Multi-level approach for parametric roll analysis

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ABSTRACT: The present study considers multi-level approach for the analysis of parametric roll phenomena. Three kinds of computation method, GM variation, impulse response function (IRF), and Rankine panel method, are applied for the multi-level approach. IRF and Rankine panel method are based on the weakly nonlinear formulation which includes nonlinear Froude-Krylov and restoring forces. In the computation result of parametric roll occurrence test in regular waves, IRF and Rankine panel method show similar tendency. Although the GM variation approach predicts the occurrence of parametric roll at twice roll natural frequency, its frequency criteria shows a little difference. Nonlinear roll motion in bichromatic wave is also considered in this study. To prove the unstable roll motion in bichromatic waves, theoretical and numerical approaches are applied. The occurrence of parametric roll is theoretically examined by introducing the quasi-periodic Mathieu equation. Instability criteria are well predicted from stability analysis in theoretical approach. From the Fourier analysis, it has been verified that difference-frequency effects create the unstable roll motion. The occurrence of unstable roll motion in bichromatic wave is also observed in the experiment.

KEY WORDS: Parametric roll; Motion stability; Difference-frequency effect; GM variation; Impulse response function; Rankine panel method.

INTRODUCTION

Roll motion of ships is one of the most crucial factors affecting structural loads, dynamic motion stability, and passenger comfort, so that it has been of great interest. Since modern commercial ships have become faster and larger over recent decades, the nonlinearity of roll motion becomes an essential element of large ship design. Many researchers have proved the possibility of the occurrence of very large roll angle in head or following waves when the wave encountering frequency is twice roll natural frequency. This nonlinear phenomenon, the so-called parametric roll, is of great concern for modern large ships. The mathematical definition of ‘parametric’ indicates self-excitation or parametric excitation which can be used in the Mathieu equation.

The analysis of parametric roll requires considering the actual wetted ship surface in motion analysis, since the temporal variation of restoring force is a crucial factor of the occurrence of parametric roll motion. To this end, earlier studies included the second-order nonlinear restoring component in the equation of motion. The harmonic variation of metacentric height, GM, leads the classical equation of motion to the Mathieu equation. Pauling and Rosenberg (1959), and Nayfeh (1988) solved the Mathieu equation, and showed the possibility of a large roll angle in head or following waves. Dunwoody (1989) used the spectral form of the GM variation to consider the wetted ship surface more accurately. In the case of two-dimensional analysis, Tanizawa and Naito (1998) introduced an excellent study by using not only nonlinear numerical method but also experimental approach.

In computational approach, many of time-domain simulation code for nonlinear motion analysis are applicable for parametric roll prediction. Three-dimensional panel methods, which have been widely used in ship motion analysis in recent years, make it possible to consider the effect of nonlinear restoring force in more accurate manner. In this case, the nonlinear motion analysis should be carried out in the time domain. France, et al. (2003) and Shin et al. (2004) applied Rankine panel method in parametric roll analysis, and showed favorable results. However, the three-dimensional panel method is still somewhat expensive in terms of computation time. The impulse response function (IRF) approach formulated by Cummins (1962) can be a candidate to compromise the accuracy and efficiency of numerical computation. This approach solves the equation of ship motion by using pre-computed hydrodynamic coefficients. Similar to the three-dimensional panel method, the IRF approach can also consider the nonlinear restoring force on an instantaneous wetted surface (Ballard et al.,

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2003). Using this advantage, Spanos and Papanikolaou (2007) have applied the IRF approach in the parametric roll analysis of a fishing vessel.

In the present study, multi-level approaches using GM variation, IRF and Rankine panel method are applied for the analysis of parametric roll. Single degree of roll motion including harmonic variation of restoring force is solved by numerical computation in the GM-variation approach. In the case of Rankine panel method, a high-order B-spline function is applied to approximate physical parameters such as the velocity potential and wave elevation. For the computation of retardation function which is necessary for the IRF approach, a set of hydrodynamic coefficients is obtained by using a frequency-domain computational program which is based on a three-dimensional Green function method. Both the IRF and Rankine panel method apply a weakly nonlinear method which solves the linear free-surface boundary condition but includes the nonlinear Froude-Krylov and restoring forces in the equation of motion. Therefore, this method combines the linear and nonlinear components of excitation force, and the nonlinear components are mostly from ship geometry. This scheme is popular in recent seakeeping analysis for ships and offshore structures, and is sometimes called as ‘blended method’. In the parametric-roll occurrence test in regular waves, the IRF and Rankine panel methods show similar results. Although the GM-variation approach predicts the occurrence of parametric roll at the twice that of roll natural frequency, its frequency criteria show a little difference from others.

