each relay is the same, we can get

$$G_i^2 P_{ni} + P_{bi} = P_i, i = 1, \ldots, N - 1, \quad (14)$$

where $P_{ni} = h^2 \delta(t) P_t + P_{bi}$ is the power of the received optical signal at the $i$th relay. Note that when the channel goes through a deep fading, the gain of the relay will increase. Meanwhile the background noise will be amplified, which deteriorates the performance of the system. Therefore, the instantaneous amplification gain can be written as:

$$G_i^2(t) = \frac{P_i + h \alpha_0 n_{sp} B_0}{P_i \langle h_i(t) \rangle + N_i B_0 + h \alpha_0 n_{sp} B_0}. \quad (15)$$

As the CSI changes slowly compared to the transmitted RF signal, the amplification gain is slowly variable.

In a practical FSO system, the power density of the background radiation can be $N_0 = 2 \times 10^{12}$W/Hz and the ASE parameter can be $n_{sp} = 5$ [22]. Compared to the background noise, the ASE noise can be neglected in Eq. (12). Here, we can assume the transmission power of the source node $S$ is the same as the power of relays. So the Eq. (12) can be simplified as:

$$\text{SNR}_e = \left\{ \prod_{i=1}^{N} \left( \text{SNR}_n \langle h_i(t) \rangle^2 \right) \right\}^{-1}, \quad (16)$$

where $\text{SNR}_n = \frac{P_t}{N_0 B_0}$ represents the SNR in the ideal channel when no atmospheric fading and attenuation exist.

### 3.2. BER Performance and Outage Probability

In this section, we derive the lower bound expression for BER and outage probability of the all-optical multihop RoFSO communication system. Since the PDF of the end-to-end SNR expression given by Eq. (16) is difficult to derive, the upper bound for Eq. (16) can be derived using the similar approach as in [19]. Using the inequality

$$\text{SNR}_e \leq \text{SNR}_o = \frac{\prod_{i=1}^{N} (\text{SNR}_t(t))^2)^{1/N}}, \quad (17)$$

where the equality holds when $h_i(t) = h_2(t) = \cdots = h_6(t)$.

Assuming $Y = \frac{1}{N} \prod_{i=1}^{N} (\text{SNR}_t(t))^2$, the PDF of $Y$ is given by [19]

$$f_Y(y) = \frac{2}{N} \frac{y^{N-4}}{N^{N-1}} \prod_{i=1}^{N} G_{b,2N}^{2N,0} \left[ \left( \prod_{j=1}^{N} \alpha_j \beta_j \right) (Ny)^{2N} \right]^{-1} \left( \text{c}_{2N} \right)_{1/2}, \quad (18)$$

where $\alpha_i$ and $\beta_i$ are the distribution parameters at the $i$th relay, $G$ is the Meijer G-function. In Eq. (18), $\Xi_i$ and $\text{c}_{2N}$ are given by

$$\Xi_i = \frac{(\alpha_i \beta_i)^{\frac{\alpha_i + \beta_i}{2}}}{\Gamma(\alpha_i) \Gamma(\beta_i)}, \quad (19)$$

$$\text{c}_{2N} = [\alpha_N - \frac{\alpha_N + \beta_N}{2}, \beta_N - \frac{\alpha_N + \beta_N}{2}, \cdots, \alpha_i - \frac{\alpha_i + \beta_i}{2}, \beta_i - \frac{\alpha_i + \beta_i}{2}], \quad (20)$$

From Eq. (17), the lower bound for BER of the all-optical multihop RoFSO communication system using BPSK signaling format can be given by

$$P_{\text{eb}} = \int_0^{\text{SNR}_o} f_Y(y) \langle N \rangle dy. \quad (21)$$

The outage probability is another important metric to evaluate the performance of multihop FSO system in fading channels. The instantaneous BER will significantly rise at a certain moment because of the deep fading for a
communication system with an adequate average BER. The outage probability is defined as the probability that the instantaneous SNR of the system falls below the threshold SNR. The outage probability is defined as [25]:

$$ P_{out} = \Pr(\text{SNR}_e < \text{SNR}_{th}), $$  \hspace{1cm} (22)

where SNR$_e$ is the threshold SNR for a given BER. Using the PDF given in Eq. (18), the lower bound of outage probability is expressed as:

$$ P_{outb} = \int_0^{y_{th}} f_Y(y|N) dy, $$  \hspace{1cm} (23)

where $y_{th} = \text{SNR}_{th} / \text{SNR}_0$.

