Dispersion constraints and the Hilbert transform for electromagnetic system response validation

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Abstract. As a check on calibration and drift in each discrete sub-system of a commercial frequency-domain airborne electromagnetic system, we aim to use causality constraints alone to predict in-phase from wide-band quadrature data. There are several possible applications of the prediction of in-phase response from quadrature data including: (1) quality control on base level drift, calibration and phase checks; (2) prediction and validation of noise levels in in-phase from quadrature measurements and vice versa and in future; and (3) interpolation and extrapolation of sparsely sampled data enforcing causality and better frequency-domain–time-domain transformations. In practice, using tests on both synthetic and measured Resolve helicopter-borne electromagnetic frequency domain data, in-phase data points could be predicted using a scaled Hilbert transform with a standard deviation between 40 and 80 ppm. However, relative differences between base levels between flight could be resolved to better than 1 ppm, which allows an independent quality control check on the accuracy of drift corrections.

Key words: calibration, dispersion, drift, electromagnetic, HEM, Hilbert transform, Kramers–Kronig.

Introduction

Frequency domain airborne electromagnetic (AEM) systems consist of several independently calibrated sub-systems, each operating at a single frequency (Valleau, 2000). The frequency domain helicopter AEM systems are commonly known as helicopter-borne electromagnetic (HEM) systems. It is well known that imperfect calibration of separate HEM sub-systems leads to incompatibilities between data and forward models (Fitterman, 1998), although calibration methods have improved considerably since Fitterman’s analysis. The incompatibilities between data and models are inferred to be calibration errors, and prior to the development of inversion methods were addressed through transformation to apparent resistivity, with levelling processes that distribute systematic errors to apparent depths or make adjustments in base levels so that resistivity-depth sections seem reasonable. Brodie and Sambridge (2006) and Deszcz-Pan et al. (1998) have presented methods to adjust calibration parameters so that data are consistent with external conductivity constraints from water sampling or boreholes. Brodie and Sambridge also allow for slow drifts and bias in holistic inversion procedures. When no earth constraints are available, Ley-Cooper and Macnae (2007) showed that a stochastic approach to calibration could be taken using the average error in layered earth-fitting to ‘similar’ data clusters, assuming that the geology (earth conductivity) varied but that calibration error was constant. In each of these methods, calibration errors are identified through comparison with a (piecewise) 1D earth conductivity model, even though the earth may have a 3D conductivity structure. Two research examples of the use of wire loop conductors in HEM calibration are given by Yin and Hodges (2009) and Davis and Macnae (2008), and their methodologies have seen regular subsequent use in field calibration. The aim of this paper to look at calibration constraints that are independent of any earth-model.

In an HEM system, sensor coils measure the rate of change of the EM magnetic component, and either a bucking transmitter or receiver is set up so that the large primary field is essentially nulled before A/D conversion and recording. Geometrical and electronic changes with temperature lead to changes in the effectiveness of the bucking, affecting mainly the in-phase component (Valleau, 2000). These drifts are corrected to first order using the results of high-altitude excursions, but correction is never perfect, with zero level drifts in the tens to few hundred ppm. The main contribution to these drifts are distortions in the relative geometry of the bucking coil/sensor that are sub-millimetre. Secondary fields are typically less than 4000 ppm of the primary field. Quadrature data are more stable than in-phase data, being independent of bucking effectiveness to a first order approximation.

While electromagnetic noise may be stochastic, controlled-source geophysical signals are inherently causal. For any causal response there is no output before excitation. Causal responses were first mathematically characterised by Kramers (1927) and independently by Kronig (1926) in optical dispersion applications. The Kramers–Kronig (KK) relationship implies that the in-phase component \( R(\omega) \) of a causal response at frequency \( \omega_0 \) can be predicted from a complete knowledge of the quadrature component \( Q(\omega) \) at all angular frequencies \( \omega \) through the ‘KK’ integral

\[
R(\omega_0) = \frac{2C}{\pi} \int_{0}^{\infty} \frac{\omega Q(\omega)d\omega}{(\omega^2 - \omega_0^2)},
\]

where \( C \) is a constant (the Cauchy principal value), obtainable analytically by contour integration (King, 2009). There are several forms of the KK equation, and we have chosen to use a form suitable for physical measurements with the infinite integral taken over the positive frequency range only. In practice, HEM and other frequency domain geophysical data
do not cover the range from zero to infinite frequency, but consist of a small number of discrete frequencies, spaced quasi-logarithmically. The question arises: can the KK integral be evaluated numerically on such sparse data, and can the result be used to predict inconsistencies in EM system calibration?