During the present numerical analysis, large roll motion is observed in the bichromatic waves. To analyze more theoretically on this motion, it is assumed that the restoring coefficient have two harmonic variation components. Then this assumption leads the equation of motion to a form of the quasi-periodic Mathieu equation. For the quasi-periodic Mathieu equation, mathematical and numerical approaches have been introduced by Zounes and Rand (1998) to achieve the solution and the same stability with the same amplitude of fluctuation in two components. Further improvements are achieved in different amplitude of fluctuation (Rand et al., 2003), in damped and nonlinear restoring models (Guennoun et al., 2001; Abouhazim et al., 2005). Based on such mathematical background, stability diagram of roll motion in bichromatic waves is developed. From the observation of such stability diagram, unexpected instability region is found in which it does not correspond to that of the Mathieu equation for single component. If this mechanism causes unstable roll motion, the roll motion in irregular waves should be carefully observed, particularly when the peak of the second-order wave spectrum is near the frequency which is that of roll natural frequency. In irregular waves, the problem becomes more complicated because both the sum- and difference-frequency components can be involved. To validate the present instability, an experimental study has been also carried out, and the occurrence of large roll motion has been found when the difference frequency of two waves is that of roll natural frequency. This indicates that the study of unstable roll needs to be expanded to include wave interference effects.

### MATHEMATICAL BACKGROUND

#### Multi-Level Approach: GM Variation

Let us consider a ship moving with a steady speed \( \bar{U} \), as shown in Fig. 1. The occurrence of parametric roll can be easily predicted by using the resonance analysis which includes the second-order property of wetted surface variation. The simple approximation of metacentric height in single harmonic wave involves adding a small periodic variation to the metacentric height in calm water value, \( \overline{GM} \), as written in Eq. 1.

![Fig. 1 Coordinates and notations.](image)

\[
\overline{GM} = \overline{GM}_0 + \overline{GM}_a \cos(\omega_t)
\]  

(1)

where \( \overline{GM}_0 \) is mean value and \( \overline{GM}_a = \left( \overline{GM}_{\text{Max}} - \overline{GM}_{\text{Min}} \right)/2 \). \( \omega_t \) denotes encounter frequency. Single-degree equation of roll motion can be written as follows including the variation of metacentric height:

\[
\ddot{x}_4 + 2\delta \dot{x}_4 + (\omega_n^2 + \omega_a^2 \cos(\omega_t))x_4 = F_4(t)
\]  

(2)

where \( x_4 \) is roll motion, and \( \delta \) is damping coefficient normalized by the sum of mass and added mass moment of inertia. \( F_4 \) means the normalized exciting roll moment, which becomes zero at head and following wave. \( \omega_n \) is the roll natural frequency as represented in Eq. 3. Here, \( \Delta, I_{44} \) and \( I_{a,44} \) are ship mass, the mass moment of inertia and the added mass moment of inertia, respectively. \( \omega_a \) corresponds to fluctuation component.

\[
\omega_n^2 = \frac{\Delta \overline{GM}_0}{I_{44} + I_{a,44}} \quad \text{and} \quad \omega_a^2 = \frac{\Delta \overline{GM}_a}{I_{44} + I_{a,44}}
\]  

(3)
Eq. 2 is the second-order linear differential equation, and its solution is easily obtained by numerical integration. Fig. 2 shows two example solutions of the equation with certain coefficients. These solutions show bounded and unbounded behaviors according to its coefficients.

![Fig. 2 Bounded and unbounded solution of Mathieu equation.](image)

Theoretical approach is also useful for proving the roll stability. For simple estimation, homogenous and undamped equation is considered.

\[
\ddot{\xi} + (\omega_n^2 + \omega_o^2 \cos(\omega t)) \dot{\xi} = 0
\]

(Nondimensional time scale parameter is introduced for normalizing the equation.

\[\tau = \omega t\]

Substituting Eq. 5 to Eq. 4 leads to classical Mathieu equation.

\[
\frac{d^2x}{dt^2} + \left[p + q \cos(\tau)\right]x = 0
\]

where \(p = \omega_n^2 / \omega_o^2\) and \(q = \omega_n^2 / \omega_o^2\). Analytic approach presents the type of solution as a diagram with respect to \(p\) and \(q\). After two values are determined, easy estimation of stability is possible at an initial stage.