### IV. RESULTS AND DISCUSSION

In this section, we present the simulation results for the BER performance and outage probability of the all-optical multihop RoFSO communication system in GG fading channel. The results are obtained via Monte Carlo simulations based on Eq. (16). The lower bound curves for BER and outage probability are plotted based on Eq. (21) and Eq. (23). In this paper, we assume the multihop AF RoFSO system with $\lambda = 1550$ nm operating under various weather conditions and visibilities. The simulation parameters of the proposed RoFSO system are shown in Table 1. The exact results and lower bounds for the BER and outage probability against SNR$_0$ with $N-1$ relays are shown in Fig. 3 and Fig. 4. In Fig. 3 and Fig. 4, we set the visibility $V$ to be 20 km for clear weather. The direct transmission link is also considered with $N = 1$ for comparison purposes. As shown in Fig. 3 and Fig. 4, the difference between the exact results and the lower bounds becomes larger with the increments of SNR$_0$.

In Fig. 5 the exact end-to-end BER is plotted against SNR$_0$ using different numbers of relays under various visibilities. In this paper, we assume the visibility $V$ to be 20 km, 2 km and 0.6 km for clear weather, light fog and moderate fog respectively. As clearly seen from Fig. 5, the BER performance of the system improves as the number of relays increases. However, the BER performance will

### TABLE 1. System parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>optical wavelength</td>
<td>1550 nm</td>
</tr>
<tr>
<td>$d_{sd}$</td>
<td>source-destination link range</td>
<td>3000 m</td>
</tr>
<tr>
<td>$D_{TX}$</td>
<td>diameter of the transmitter aperture</td>
<td>10 cm</td>
</tr>
<tr>
<td>$D_{RX}$</td>
<td>diameter of the receiver aperture</td>
<td>10 cm</td>
</tr>
<tr>
<td>$\theta$</td>
<td>beam divergence</td>
<td>0.5 mrad</td>
</tr>
<tr>
<td>$C_n^2$</td>
<td>refractive index structure parameter</td>
<td>$1 \times 10^{-13}$ m$^{-2/3}$</td>
</tr>
</tbody>
</table>
be deteriorated due to low visibility. Particularly, for a BER of $10^{-5}$, we observe performance improvements of $-6.5$ dB, $-11.1$ dB and $-14.3$ dB for $N=2, 3$ and $4$ with respect to the direct transmission under the visibility $V=20$. Additional power gain can be achieved by using extra relays. The BER performances using different numbers of relays under various weather conditions are shown in Table 2. For a fixed number of $N$, the BER performance deteriorates as the visibility becomes worse. This is because the average received optical power is affected by the weather conditions, such as atmospheric attenuation.

The exact end-to-end outage probability of the all-optical multihop RoFSO system is plotted against SNR with $N-1$ relays under various visibilities in Fig. 6. In Fig. 6, SNR is $12.6$ dB is required to achieve an average BER of $10^{-5}$. The performances of outage probability of the system improve as the number of relays or the visibility is increased in Fig. 6. For an outage probability of $10^{-5}$, we observe performance improvements of $-7.1$ dB, $-14.6$ dB and $-19.1$ dB for $N = 2, 3$ and $4$ with respect to the direct transmission under clear weather. The performances of outage probability using different number of relays under various visibility are shown in Table 2. For a fixed value of $N$, the performances of outage probability will be increased as visibility becomes worse.

**TABLE 2. Improvements (dB) at BER and outage probability of $10^{-5}$**

<table>
<thead>
<tr>
<th>Performance</th>
<th>Weather conditions</th>
<th>Number of relays</th>
</tr>
</thead>
<tbody>
<tr>
<td>BER</td>
<td>clear</td>
<td>1   2   3</td>
</tr>
<tr>
<td></td>
<td>light fog</td>
<td>6.5 11.1 14.3</td>
</tr>
<tr>
<td></td>
<td>moderate fog</td>
<td>11.8 19.3 23.1</td>
</tr>
<tr>
<td>Outage probability</td>
<td>clear</td>
<td>38.2 60.4 71.5</td>
</tr>
<tr>
<td></td>
<td>light fog</td>
<td>13.3 23.3 28.8</td>
</tr>
<tr>
<td></td>
<td>moderate fog</td>
<td>36.4 58.2 77.4</td>
</tr>
</tbody>
</table>

**FIG. 6. Outage probability of all-optical multihop RoFSO system with $N$-1 relays under different visibilities.**

**V. CONCLUSION**

In this paper, we have studied the performances of the all-optical multihop RoFSO system using AF relays. Both the BER performance and outage probability have been analyzed using different numbers of relays under various weather conditions. The proposed system employs all-optical AF relays which eliminate the OE/EO conversions and have a variable gain with the full-CSI. Results demonstrate the performance of the proposed system will be improved by using additional relays. The BER performance improves $-6.5$ dB, $-11.1$ dB and $-14.3$ dB for $N = 2, 3$ and $4$ with respect to the direct transmission under clear weather for a BER of $10^{-5}$. For a fixed value of $N$, the BER and outage probability will be reduced with higher values of visibility. The novel system model proposed in this paper can provide a guideline so that the optimum number of relays can be chosen to achieve the optimal performance when the visibility is known.

**REFERENCES**