**Method**

The desired physical property detected by AEM is nominally the earth resistivity (conductivity). Experimental measurements sample the response function of the conductive materials within the earth when excited by a transmitter. The data collected includes two receiver measurements per frequency, the in-phase and quadrature components of the field.

Our first tests of using the KK relationship to predict in-phase from quadrature data were done on synthetic EM data calculated over a layered earth with the program LEMEM, a layered-earth modelling code written by Terry Robb for AMIRA project P407b based on the formulation by Fitterman and Yin (2004). Figure 1 presents the in-phase and quadrature response over an example two-layer model calculated at five frequencies per decade between 10 Hz and 1 MHz. Note that the KK relationship implicitly defines the in-phase $R(\omega)$ as the complement (with respect to the inductive limit) of the usual geophysical definition of in-phase response $P(\omega)$, so that $P(\infty) - R(0) = 0$, and $P(0) - R(\infty)$ which is the inductive limit (West and Macnae, 1991).

We first attempted to predict the in-phase data plotted in Figure 1 from the quadrature data using standard Gauss–Kronrod quadrature integration, using the quadk routine in MATLAB. This attempt was unsatisfactory due to the strength of the singularity in equation 1 at $\omega = \omega_0$. To address this numerical instability we rearranged the KK equation into the form of a Hilbert transform, specifically:

$$R(\omega_0) = \frac{2C}{\pi} \int_0^\omega \frac{Q(\omega)(1 + \omega_0/\omega)^{-1}}{(\omega - \omega_0)} d\omega. \quad (2)$$

$$= 2H(u). \quad (3)$$

where $u = Q(\omega)(1 + \omega_0/\omega)^{-1}$ is the kernel of a standard Hilbert transform $H(u)$. Complex analysis and the relationship between Hilbert and Fourier transforms are established mathematical procedures, and the interested reader is referred to King (2009). A Fourier transform equivalent (e.g. Macnae, 1984) can be used to evaluate equation 2 in the form:

$$R(\omega_0) = \Im[-j \text{sgn}(\omega) F(u)], \quad (4)$$

where $F$, $\Im$ are the Forward and Inverse Fourier transforms respectively, $j$ is the unit imaginary number (square root of $-1$) and sgn is the signum (sign) operator.

To calculate the in-phase response using fast discrete Fourier transform (FFT) algorithms, it is first necessary to interpolate the quadrature data to linear frequency spacing, including extrapolation of the responses to zero frequency and at a substantially higher frequency than collected in order to be ‘close to infinity’. This interpolation is best done using smooth splines in log-space (Boerner and West, 1984). Both $Q$ and $R$ at infinity are necessarily zero, but a discrete FFT is limited to a finite upper frequency. It is worth noting that the Hilbert kernel ($u$) is

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**Fig. 1.** Coplanar HEM system response over a two layer (10 m thick sea-ice) model. The in-phase component $P(\omega)$ as plotted is the complement $R(\omega) = R(\omega)$ of the Kramers–Kronig relationship.

**Fig. 2.** Successful prediction of in-phase data in the 100 Hz to 100 kHz range within 30 ppm using only the quadrature data as input, with causality enforced through the Kramers–Kronig relationship.

**Fig. 3.** Successful prediction of in-phase data within 80 ppm using only two quadrature data per decade as input.
weighted by a term dependent on $\omega_0$, which implies that separate FFT and inverse FFT pairs need to be performed for each $\omega_0$ to calculate the Hilbert transform. The algorithm therefore makes the prediction one frequency at a time. Results of the prediction are compared with the forward calculation in Figure 2, and are seen to have a prediction error of up to 30 ppm using five frequency quadrature data per decade as input.

The next step was to determine if a successful prediction could be made using fewer, sparsely spaced frequencies. Figure 3 shows that nine data points, roughly two per decade between 10 Hz and 1 MHz are indeed sufficient for a reasonably successful prediction, with prediction errors within 80 ppm. These input data are a subset of the data presented in Figure 2. Errors of this magnitude were typical over the limited range of layered earth models tested.