**Multi-Level Approach: Impulse Response Function**

This approach is basically the conversion of the frequency-domain solution into the time domain. Particularly, in this study, the conversion is limited to radiation force, and the excitation force includes the nonlinear Froude-Krylov force and restoring force on an instantaneous wetted surface as well as linear diffraction force. The wetted surface in the present computation is defined as the hull surface wetted by the body motion and incident wave (see Fig. 3).

![Fig. 3 Definition of the wetted body surface.](image)

When the frequency-domain solution is known, the radiation force, \(F_{\text{Rad}}(t)\), can be calculated from the convolution integration of retardation function, \(R(t)\), as follows:

\[
F_{\text{Rad}}(t) = -M_\infty \ddot{\xi}(t) - \int_0^t R(t-\tau) \dot{\xi}(\tau)d\tau
\]

(The infinite-frequency added mass, \(M_\infty\), and the retardation function can be obtained from pre-calculated hydrodynamic coefficients, as shown in Eqs. 8 and 9. Either added mass or damping coefficient can be used to obtain the retardation function. In the present study, the retardation function is obtained by using the damping coefficient.

\[
M_\infty = M(\omega) + \frac{1}{\omega^2} \int_0^\infty R(t)\sin(\omega t) dt
\]

\[
R(t) = \frac{2}{\pi} \int_0^\infty b(\omega) \cos(\omega t) d\omega
\]

Including the nonlinear Froude-Krylov and restoring forces, the equation of motion based on the IRF formulation can be written as follows:

\[
(M + M_\infty) \ddot{\xi} + \int_0^t R(t-\tau) \dot{\xi}(\tau)d\tau = (F_{\text{FK}})_{\text{nonlinear}} + (F_{\text{Diff}})_{\text{external}} + F_{\text{viscous}} + F_{\text{external}}
\]

where the force terms consist of Froude-Krylov, restoring, diffraction, viscous, and external forces. The diffraction force can be converted from the frequency-domain solution, and viscous force is added to the roll excitation component. In this study, an equivalent linear damping mechanism is applied. The external force includes the soft spring mechanism for non-restoring motions, but no such external force is needed for roll motion. Eq. 10 can be solved by using a multi-step predictor-corrector method.

The amplitude of roll angle is sensitive to the viscous effect. In the actual physical problem, the viscous damping force is proportional to the quadratic of roll angular velocity, but the concept of an equivalent linear damping is popular for the ship motion analysis, e.g. Himeno (1981). The equivalent damping coefficient is defined as follows:

\[
F_{\text{viscous}} = -b_{\text{equiv.viscous}} \ddot{\xi}
\]

\[
b_{\text{equiv.viscous}} = b + \frac{8}{3\pi} \omega \phi \beta_{\text{equiv.viscous}}
\]

\(\phi\) and \(\omega\) are the amplitude of roll motion and wave frequency, respectively. In addition, \(b\) is the wave damping coefficient. The choice of the viscous damping coefficient would not be easy, because it is dependent on body shape, ship speed, wave frequency and so on. For easy numerical implementation, the equivalent linear viscous damping force defined in Eq. 13 is adopted in this study. Here, \(\gamma\) means
the ratio with respect to the critical damping coefficient, and
is generally in the range of 0.05–0.1. C refers to the restoring
coefficient of the considered motion, i.e. roll in this case.

\[
B_{eq_H \_viscous} = b + 2\gamma \sqrt{(M + M_a)C}
\]

The IRF approach requires a set of pre-computed
hydrodynamic coefficients; however, when these are given, it
has a strong advantage of fast computational time. This merit
is important particularly for long-time prediction or many
simulation cases.

**Multi-Level Approach: 3D Rankine Panel Method**

In the present study, a three-dimensional Rankine panel
method is also applied to observe nonlinear roll motions. Recently, Kim et al. (2008) introduced a new computer
program based on a time-domain Rankine panel method,
called WISH, under the support of several large shipbuilding
companies. This program is used for the simulation of
nonlinear ship motions in waves.

In this method, the total velocity potential is decomposed
into three components as follows:

\[
\phi(x,t) = \Phi(x,t) + \phi_i(x,t) + \phi_d(x,t)
\]

where \( \Phi \), \( \phi_i \), \( \phi_d \) are the basis flow, incident wave, and
disturbed wave velocity potentials, respectively. In the
present weakly-nonlinear approach, the disturbed component
of wave and velocity potential is assumed to be small. Then,
the kinematic, dynamic free surface boundary conditions and
body boundary condition can then be linearized as follows:

\[
\begin{align*}
\frac{\partial \zeta_d}{\partial t} &- (\nabla \Phi) \cdot \nabla \zeta_d \\
&= \frac{\partial^2 \Phi}{\partial z^2} \zeta_d + \frac{\partial \phi_i}{\partial z} + (\nabla \Phi) \cdot \nabla \zeta_i \quad \text{on } z = 0 \\
\frac{\partial \phi_i}{\partial t} &- (\nabla \Phi) \cdot \nabla \phi_i = - \frac{\partial \Phi}{\partial t} - g \zeta_d \left[ \nabla \cdot \nabla \Phi - \frac{1}{2} \nabla \Phi \cdot \nabla \Phi \right] \\
&+ (\nabla \Phi) \cdot \nabla \phi_i \quad \text{on } z = 0
\end{align*}
\]

\[
\frac{\partial \Phi}{\partial n} = \vec{U} \cdot \vec{n} \quad \text{on } S_h
\]

\[
\frac{\partial \phi_i}{\partial n} = \sum_{j=1}^{l} \left( \frac{\partial \xi_j}{\partial t} n_j + \xi_j m_j \right) - \frac{\partial \phi_i}{\partial n} \quad \text{on } S_h
\]

\[
(n, n_z, n_k) = \vec{n}, \quad (m, m_z, m_k) = \vec{m} \times \vec{n}
\]

\[
\begin{align*}
(n, n_z, n_k) &= \vec{n}, \quad (m, m_z, m_k) = \vec{m} \times \vec{n} \\
(m, m_z, m_k) &= \vec{m} \times (\vec{U} - \nabla \Phi)
\end{align*}
\]

where the subscripts, \( l \) and \( d \), refer to the incident wave and
disturbance components, respectively. In addition, \( \vec{n} \)
indicates the normal velocity on the body surface. The m-
terms in Eq. 18, \( m_j \), are hard to compute, since these require
the second-order differentials of the basis flow. In this
computation, the second-order differentials are converted to
the first-order differentials by using Stoke’s theorem, as used
by Nakos (1990). The present study adopts the bi-quadratic
B-spline basis function for physical variables, so that the
variables can be written as follows:

\[
\left[ \begin{array}{c}
\phi_d \\
\vec{\xi}_d
\end{array} \right] (x,t) = \sum_j \left[ \begin{array}{c}
\phi_d \\
\vec{\xi}_d
\end{array} \right] (t) B_j (x)
\]

\[
\phi_d + \int \phi_i \frac{\partial G}{\partial n} dS - \int \frac{\partial \phi_i}{\partial n} dS = \int \frac{\partial \phi_i}{\partial n} G dS - \int \phi_i \frac{\partial G}{\partial n} dS
\]

In time-marching, the instantaneous wave elevation can
be obtained by the time integration of Eq. 15, and the
velocity potential on a free surface can be obtained from Eq.
16. The hydrodynamic forces due to radiation and diffraction
can be obtained by direct integration of pressure on the hull
surface. Similarly to the IRF approach, the nonlinear Froude-
Krylov and restoring forces are obtained by taking into
account the actual wetted hull surface.