Results

The next step was to apply the algorithm to data obtained from field measurements. We used six frequency Resolve (Yin and Hodges, 2009) data from a large survey over the Chowilla flood plains in South Australia, surveyed in 2005. This system has five coplanar and one coaxial coil-pairs, collecting data at six different frequencies. Data had the usual calibration and levelling processes applied to it by Fugro, the contractor. We then used the methods of Ley-Cooper and Macnae (2007) to make an overall recalibration of the entire dataset, and will call the output the ‘recalibrated data’. The expectation of this recalibration process was that residual phase and gain errors would be reduced in the recalibrated data. As part of this processing step, data from the coaxial pair operating at 3242 Hz were scaled to equivalent coplanar amplitudes. The six recalibrated quadrature data at each fiducial were then used to predict the in-phase.

The results from our prediction at a frequency of 8177 Hz using the Hilbert transform (2) and the recalibrated in-phase data are shown in Figure 4, with little difference evident on the plot with its colour bar between 0 and 4500 ppm. The relatively

![Fig. 4. 8177 Hz in-phase Resolve data (a) recalibrated and (b) as predicted from recalibrated quadrature data using equation 2.](image)

![Fig. 5. The difference in ppm between the delivered and the predicted in-phase data at the six measured frequencies. Systematic differences occur between flights, with the outlines of flights 15, 16 and 17 shown.](image)
A conductive flood-plan is bounded by the Murray river to the south and hills to the north and east. Differences between the images are enhanced if the prediction is subtracted from the recalibrated data before plotting. A set of difference plots are presented in Figure 5 for all six frequencies. The coaxial data has been scaled by a factor of 3.2 to account for the difference in geometry (orientation alone leads to a factor of 4 and the remainder is due to a coil spacing change compared to the coplanar pairs). The difference plot shows spatially consistent patterns: individual flights can be easily identified through consistent level-shifts, with the areas covered by flights 15, 16 and 17 outlined.

**Fig. 6.** Histograms of the differences (in 5 ppm cells) between recalibrated and KK predicted in-phase results at the six frequencies of the Resolve survey.

**Fig. 7.** Scatterplot of the causality-predicted amplitude (vertical scale) against the recalibrated amplitude (horizontal axis). Three separate frequencies are shown on offset horizontal axes, specifically (a) the highest, (b) an intermediate and (c) the lowest frequency. Note that the differences do not appear to be proportional to amplitude in that the envelope does not expand towards higher amplitudes. The colourbar is the number of samples in $10 \times 10$ ppm cells.

**Fig. 8.** Mean difference between prediction and in-phase data as a function of flight number and frequency. Flights 1–14 and 19 were outside the Chowilla area.
models, we are not confident that the KK transform predicts in-phase results well enough to use as a ‘true’ value for zero-level drift estimation. However, with similar geology as expected in the flood-plan area, systematic differences between flights are evident. The mean difference for the 13 flights and six frequencies is imaged in Figure 8, and inspection of Figure 5 shows that it has provided an estimate of the relative differences between flights. These differences could be taken to be estimates of the average of residual zero level drift, after conventional drift correction, for each flight. The best agreement between the empirical and numerical data are at the five lowest frequencies (Figures 5–8). Prediction errors at the highest frequency (Figure 8) are larger than at the mid frequencies. Data at the highest frequency shows horizontal bands coincident with the spatial boundaries between flights, each of which has independent base-level and drift estimates in contractor processing.

It appears that the differences between recalibrated and predicted data are not clearly proportional to amplitude, and thus optimally expressed as a percentage error. Figure 7 shows that at the highest frequency data (group a), the error distribution is most spread out at mid frequencies. At the lowest frequency data (group c), the differences appear to be additive, in that they do not reduce at small amplitudes.

Discussion

Because of systematic errors in prediction using synthetic models, we are not confident that the KK transform predicts in-phase results well enough to use as a ‘true’ value for zero-level drift estimation. However, with similar geology as expected in the flood-plan area, systematic differences between flights are evident. The mean difference for the 13 flights and six frequencies is imaged in Figure 8, and inspection of Figure 5 shows that it has provided an estimate of the relative differences between flights. These differences could be taken to be estimates of the average of residual zero level drift, after conventional drift correction, for each flight. The best agreement between the empirical and numerical data are at the five lowest frequencies (Figures 5–8). Prediction errors at the highest frequency (Figure 8) are larger than at the mid frequencies. Data at the highest frequency shows horizontal bands coincident with flight boundaries, either indicating inconsistencies in zero-level and amplitude adjustments between flights or possibly problems with numerical extrapolation for the KK algorithm.