**Difference Frequency Induced Parametric Roll**

Following similar perturbation manner in regular waves,
equation of roll motion can be written with two component
harmonic variation.

\[
\ddot{\xi}_4 + 2\omega_4 \xi_4 + \left( \omega_4^2 + \omega_5^2 \cos(\omega_5 t + \alpha) \right) = F_4
\]

where \( \bar{\omega}_n \) is mean value and \( \omega_4 \) and \( \omega_5 \) are fluctuation
components similarly as defined in Eq. 3. Homogenous and
undamped equation is considered for the simple estimation of
solution. Phase difference, \( \alpha \), is ignored because it does not
affect the stability.

\[
\ddot{\xi}_4 + \left( \omega_4^2 + \omega_5^2 \cos(\omega_5 t + \alpha) \right) = 0
\]

Non-dimensional time scale parameter is introduced to one
frequency component for normalizing the equation.

\[
\tau = \omega_4 t
\]
Substitution of Eq. 23 to 22 leads the quasi-periodic Mathieu equation.

\[
\frac{d^2x}{dt^2} + \left[ v + \varepsilon \cos(\tau) + \mu \cos(1 + \Lambda) \right] x = 0
\]  

(24)

where \( v = \frac{\omega_2^2}{\omega_n^2} \), \( \varepsilon = \frac{\omega_2^2}{\omega_1^2} \), \( \mu = \frac{\omega_3^2}{\omega_2^2} \) and \( \Lambda = \left( \omega_2 - \omega_1 \right) \omega_n^2/\omega_2^2 \). Perturbation and numerical approach are introduced by Rand et al. (2003). Two examples of stability diagram are plotted in Fig. 4 by using numerical integration. The results are similar to those of Rand et al. (2003).

Instability zone is possible even in head or following wave because the quasi-periodic Mathieu equation is homogeneous, physically without excitation force. Therefore wave frequencies in certain criterion, where each single wave component gives stable solution, can be another source of parametric roll.

NUMERICAL RESULTS

Ship Model

A large containership is considered for numerical computation, since this type of ship has a higher possibility of parametric roll in ocean waves than other types of ship. Fig. 5 shows nonlinear panel model and Table 1 shows the principal dimensions of the ship.

![Fig. 5 Panel model of 6500 TEU container ship.](image)

Table 1 Principal dimensions of 6500 TEU container ship.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LBP (m)</td>
<td>286.3</td>
</tr>
<tr>
<td>Beam (m)</td>
<td>40.3</td>
</tr>
<tr>
<td>Draft (m)</td>
<td>13.127</td>
</tr>
<tr>
<td>( \overline{GM} ) (m)</td>
<td>1.14</td>
</tr>
<tr>
<td>Displacement (m³)</td>
<td>92,952</td>
</tr>
<tr>
<td>Natural frequency ( (\omega_n = (L/g)^{1/2}) )</td>
<td>1.10</td>
</tr>
</tbody>
</table>

Free Decay

Roll motion is sensitive to viscosity as well as wave damping. For the fair comparison, damping should be adjusted to three approaches. Actual experimental results would give exact damping value, but available result is hard to find. Therefore, damping is modified as each numerical result shows same free decay result. Based on the damping value of general ship, viscous damping is determined as 5% of critical damping in Rankine panel method. Other two methods are fitted to the result of Rankine panel method. Although calculation of motion with 6 degree of freedom is possible in IRF and Rankine panel method, heave, roll and
pitch motions are just allowed at these tests to forbid the effect of soft mooring. Fig. 6 shows the free decay result of 6500 TEU container ship. Three methods show almost identical decaying behavior.

![Fig. 6 Free Decay Test](image)

### Regular Wave

To compare the three approaches, the occurrence of parametric roll in different wave frequencies and amplitudes is tested. The ship is under excitation in head sea condition, and an impulsive disturbance is imposed in the transverse direction during regular motion. The time histories of nonlinear roll in different wave frequencies are shown in Fig. 7-(a) and the coefficients in GM variations are plotted in Fig. 7-(b).

All three methods show that parametric roll does not develop as the normalized wave frequency exceeds 2.55. The IRF method and Rankine panel method predict the occurrence of parametric roll at 2.55, though the GM-variation method does not. The GM-variation method predicts that parametric roll develops even in the small frequencies of 1.75 and 1.91. As the solution of GM variation diverges in these cases, the simulation is stopped when it shows too large motion.

The Rankine panel and IRF methods show similar frequency criteria for these test cases. It implies that they are in the similar degrees of accuracy. The GM-variation approach also predicts the occurrence at 2.23, which is very close to the twice that of roll natural frequency. However, it loses accuracy in smaller and higher frequencies. There could be several reasons: limitation of harmonic GM variation, coupling effect with other motions, and other limited assumption.

Although the Rankine panel and IRF approach have an adjusted damping coefficient, increasing ratio shows a little different behavior. At the frequency of 2.23 which is close to twice of natural frequency, the two methods show similar increasing ratio. However, the Rankine panel method shows slower increasing motion than that of the IRF approach at 2.55. Decaying behaviors at 1.91 and 2.72 are also somewhat different. These differences would imply the Rankine panel and the IRF methods have a little different radiation force components in this case. However, these are acceptable difference when their linear approaches of the radiation problem in highly-nonlinear roll motion are considered.

![Fig. 7 Development of parametric roll at different frequencies](image)

As well as wave frequencies criteria, a certain amount of excitation is required for the development of parametric roll. Fig. 8-(a) shows time histories of roll motion for different wave amplitudes. Roll develops at the wave amplitudes of 0.01 and 0.015, which are normalized to ship length. All the three approaches show same results of parametric roll occurrence. These test conditions are plotted in stability diagram in Fig. 8-(b). Though the conditions of 0.0025 and 0.005 locate near unstable boundary, roll is not increased by damping effect.
Bichrometic Wave

Although the present nonlinear methods do not consider a complete set of the second- or higher-order components, the nonlinear solutions include a part of the second- and higher-order effects. There are two distinct effects in the second-order component: sum-frequency and difference-frequency effects. There is a possibility that the sum- and/or difference-frequency excitations can trigger the unstable roll behavior.

Fig. 9 shows the linear and nonlinear roll motions in a single component and two-component waves. When the ship is under the single wave excitation at the frequency of 2.88 and 4.13, both the linear and nonlinear roll motions are very small and regular without any significant development of large roll motion. However, when the two-component waves are considered at the same time, i.e. in the case of bichromatic waves, the linear and nonlinear roll motions are significantly different. In particular, the nonlinear solution shows very large roll motion which must be generated by the high-order nonlinear effect.

It should be mentioned that both the single-wave cases have the wave amplitude of 2% of the ship length, while each component of the bichromatic waves has the wave amplitude of 1% of the ship length. This means that the bichromatic wave cannot exceed $A/L=0.02$, and mostly wave elevation peaks are less than $A/L=0.02$ since the two components have different frequencies. Therefore, Fig. 9 implies much stronger nonlinear effect on roll motion in weaker wave excitation.

![Fig. 8 Development of parametric roll at different wave amplitude: $F_n=0.049$, $\beta=180^\circ$, $\omega_r=(L/g)^{1/2}=2.23$.](image)

Fig. 8 Development of parametric roll at different wave amplitude: $F_n=0.049$, $\beta=180^\circ$, $\omega_r=(L/g)^{1/2}=2.23$.

![Fig. 9 Linear and nonlinear motions in the single and bichromatic waves: $F_n=0.049$, $\beta=120^\circ$, $\omega_r=(L/g)^{1/2}=2.88$ and $\omega_r=(L/g)^{1/2}=4.13$.](image)

![Fig. 10 Nonlinear roll time signal at various wave amplitudes](image)
research will be focused to the occurrence for various hull model based on this stability analysis.

Fig. 10 Nonlinear restoring (top) and Froude-Krylov (bottom) moments in bichromatic waves: \( F_n=0.049 \), \( \beta=120^\circ \), \( \omega_r=(L/g)^{1/2}=2.88 \) and \( \omega_e=(L/g)^{1/2}=4.13 \), \( A/L=0.01 \) for each component.

All of these appear in Fig. 11-(a), except for the component of 8.26 which is in the out of frequency range. This result may be somewhat surprising, since the second-order component is generally much less than the linear components. However, in this case, the large second-order component must be due to the large roll motion at frequency 1.25, which is near the roll natural frequency. This explains that the difference-frequency effect triggers the unstable roll motion.

It is also interesting that there is a non-ignorable component at frequency 1.63, and this frequency is identical to the difference frequencies of 5.76 and 4.13. This means that the third-order component, particularly the difference-frequency effect, may provide some contribution to nonlinear restoring moment.

The FK moment in Fig. 11-(b) doesn't show a significant contribution by difference frequency, and the linear components are primary. Therefore, in this case, it can be concluded that most nonlinear effects are from the restoring moments and their interaction with nonlinear roll motion. As a result, the nonlinear roll signal has a dominant component at the difference frequency, as shown in Fig. 11-(c).