Figure 9 presents a summary scatterplot of the differences at all frequencies as a function of amplitude, which shows that the prediction error is not clearly correlated with data amplitude, and that the standard deviation of the prediction averages to 63 ppm. There may be some small systematic effects seen in Figure 9, in a subtle rising trend of the most common difference towards high frequency. Using standard statistics on the differences, the standard error of the mean difference value for each frequency and flight is however less than 1 ppm. We would suggest that, rather than using this method to provide an additional correction to data, it could be used as a quality control estimate on levelling and drift errors. In our opinion, the method of Brodie and Sambridge (2006) is better suited to drift corrections than our KK prediction.

There are several possible applications of the prediction of in-phase response from quadrature data including: (1) quality control on base level, calibration and phase checks; (2) prediction and validation of noise levels in in-phase from quadrature measurements, and vice versa; and, although not detailed here, (3) interpolation and extrapolation of sparsely sampled data enforcing causality and better frequency-domain – time-domain transformations. This last application could be used, for example, to speed up forward and inverse modelling without loss of accuracy, and will be the subject of further investigation.

Conclusions

Causality as mathematically enshrined in the KK relationship can be exploited to predict in-phase from quadrature data for any causal system, of which frequency domain EM and induced polarisation responses are geophysical topics of interest. In practice, using recalibrated Resolve HEM data, in-phase data points can be predicted with an error of zero mean and a standard deviation of less than 40–50 ppm at most frequencies, and ~80 ppm at the highest frequency. In similar geology, these systematic differences are stable, and averages during each flight of the difference are determined to better than 1 ppm, which is stable enough to identify the presence of base level drift correction errors in HEM data. This detection is best used for quality control rather than data correction.

References

電磁探査システムの応答検証のための分散関係およびヒルベルト変換

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要旨：本文では、商用の周波数領域エリアーポン電磁探査システムのキャリブレーションおよびドリフト誤差のチェックとして、因果関係制約の下で広帯域の微分成分データから同相成分の予測を試みた。微分成分データから同相成分データを予測するには、以下のいくつかの手法がある：（1）ベースレベルドリフト誤差、キャリブレーションおよび位相をチェックし、その性状より予測する。（2）同相成分の観測データから同相成分の予測およびその検証、また将来的には、逆に同相成分の観測データから同相成分の予測およびその検証により予測する。（3）因果関係およびより良い時系列から周波数領域への変換を適用し、観のサンプリングされたデータを補間・外挿して予測する。実験では、数値モデルデータおよびFugro社のヘルコプターポン EMシステム Resolve®で計測された周波数領域電磁気データを用いたテストにより、同相成分データを規格化ヒルベルト変換によって予測した。その予測結果は、標準偏差40-80 ppmの範囲であった。しかしながら、各フライスベースレベル間の相対誤差は1 ppm以下の解像度があるので、ドリフト誤差補正の精度をスクリーンで独立に性状検査するのに十分である。

キーワード：キャリブレーション、分散、ドリフト誤差、電磁気、ヘルコプターポン EM、ヒルベルト変換。

克拉マス・クロニュッヒの関係式

変位機タマシステム反応のタダゼの確率を推定形の分散関数とヒルベルト変換

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要約：数値的なソーパス映り水平調査システムのキャリブレーションと補正にための検証方法により、線形的な影響の制限を考慮する。状態変数の回復を求める方法を次に示す：(1) 基線図変換、保択、位相の検討への変換関数、(2) 状態変数回復に伴う状態変数の回復変換および検証と、(3) 状態変数との間の変換を考慮する。変換を行うシステムは、変換された変換から、時系列情報が与えられる。実験の結果、ヒルベルト変換とから40 ppmの変換点の有する変換が適用された。したがって、変換の基準を有する一定の変換点の変換は1 ppmよりはるかに正確であるが、これはキャリブレーション補正の終点で、ドリフト計測を必要とする。

主なキーワード：保択、分散、キャリブレーション、HCM、HILBERT変換、Kramers-Kronig

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