According to the present numerical test, the difference-frequency-induced unstable roll motion is dependent on wave condition. Although the difference frequency of two wave components is near the roll resonance frequency, triggering the occurrence of parametric roll seems to require a certain magnitude of difference-frequency excitation, similarly to the single wave cases shown in Fig. 8. For example, for this containership, the occurrence of difference-frequency-induced parametric roll is hardly found in head seas. Therefore, a small magnitude of the excitation cannot generate the development of parametric roll. However, the homogeneous quasi-periodic Mathieu equation implied unbounded solution is possible even in head or following seas similar to oblique condition in Figs. 9 and 12. Future
Linear Stability diagram quasi-periodic Mathieu equation: \( \varepsilon=0.100, \mu=1.670 \), case in Fig.12. stable zone in shaded area, unstable zone in bright area. The rectangular mark corresponds to the bichromatic-wave case in Fig.12.

Fig. 14 Fourier components of roll motions in bichromatic waves: \( F_n=0.049, \beta=120^\circ \).

Polar Diagram

Numerical simulation can predict nonlinear ship motion in irregular waves. Then it gives statistical values available in actual ship sailing. Fig. 15 shows a polar diagram which contains predicted peak value of roll motion for 30 minutes. ISSC spectrum is used and it divided by 100 components of wave frequency. Radial direction means sea state from 1 to 8, and angular direction means heading angle. This diagram shows that excessive roll motion over 20 degrees is predicted around head and following waves in sea state 8. However, peak value decreases in oblique sea around 90 or 270 degrees. This kind of diagram is useful for ship operation.

EXPERIMENTAL VALIDATION

A set of model tests for the cruise ship has been carried out at the ocean basin in University of Ulsan. Table 2 shows the main dimensions of the parent ship. An experimental model of 1/100 scale, shown in Fig. 16, has been towed in the presence of regular and irregular waves, and their motion responses have been measured. This experiment has been focused on global motion, not specifically on parametric rolling. However, a series of test have been carried out to observe the occurrence of parametric roll in regular and bichromatic waves. The details of the experimental setup and results can be found in the report of Kim and Kim (2010), and this paper includes only the results of the parametric roll.

Fig. 16 Experimental model of cruise ship in towing condition.
Fig. 17 shows the comparison of motion RAOs between the present experiment and the computation based on the Rankine panel method. In general, the agreements of the heave and pitch motion RAOs are very fair, as shown in Fig. 17-(a). The roll motion RAOs do not have the same degree of agreement as the heave and pitch RAOs, but the overall trend is acceptable. Typical roll motion has narrow-banded RAOs, so that a small difference of physical parameters involved in roll motion can affect the location of motion peak.

![Graph showing comparison of motion RAOs between experiment and computation.](image)

(a) Heave and pitch: $F_n=0.127$, $\beta=180^\circ$

(b) Roll: $F_n=0.106$, $\beta=150^\circ$

**Table 2 Principal dimensions of the cruise ship.**

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LBP (m)</td>
<td>242</td>
</tr>
<tr>
<td>Beam (m)</td>
<td>36</td>
</tr>
<tr>
<td>Draft (m)</td>
<td>8.39</td>
</tr>
<tr>
<td>$\bar{G}M$ (m)</td>
<td>2.34</td>
</tr>
<tr>
<td>Displacement ($m^3$)</td>
<td>49,756</td>
</tr>
<tr>
<td>Natural frequency ($\omega_n=(L/g)^{1/2}$)</td>
<td>1.49</td>
</tr>
</tbody>
</table>

Fig. 18 Parametric roll observed in experiment: $F_n=0.053$, $\beta=180^\circ$, $\omega_e=(L/g)^{1/2}=3.10$.

Fig. 18 shows some snapshots of the experimental model with roll motion. Experiment was carried out at the normalized test frequency of 3.10, which is twice that of roll natural frequency. In the experiment, the amplitude of roll motion became very large when parametric rolls occurred. So, unfortunately, the time-histories of roll motion have not been measured due to the possibility of damage in motion sensors.
Table 3 summarizes the test conditions to observe the occurrence of parametric roll. In fact, more conditions have been tested; however all the cases were not successfully carried out due to the experimental errors or the shortage of measuring time. Also, the occurrence of parametric roll was not observed in all the conditions for the wave amplitudes less than $A/L=0.012$. Therefore, the conditions in Table 3 are the cases which are physically meaningful. In the case of single waves, the strong possibility of occurrence is clear at the considered frequencies. The two bichromatic cases in Table 3 are only the cases that experiment was successfully carried out. In other bichromatic wave cases (not shown in Table 3), the generated waves were not in an acceptable range of error.

Table 3 Test conditions for parametric roll test.

<table>
<thead>
<tr>
<th>Heading (deg)</th>
<th>Froude Number</th>
<th>Encounter Frequency $(\omega_e=(L/g)^{1/2}$)</th>
<th>Expected wave Amplitude $(A/L)$</th>
<th>Occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>0.053</td>
<td>3.075</td>
<td>0.021</td>
<td>O</td>
</tr>
<tr>
<td>120</td>
<td>0.053</td>
<td>3.075</td>
<td>0.029</td>
<td>X</td>
</tr>
<tr>
<td>150</td>
<td>0.053</td>
<td>3.075</td>
<td>0.021</td>
<td>X</td>
</tr>
<tr>
<td>150</td>
<td>0.053</td>
<td>3.075</td>
<td>0.029</td>
<td>O</td>
</tr>
<tr>
<td>180</td>
<td>0.053</td>
<td>3.103</td>
<td>0.021</td>
<td>X</td>
</tr>
<tr>
<td>180</td>
<td>0.053</td>
<td>3.103</td>
<td>0.029</td>
<td>O</td>
</tr>
<tr>
<td>180</td>
<td>0.106</td>
<td>3.074</td>
<td>0.021</td>
<td>O</td>
</tr>
<tr>
<td>180</td>
<td>0.106</td>
<td>3.074</td>
<td>0.029</td>
<td>O</td>
</tr>
<tr>
<td>120</td>
<td>0.053</td>
<td>2.24, 3.77</td>
<td>0.012, 0.012</td>
<td>O</td>
</tr>
<tr>
<td>120</td>
<td>0.053</td>
<td>2.24, 3.77</td>
<td>0.021, 0.021</td>
<td>O</td>
</tr>
</tbody>
</table>

Fig. 19 shows the numerical simulation or roll motion when the two waves of amplitude $A/L=0.012$ are considered. As the numerical result shows, the roll motion in the bichromatic waves is much larger than the linear solution. Therefore, the occurrence of the difference-frequency-induced parametric roll is obvious.

![Fig. 20 Experiment of difference-frequency-induced parametric roll for cruise ship: $Fn=0.053$, $\beta=120^\circ$, $\omega_e=(L/g)^{1/2}=2.24$ and $\omega_e=(L/g)^{1/2}=3.77$, $A/L=0.012$ each component.](image)

Fig. 20 Experiment of difference-frequency-induced parametric roll for cruise ship: $Fn=0.053$, $\beta=120^\circ$, $\omega_e=(L/g)^{1/2}=2.24$ and $\omega_e=(L/g)^{1/2}=3.77$, $A/L=0.012$ each component.

Fig. 20 shows the measured wave elevation, the Fourier component of measured waves, and the snapshots of ship rolling. Although the same amplitude of the two components was expected, it did not occur in the actual experiment. However, the occurrence of parametric rolling in the
bichromatic wave is obvious. Again, the experiments in other conditions were carried out, but measurement was not successful mostly due to the poor generation of bichromatic wave. Therefore, more systematic and thorough experimental study is needed for further observation on the difference-frequency-induced parametric roll.

CONCLUSION

The parametric roll motions of a large containership is studied by the GM-variation approach, the IRF approach and Rankine panel method. Experimental validation is carried out for a cruise ship. From this study, the following conclusions are suggested:

- The occurrence of parametric roll in regular wave is similarly predicted by the IRF and Rankine panel methods.
- The development of parametric roll depends on the wave condition, and a certain amount of excitation force or moment is need for the generation of parametric roll.
- From the theoretical approach by using the quasi-periodic Mathieu equation, it is shown that the second-order difference-frequency effect can be another source of parametric roll. This possibility has been proved by numerical simulation and also experimental observation.
- According to the numerical simulation based on the weakly nonlinear approach, the nonlinear restoring moment plays the key role in the generation of the difference-frequency-induced parametric roll.
- To observe the effects of the difference-frequency-induced parametric roll, more systematic experimental study should be carried out. Furthermore, the effects on the ship motion in irregular ocean waves should be carefully observed.

REFERENCES


